

GREGG M. SINCLAIR LIBRARY
UNIVERSITY OF HAWAII

8-7-63

NUCLEAR INSTRUMENTS & METHODS

A JOURNAL ON ACCELERATORS, INSTRUMENTATION
AND TECHNIQUES IN NUCLEAR PHYSICS

EDITOR: K. SIEGBAHN - UPPSALA

CONTENTS

PREFACE	245
CONTENTS OF THE FUSION ISSUE	246
1. R. LÜST, Notes on some theoretical investigations of the physics of high temperature plasmas	247
2. C. F. WANDEL, T. HESSELBERG JENSEN and O. KOFOED-HANSEN, A compilation of some rates and cross sections of interest in controlled thermonuclear research	249
3. R. LATHAM and J. A. NATION, Report on linear pinch devices	261
4. R. J. BICKERTON, Brief review of the toroidal stabilised pinch	273
5. L. BIERMANN, Remarks on the stellarator-scheme	279
6a. M. BINEAU, T. CONSOLI, P. HUBERT, F. PRÉVOT, P. RICA TEAU et A. SAMAIN, Les machines à miroirs magnétiques. Résumé de l'état de la question et esquisse d'un programme de recherches	282
6b. M. BINEAU, T. CONSOLI, CH. MAISONNIER, F. PRÉVOT et P. RICA TEAU, Les machines à miroirs magnétiques	290
7. H. L. JORDAN, H. KEVER and K. SCHINDLER, Notes on fast magnetic compression of plasmas	322

(continued on p. 3 of cover)



NUCLEAR INSTRUMENTS AND METHODS

A journal on accelerators, instrumentation and techniques in nuclear physics

EDITOR: K. SIEGBAHN, UPPSALA

EDITORIAL BOARD:

J. B. ADAMS (CERN)	S. KIKUCHI (Tokyo)
F. A. ALSINA (Buenos Aires)	J. KOCH (Copenhagen)
E. AMALDI (Rome)	M. S. KOZODAEV (Moscow)
H. L. ANDERSON (Chicago)	S. LIVINGSTON (Cambridge, Mass.)
K. T. BAINBRIDGE (Cambridge, Mass.)	E. M. MCMILLAN (Berkeley)
C. J. BAKKER (CERN)	M. G. MEŠČERJAKOV (Moscow)
P. R. BELL (Oak Ridge)	P. B. MOON (Birmingham)
R. E. BELL (Montreal)	M. L. OLIPHANT (Canberra)
A. BERTHELOT (Saclay)	W. PAUL (Bonn)
H. BRINKMAN (Groningen)	A. S. RAO (Bombay)
E. H. COOKE-YARBOROUGH (Harwell)	G. SALVINI (Rome)
S. DE BENEDETTI (Pittsburgh)	P. SAVIĆ (Belgrade)
M. DE HEMPTINNE (Louvain)	P. SCHERRER (Zürich)
A. DE-SHALIT (Rehovoth)	CH. SCHMELZER (CERN)
M. DEUTSCH (Cambridge, Mass.)	H. W. B. SKINNER (Liverpool)
S. DU TOIT (Pretoria)	R. TANGEN (Oslo)
B. S. DŽELEPOV (Leningrad)	D. TAYLOR (Northampton)
W. GENTNER (CERN)	V. I. VEKSLER (Moscow)
P. C. GUGELOT (Amsterdam)	O. WERNHOLM (Stockholm)
H. HALBAN (Paris)	G. VON DARDEL (CERN)
O. HAXEL (Heidelberg)	R. R. WILSON (Cornell)
P. HUBER (Basel)	S. D. WINTER (Saclay)
D. J. HUGHES (Brookhaven)	

INSTRUCTIONS FOR CONTRIBUTORS

Contributions may be sent to the Editor, Professor K. Siegbahn, Institute of Physics, University of Uppsala, Sweden, either directly or through the intermediation of one of the members of the Editorial Board. Contributions may be written in English, French or German but should always, except for very short papers, be supplied with a brief abstract in English. Figures should be in inked original drawings or glossy prints and they should be sufficiently large. Contributors should make sure that important details in drawings and photographs will show up clearly in the diminished copy used by the publisher.

Books for review should be sent to Professor Siegbahn.

Objective and descriptive material on endeavours and product developments from the industry for the section "Spotlight on Industry" should be sent directly to the publishers, North-Holland Publishing Company, P.O. Box 103, Amsterdam.

Nuclear Instruments and Methods is published monthly. The subscription price of a volume of 360 pages is \$ 17.00 (130 s), postage free.

Subscriptions should be sent to the publishers, North-Holland Publishing Company, P.O. Box 103, Amsterdam or to any subscription-agent.

No part of this issue may be reproduced in any form, by print, photoprint, microfilm or any other means without written permission from the publisher. Reprints, photoprints or microfilms are obtainable at cost from the publisher.



PHILIPS

equipment for Nuclear Physics and Atomic Energy



Transistorized
Pocket Radiation Monitor
type PW 4014

- Civil defence instruments.
- Linear electron accelerators.
- Generators for very high d.c. voltages.
- Synchro-cyclotrons for various final energies.
- Cockcroft-Walton accelerators.
- Radiation measuring and detection equipment.
- Area monitoring instrumentation.
- Reactor control instrumentation.
- Geiger counter tubes.
- Radio-active isotopes.
- Air liquefiers.

Outstanding features such as:

- Lowest measuring range: 0-3 mr/h (Ra- γ)
- Provision for connecting separate G.M. Probes and earphone
- Low power consumption
- Printed wiring
- Tropic and waterproof construction

make this new all-transistorized pocket radiation monitor-incorporating the rugged Philips Geiger Müller counter tube type 18503 - the ideal instrument for measuring and locating radioactivity.

PHILIPS

Nuclear Equipment

For further information please apply to one of the following addresses:

N.V. PHILIPS' GLOEILAMPENFABRIEKEN, Scientific Equipment Department, Eindhoven-Holland
 PHILIPS ELECTRICAL LTD., Research and Control Instruments Division, Shaftesbury Avenue, London W.C. 2
 PHILIPS ELECTRONICS INC., Instruments Division, 750. South Fulton Avenue, Mount Vernon N.Y. U.S.A.



SCINTILLATION SPECTROMETRY



NON-OVERLOADING LINEAR PULSE AMPLIFIER NE 5202:

with 5 microsecond recovery for 100 times overload. Designed for scintillation spectrometry and pulse height analysis studies, this amplifier features high gain (50,000), high gain stability (better

than 0.25% per week), good linearity, low noise (less than 40 microvolts at input), excellent overload characteristics, and operation at high duty cycles. Rise time 0.2 microseconds and integral linearity 0.15%. Preamplifier NE 5202A also available. Based on the double delay line differentiated Fairstein system, it consists of three high gain feedback groups, a white cathode follower output.

Resolving time 1.2 microsecs; 3–140 volts operating range with energy zero drift less than 0.05 volts per day; 0 to 10 volt gate width with less than 0.01 volts per day drift; maximum counting rate, 2×10^6 c.p.s.

SINGLE CHANNEL PULSE HEIGHT SELECTOR NE 5102:

E.H.T. SUPPLY UNIT NE 5302:

E.H.T. SUPPLY UNIT NE 5303:

with minimum of noise and ripple; output stability 0.01%; low noise and ripple—less than 1 mV RMS; output voltage continuously variable from 500–1,800 V; output current 5mA.

As NE 5302, but with output voltage range from 1000–5000V.

containing white cathode follower preamplifiers with noise level of 25 microvolts; photo-multipliers of 1" or 2" can be accommodated; for general use or for C14 and Tritium investigations with special rotary light locking device.

SHIELDED SCINTILLATION HEAD UNITS NE 5501 & 5502:

Other Units available:

Precision Ratemeter NE 5401 with eight ranges and six time constants Scanning scintillation spectrometer NE 8601. Multi-channel scintillation spectrometer NE 8201 for industrial tracer investigations. Scintillometers for geophysical exploration.

C14 and H3 counting unit NE 8301. Moisture and density gauge NE 8401.

Exclusive concessionaire for C.D.C./Bendix 100-channel Transistorized Kicksorter in Great Britain.

Sole agent for Harshaw Sodium Iodide Crystals in the United Kingdom.

For full detail of the above and of our SCINTILLATION PHOSPHOR PRODUCTS (plastics, liquids, loaded liquids, crystals, gels, neutron detectors, and chemicals) write to

NUCLEAR ENTERPRISES (G.B.) LTD.
SIGHTHILL, EDINBURGH, 11, SCOTLAND — TEL.: CRA 5262

Associate Company: Nuclear Enterprises Ltd.
1750 Pembina Highway, Winnipeg, Canada

ndh 708

the transistorized TMC model sg-10a serviscaler

TMC designed the new "Serviscaler" specifically for college classroom training or laboratory use . . . moderately priced, dependable and compact—yet with performance specifications more than sufficient for training and monitoring purposes. The transistorized SG-10A costs only \$350 f. o. b. New Haven.

The transistors used throughout all scaling stages (the high voltage regulator is the only tube) increase operating life and over-all reliability and make the Serviscaler more rugged. It weighs only 12 lbs., measures $12\frac{3}{4}$ " x 9" x $5\frac{7}{8}$ " high so that it can be easily carried by anyone. Power consumption of 20 watts means that a number of units can be used in a single room without "power" problems.

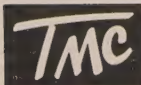
In actual operation the Serviscaler scales data on five decade glow tubes with a maximum capacity of 99,999 counts. And high voltage is continuously adjustable between 600 and 1600 volts.

... especially for training



operating specifications

resolving time: better than 200 microseconds
input sensitivity: adjustable internally 50
millivolts to 200 millivolts
maximum load current: 100 microamperes
high voltage stability: 1.0%
line regulation (105-125 v.): 0.1%
power: 115 v. 50-60 cycles (230 v. on order)
connectors: detector input, timer output,
remote control



TECHNICAL MEASUREMENT CORPORATION

441 WASHINGTON AVENUE, NORTH HAVEN, CONNECTICUT



Unseen danger made visible

Isotope labs can be fraught with danger, but not to those who regularly check the dosage they received with a Philips Pocket Dosimeter.

Sturdily constructed to resist rough handling without loss of accuracy, these dosimeters require no separate charging device. Little larger than a fountain-pen they contain:

- a sensitive quartz-fibre electrometer
- an optical system that permits readings to be taken simply and directly
- a mercury-drop charging device

HERMETICALLY SEALED
Measuring ranges



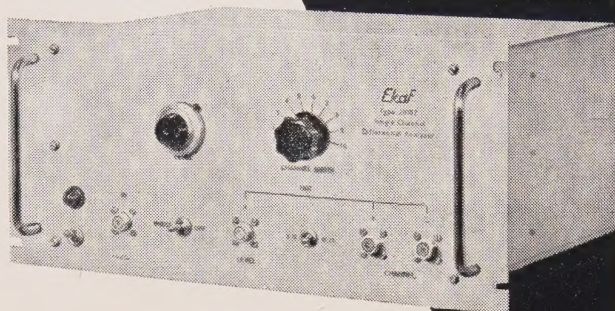
Q 4423 C Q 4424 C
250 mr 20 r



PHILIPS

pocket dosimeter

For further information please apply to:
N.V. PHILIPS' GLOEILAMPENFABRIEKEN,
Scientific Equipment Department, Eindhoven-Holland



The EKA Single Channel Discriminator, Model 20102, is based on the latest Fairstein data, but with increased channel width (adjustable 1-20 V) and equipped with improved, stabilised power supply.*

The short dead time and minimal drift guarantee practically perfect precision.

**) Rev. Sc.Instr. 27 (1956) 549*

Brief specifications:

Input Pulses:	10-110 V
Channel Width:	Positive
	1-20 V in steps of 1 V
	(or continuously as extra)
Dead Time:	< 2 μ sec
Output Pulses:	positive and negative,
	width 1.2 μ sec
	height 24 V
Rise time:	0.2 μ sec
Drift in Level Zero:	0.05 V/day
Drift in Channel Width:	10 mV/day.

This instrument is one from a wide range of EKA precision instruments for nuclear physical research and for every stage of medical-therapeutical and -diagnostical use of radioactive isotopes.

EKA

**ELEKTRONISCH-KERNFYSISCH
APPARATENFABRIEK N.V.**

Sales Office: 74, Laan Copes van Cattenburch,
The Hague, Holland.
Tel. 070-630977

INTERNATIONAL

INTERNATIONAL

advertise

YOUR NUCLEAR PRODUCTS IN

sell

YOUR NUCLEAR PRODUCTS

reap

YOUR NUCLEAR SALES
REWARDS

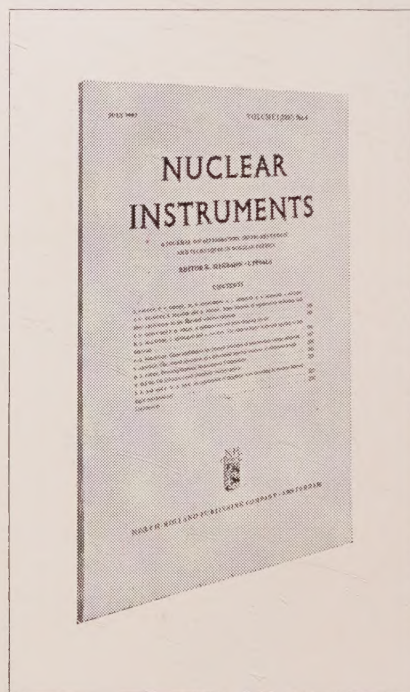
world wide

KEY NUCLEAR MANAGEMENT
NUCLEAR SCIENTISTS
NUCLEAR ENGINEERS
NUCLEAR PRODUCTION
PERSONNEL

SUBSCRIBE TO

you reach

THE WHOLE
NUCLEAR SALES WORLD IN



Entirely international in scope Nuclear Instruments and Methods is the one nuclear scientific journal that has an established subscription clientele throughout the whole world—where-ever nuclear men are at work—they want your nuclear products—they need your nuclear products—advertise in Nuclear Instruments and tell them WHAT—HOW—WHERE they can get your nuclear products

GO INTERNATIONAL WITH YOUR SALES PROGRAM TODAY WRITE

WILLIAM F. BAARS - ADVERTISING MANAGER NORTH-HOLLAND PUBLISHING COMPANY

P.O. BOX 103 - AMSTERDAM, HOLLAND

**FASTEST DELIVERY
ANYWHERE IN THE WORLD!**

**ONE Complete
OVERSEAS
SOURCE**

for America's Finest
**NUCLEONIC &
ELECTRONIC
Components
and Equipment**



Exclusive Export Sales Agents and Export Distributors

TERMINAL RADIO INTERNATIONAL LTD.

TRIL—the important export suppliers of Nucleonic and Electronic Components and Equipment.

TUBES:

- Photomultiplier
- Electrometer
- GM
- Diodes
- Fission
- Voltage Regulators
- Transistors

SCINTILLATION PHOSPHORS Including:

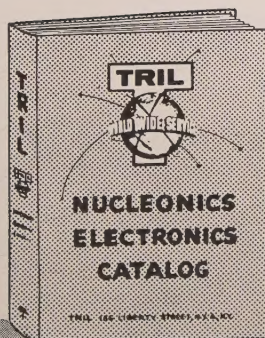
- Thallium Activated Sodium Iodide
- Thallium Activated Potassium Iodide
- Europium Activated Lithium Iodide
- Anthracene
- Thallium Activated Cesium Iodide
- Pure Sodium Iodide

REACTOR INSTRUMENTATION and RADIATION MONITORING:

- Area Monitors
- High Voltage Power Supplies
- Pulse Height Analyzers
- Air Particle Monitors
- Radiation Dose Control Equipment
- Linear Amplifiers
- Ion Chambers, Laboratory Monitors
- Radiation Detectors
- Preamplifiers
- Geiger and Scintillation Survey Meters
- Personnel and Area Dosimeters
- Scalers
- Neutron Detectors
- Pulse Generators

Electronic Analog Computing Equipment.

Your request for Price quotations on your requirements will receive our prompt attention.



NEW! BIG! TRIL CATALOG

with complete technical data **FREE** on request.

Write or cable for your copy now!

TRIL

135 Liberty Street, New York 6, N.Y., U.S.A.

Cable Address: TRILRUSH, NY, NY (USA)



**ELECTROMAGNETIC
ISOTOPE SEPARATORS AND APPLICATIONS
OF ELECTROMAGNETICALLY
ENRICHED ISOTOPES**

By J. KOCH (Editor),

R. H. V. M. DAWTON, M. L. SMITH and W. WALCHER

6 x 9" xii + 314 pp. 156 illus. 53 s.

"This is a very welcome book on a subject which had its beginnings 24 years back, which has grown to undreamt of proportions during and after the war, and of which this is the first comprehensive monograph. It is written by a team of experts with first-hand experience in electromagnetic separators... it tells in a masterly way the fascinating "success story" of a physical instrument which has grown into a giant". (D. GABOR in *British Journal of Applied Physics*)

Sole Agency for U.S.A.:

Interscience Publishers Inc. New York

**NORTH-HOLLAND PUBLISHING COMPANY
AMSTERDAM**

MODEL 412 MERCURY RELAY

PRECISION SLIDING PULSER

for Precise Calibration of Single & Multi-Channel
Pulse Height Analyzers

Determines Window Widths
Using Principle of
Constant Angular Velocity

AUTOMATICALLY
10 × more accurately
40 × faster than conventional
pulsers

Incorporates
Both Manual Control
and
Precision Motor Drive

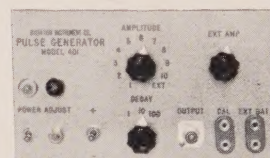
Simulates Pulses from
Scintillation,
Proportional or
Geiger Counters



MERCURY PULSER, Model 401

Featuring Versatility
and Precision

Specifically designed
for the
Nuclear Physics
Laboratory



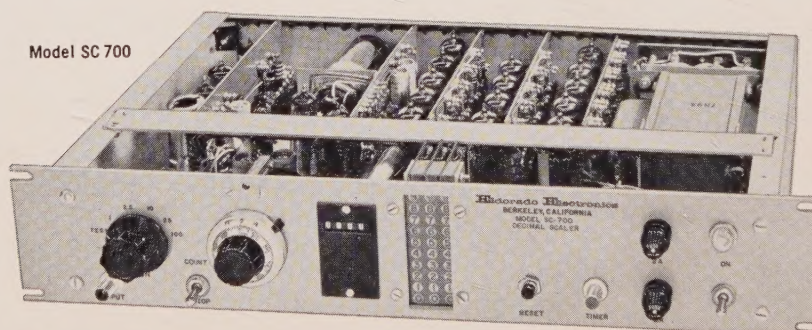
Fast rise and exponential decay simulates scintillation, proportional or geiger counter pulses for test and calibration of amplifiers, pulse height analyzers, scalars, etc.

MODEL 412 . . . \$450.00 — MODEL 401 . . . \$125.00

RADIATION INSTRUMENT CO.

P. O. BOX 733, SILVER SPRING, MARYLAND, U.S.A.

Model SC 700



**New Eldorado SC 700 Decimal Scaler with
better than 1 microsecond resolution**

Designed for general radiation counting, Eldorado's compact new SC 700 Decimal Scaler provides high performance at a budget price, \$595.

FEATURES

- Small Size — 3 1/2" panel height, weighs only 16 pounds.
- 25 Millivolt Sensitivity
- Seven Decades of Count Storage
- Calibrated Input Amplifier

- Precision Integral Discriminator
- Building Block Construction
- Built-in Test Circuit
- Provision for External Timer

For complete
technical information,
please address Dept. _____

**Eldorado
Electronics**

2821 TENTH ST. • BERKELEY 10, CALIF.
Phone THornwall 1-4613

Société d'Applications Industrielles de la Physique

S. A. I. P.

38, rue Gabriel Crié - MALAKOFF (Seine)

NUCLEAR PHYSICS

PROPORTIONAL COUNTER CPX

- Continuous purification on heated Calcium
- Safety- and isolating rings made of quartz
- Beryllium counting window
- Resolution: 15 to 25% at 6 KV
- With internal source: detection starts from 0.1 KV

OTHER PRODUCTS:

Single-channel kicksorters

Scalers

Scintillation spectrometers

50-Channel Kicksorter "SA 50"

Circulation counters

Pressurised ionisation chambers

Linear amplifiers

Vibrating condensator type D.C. amplifiers

Automatic sampling dispensers.



AGENTS:

BELGIUM:

DRUA, 205 Avenue Van Volxem, BRUSSELS

DENMARK:

LANGENBAEK & SON, Trondhjems gate 9, COPENHAGEN

NETHERLANDS:

ENRAF-EVERSHED, Van Leeuwenhoeksingel 69, DELFT

ITALY:

AMPERE, Via Felice Casati 8, MILAN

FINLAND:

ISOTEKNO, 13 A Mikontaku, HELSINKI

JAPAN:

RIKEI TRADING CO, Kosato Building, 12-4 Chome Shiba
Tamura-Cho Minato-Ku, TOKYO

SWITZERLAND:

A. MAGAT, 28, rue de Vermont, GENEVA

U.S.A.:

LEVIANT, 239 Broadway, N.Y. 7., NEW YORK

The place of the Particle Accelerator in Basic Research...

"It has long been my ambition to have available . . . a copious supply of atoms and electrons . . . transcending in energy the alpha and beta particles from radioactive substances."

Ernest Rutherford — 1927

High-energy Electrons in Solid-state Physics—I

Studies of radiation effects are vital to physicists in developing the theory of the solid state. For example: high-energy particles have been used extensively to produce defects in crystalline materials. The resulting vacancy-interstitial pair alters the mobility of the charge carriers, and therefore the electrical characteristics of the material. In conductors, this will result in decreased conductivity. In semi-conductors, the defects, which are primarily of the Frenkel type, will act as carrier traps and decrease the carrier lifetime. A knowledge of these effects which can be easily measured has been invaluable in extending the theory of crystal structure.



Lattice Bonds

Theoretical studies in ionic and co-valent crystals have shown that the lattice binding energy is about 25 ev. The energy of an electron which can cause a single displacement is therefore of the order of 0.4 Mev. This figure is confirmed by a number of experiments with electrons from Van de Graaff accelerators. Reports of these experiments have come from Purdue University using the University of Notre Dame accelerator¹, North American Aviation, Inc.², Bell Telephone Laboratories³, and others. All confirm the theory within the experimental errors. More refined measure-

ments on such aspects as crystal orientation by Bell Laboratories³, and minority carrier lifetime, by R.C.A. Laboratories⁴, are under way and will serve to extend even further our understanding of the crystal structure.

Type of Particle

Many different particles can be used for radiation studies. Their effects differ mainly in the momentum transferred to the lattice atom. Heavy particles transfer a large amount of energy. The ejected atom may have enough kinetic energy to cause subsequent displacements and produce a thermal spike in which a large number of atoms are involved. This type of damage is hard to analyze because of its complexity.

Light particles, such as electrons, can be accelerated to energies where only a single displacement will occur. It is possible to measure the binding energy of the atom in the lattice by suitable measurements following this type of bombardment.

The source of these particles can be an accelerator or a nuclear pile. The need for precise control of certain variables and the necessity for knowing the particle energy makes the accelerator a superior source. The Van de Graaff has been used extensively in these studies because its precision and energy stability best meet the requirements.

Applications

Studies of radiation effects can be applied to current problems. Nuclear pile designers must know how much radiation a material can withstand before failure, and how well important instruments will stand up under particle bombardment. In particular, sensitive electronic systems must operate safely in high-intensity radiation fields. This is particularly true for nuclear-powered aircraft and guided missiles. Much work is now going on in testing components in radiation fields provided by particle accelerators.

There have been reports of high-frequency diodes being produced by irradiating semiconductor materials to reduce the carrier lifetimes. While these have not been produced commercially, it appears that this is a simple method of producing traps in semi-conductors where they are needed.

Accelerator Versatility

The studies described above are being carried out principally with the Van de Graaff®. Capable of accelerating charged particles such as electrons and protons, this versatile source of synthetic radiation can also produce monoenergetic neutrons and high-intensity x-ray fields. The primary particle energy is continuously variable over wide ranges, and can be accurately measured for threshold studies.

¹ E. E. Klontz, K. Lark-Horowitz, Phys. Rev. 86, 643, 1952
² D. T. Eggen, M. J. Laubenstein, Phys. Rev. 91, 238, 1953

³ W. L. Brown, Bull. APS, 2, 156, 1957
⁴ P. Rappaport, Phys. Rev. 94, 1409, 1954

HIGH VOLTAGE ENGINEERING CORPORATION



BURLINGTON

MASSACHUSETTS

PREFACE

In early Spring 1958 CERN, the European Organization for Nuclear Research, initiated the forming of a study group on fusion problems. Leading representatives from the European Laboratories working in fusion research, were invited to be members of this study group. It was planned that the main objective of this group would be to discuss the fusion research work in Europe and to consider various ways of facilitating it. One of the first undertakings of the group was to summarize the results and problems—both purely scientific and technological ones—so far as they were known at that time (December 1958) in the field of controlled fusion, in a series of internal reports. As we feel that this series is well worth knowing outside the members of the group, we are pleased to be able to publish it now in its complete form. Since the authors were fully free as to the size and extent of their reports, the articles have a tendency to be somewhat heterogeneous. It is our hope, however, that this issue of *Nuclear Instruments and Methods* will be of value as a source of reference both for those already working with fusion and for those who are considering to enter this highly interesting field of research.

We wish to express our deep gratitude to the authors of the various reports for their full co-operation in making this publication possible. Finally, special thanks are due to the Chairman of the group, Mr. J. B. Adams of CERN, for his excellent leading of the sessions and for initiating the writing of the reports.

THE EDITORS

CONTENTS OF THE FUSION ISSUE

1. R. LÜST, Notes on some theoretical investigations of the physics of high temperature plasmas	247
2. C. F. WANDEL, T. HESSELBERG JENSEN and O. KOFOED-HANSEN, A compilation of some rates and cross sections of interest in controlled thermonuclear research	249
3. R. LATHAM and J. A. NATION, Report on linear pinch devices	261
4. R. J. BICKERTON, Brief review of the toroidal stabilised pinch . . .	273
5. L. BIERMANN, Remarks on the stellarator-scheme	279
6a. M. BINEAU, T. CONSOLI, P. HUBERT, F. PRÉVOT, P. RICATEAU et A. SAMAIN Les machines à miroirs magnétiques. Résumé de l'état de la question et esquisse d'un programme de recherches	282
6b. M. BINEAU, T. CONSOLI, CH. MAISONNIER, F. PRÉVOT et P. RICATEAU, Les machines à miroirs magnétiques	290
7. H. L. JORDAN, H. KEVER and K. SCHINDLER, Notes on fast magnetic compression of plasmas	322
8. C. M. BRAAMS, W. J. SCHRADER and C. J. TERLOUW, Confinement of plasma by radio-frequency electro-magnetic fields	327
9. J. G. LINHART and A. SCHOCH, Relativistic electron beam devices for fusion	332
10. A. FOLKIERSKI, Report on high-speed photography	346
11. H. WULFF, Plasma diagnostics by spectroscopical means	352
12. L. H. TH. RIETJENS and C. M. BRAAMS, Detection of heavy particles in thermonuclear reaction experiments	363
13. G. L. MUNDAY, Ultra high vacuum technology	367
14. T. CONSOLI, P. HUBERT, R. LE QUINIO et D. VÉRON, Les sources d'ions et de plasma	376
15. D. TH. J. TER HORST, Some remarks on energy storage	382
16. SETH BERGLUND, High magnetic fields.	386

1. NOTES ON SOME THEORETICAL INVESTIGATIONS OF THE PHYSICS OF HIGH TEMPERATURE PLASMAS

R. LÜST

Max-Planck-Institut für Physik und Astrophysik, München

A brief survey is given of those 1958 Geneva papers devoted to the theoretical aspects of high temperature plasma physics.

In the following short survey shall be given the theoretical papers and reports which have been presented at the Geneva Conference. There were a few papers discussing the fundamental equations of plasma physics (1). A large number of papers had the stability problem (2) as the main subject, while some other papers were dealing with equilibrium configurations (3). The motion of single particles in magnetic fields (4) is discussed in a number of further papers. Different kinds of heating mechanisms (5) was another subject, as was the problem of magnetohydrodynamic shock waves (6). Finally, theoretical papers dealing with the state of ionization and the different kinds of energy losses (7) were presented†.

(1) The theoretical structure of the plasma equations was considered by Rosenbluth (P/349) and a new method was developed for the solution of the linearized transport equations in the absence of collisions. The same subject but using different methods is treated in a Russian paper by Sagdeyev *et al.* (P/2214). In both papers also stability problems are discussed, especially for the case that the pressure is anisotropic.

(2) The general aspects of the stability as well as the stability for special equilibrium configurations are treated in a number of papers. The Princeton group makes extensive use of the energy principle which was found by that group and in Göttingen (see P/1056) and deduce general criteria for stability directly from the Boltzmann equation neglecting collisions of the particles (P/365). The influence of collisions is briefly discussed. This approach

yields a generalization of the normal energy principle for investigating the stability of hydromagnetic systems to situations where the effect of heat flow along the magnetic lines is not negligible††. In two other papers by the same group, the stability of some equilibrium configurations are treated which are of interest in connection with the Stellarator (P/364; P/1875). Special attention is given to the so-called interchange and the kink instabilities. Rosenbluth reports further investigations of the stability of the pinch effect (P/374). The same problem is the subject of some other papers from the U.S.A. (P/354; P/1861). Suydam derives a very interesting necessary condition on the shear of the magnetic lines of force for the stability of the linear pinch using the energy principle (P/354). In Harwell, Taylor continued with investigations of the stability of the pinch effect in two ways (P/33). First, using as a model a simple wire inside of a conducting cylinder, plasma dynamics is completely neglected and only the forces exerted by the electromagnetic fields are considered. In the second approach, the equations of hydromagnetics are used and the differential equations for the perturbations are solved. The diffusion of the magnetic fields due to the finite value of the electric conductivity is also treated. The problem of turbulence in connection with the stability for a Zeta-type machine has been studied by some numerical

† The numbers given below refer to the numbers of the Geneva papers.

†† It could be shown, that a configuration, which is stable according to the hydromagnetic equations, is also stable by taking into account the anisotropy of the disturbed pressure, the heat flow and the collisions.

calculations by Thompson *et al.* (P/2). In a paper by Braginsky and Shafranov the stabilisation of a pinch by weak and strong magnetic fields is discussed (P/2500). The influence of volume currents on the stability has been investigated in Göttingen (P/1056). Finally, a paper by the New York group should be mentioned in which several general stability criteria are given (P/376).

(3) The problem of hydromagnetic equilibrium for torus-like configurations is investigated with the aid of a variation principle by Kruskal and Kulsrud (P/1876). This was also a special point of investigation of the Göttingen group (P/1056). Here different methods were used. Toroidal equilibrium configurations with and without longitudinal surface currents and with longitudinal volume currents have been studied. In the paper mentioned above by Braginsky and Shafranov the equilibrium of a torus is also discussed (P/2500). Grad and Rubin deal with hydromagnetic equilibria in connection with force-free fields (P/386). The confinement of particles by cusped geometries has been discussed at New York (P/1538) and in Russia (P/2212). Also the possibility of using the radiation pressure to contain particles has been investigated in Russia (P/2500).

(4) The motion and in particular the confinement of individual charged particles in suitable electromagnetic fields has been investigated in Göttingen (P/1056). Also the constancy of the magnetic moment of a charged particle has been studied there. This question has furthermore been the subject of a paper by Garren *et al.* (P/383). These authors calculated orbits in a mirror-machine and solved the Fokker-Planck equation numerically.

(5) The heating of a plasma by Joule heat is discussed in a paper by the Princeton group (P/363), in the papers mentioned above by Braginsky and Shafranov (P/2500), by Thompson *et al.* (P/2), and by the group in Göttingen (P/1056). The mechanism of magnetic pumping

(collisional heating or gyro-relaxation, transit-time heating, acoustic heating and ion cyclotron resonance heating) is investigated in Princeton (P/357). The gyro-relaxation has also been studied in Göttingen (P/1056) and furthermore a heating mechanism by high frequency radiation (*ibid*). Several Russian papers are dealing with high frequency and plasma oscillations and the absorption of high frequency electromagnetic waves in a high temperature plasma (P/3215; P/2300). Here, especially the cyclotron and Čerenkov absorption is investigated.

(6) In another group of papers, the problem of hydromagnetic shock waves in absence of collisions is treated. Here, the work by Allen (P/1), Grad (P/374), Akhiedzev (C/2509) *et al.*, and by Göttingen group (P/1056) should be mentioned. (The problem is not yet completely solved, but the investigations show that waves with finite amplitude can exist in a plasma, which have a wave-length in the order of the geometric mean of the gyration radii of the ions and electrons.)

(7) The dependence on the physical conditions of the state of ionization of added heavier elements has been studied in Göttingen (P/1056). Also the radiative energy losses by impurities of heavy elements and the energy losses by other mechanism are investigated. The energy losses in a pinch are discussed by Braginsky and Shafranov (P/2500). The synchrotron radiation from a magnetized plasma is treated in a Russian (P/2213) and a Japanese (P/1330) paper and turns out to be larger than expected. Finally, a theoretical paper by Dreicer should be mentioned dealing with run-away electrons by solving the time-dependent Fokker-Planck equation (P/2292).

Many of the Russian papers given at the Geneva Conference are also published now in a 4 volume work "The Physics of Plasmas" edited by the U.S.S.R. Academy of Sciences. This work contains several very interesting papers concerning basic questions of plasma physics.

2. A COMPILATION OF SOME RATES AND CROSS SECTIONS
OF INTEREST IN CONTROLLED THERMONUCLEAR RESEARCH

C. F. WANDEL, T. HESSELBERG JENSEN and O. KOFOED-HANSEN

Research Establishment Risø, Danish Atomic Energy Commission

A compilation has been made of cross sections and corresponding time rates, of interest for calculations in the controlled thermonuclear field. The compilation is limited to phenomena occurring in fully ionized plasmas. It includes

the relevant cross sections for nuclear reactions, Coulomb collisions and bremsstrahlung radiation. Characteristic time rates for the above phenomena are given assuming Maxwellian velocity distributions for the particles involved.

1. Introduction

The following compilation is intended as a help to people working in the field of controlled thermonuclear research, in making preliminary calculations on the feasibility of a proposed theoretical or experimental scheme.

Because of the short time for preparation it does not claim any degree of completeness. The material included deals only with the phenomena relevant for computations on a fully ionized plasma. Accordingly, all the important phenomena occurring in a partly ionized plasma are not dealt with in this publication.

We have thus included only such processes as nuclear reactions, Coulomb collisions and emission of bremsstrahlung, while processes involving excitation, ionization and charge exchange among electrons, ions, atoms and molecules are excluded.

The applicability of the material is thus obviously restricted to the region of extremely high temperatures.

In each section we have tried to give a brief introduction including the relevant theoretical formulae for the phenomena under consideration, followed by some experimental data where possible and a few references mainly to standard review literature where further references can be found.

The compilation contains formulae and data on

- 1. Nuclear Cross Sections
- 2. Coulomb Cross Sections and
- 3. Bremsstrahlung Cross Sections.

In all cases we have also given the reaction rates corresponding to the cross sections assuming Maxwellian distributions for the reacting particles.

2. Nuclear Cross Sections

Only such processes have been considered which are thought to be of interest for controlled fusion, they are given below in table 1 together with their corresponding Q -values.

TABLE 1
The reactions of interest and their Q -values

	Reactions	$Q^\dagger$ (MeV)
1	$D + D \rightarrow T + P$	4.04
2	$D + D \rightarrow He^3 + N$	3.27
3	$T + D \rightarrow He^4 + N$	17.58
4	$He^3 + D \rightarrow He^4 + P$	18.34
5	$T + T \rightarrow He^4 + 2 N$	11.32
6	$T + He^3 \rightarrow He^4 + D$	14.31
7	$T + He^3 \rightarrow He^4 + P + N$	12.08

$\dagger$ The Q -values are taken from refs. 1) and 2).

2.1. CROSS SECTIONS

In the energy region below 100 keV the energy dependence of the nuclear reaction cross sections is dominated by the effect of the Coulomb repulsion. The cross section has then the form

1) F. Ajzenberg and T. Lauritsen, *Revs. Mod. Phys.* 27 (1955) 77.

2) D. M. van Patter and W. Whaling, *Revs. Mod. Phys.* 29 (1957) 757.

$$\sigma(E) = AE^{-1} \exp(-BE^{-\frac{1}{2}}) \quad (1)$$

where the theoretical value of B is given by

$$B = \sqrt{2\pi\alpha} (mc^2)^{\frac{1}{2}} Z_1 Z_2 \quad (2)$$

where

α = the fine structure constant ($\frac{1}{137}$)

$m = m_1 m_2 / (m_1 + m_2)$, the reduced mass of the two particles with masses m_1 and m_2 and with charges $Z_1 e$ and $Z_2 e$

$E = \frac{1}{2} mv^2$, energy available in the CM-system
 v = the relative velocity.

Above 100 keV, deviations from the form given by (1) can be expected if nuclear resonances are present in this region. Such deviations are for instance found in the case of the reactions $T(d,n)He^4$ and $He^3(d,p)He^4$ at energies above 100 keV and 400 keV respectively.

Below are given the experimental cross sections for the reactions in table 1. It should be noted that the cross sections are given as functions of the energies of the bombarding particles. The material is taken from ref. ³⁾.

2.2. MAXWELL AVERAGES

To determine the reaction rate in a plasma for particles of type i and j it is necessary to compute the average of the quantity $v\sigma(E)$ over the velocity distributions of the two kinds of particles. If one assumes the distributions to be Maxwellian with the same temperature T then the reaction rate is given by

$$R = \frac{1}{1 + \delta_{ij}} n_i n_j \langle v\sigma \rangle_T \quad (3)$$

where n_i and n_j are the number densities of the two kinds of particles i and j respectively. The Kronecker- δ takes into account the possibility that the reacting particles are of the same kind. It can easily be shown that the expression for the average in (3) is given by

$$\langle v\sigma \rangle_T = \frac{8\pi}{\sqrt{m}} \left(\frac{1}{2\pi kT} \right)^{\frac{3}{2}} \int_0^\infty \exp\left(-\frac{E}{kT}\right) E \sigma(E) dE \quad (4)$$

(see ref. ⁴⁾).

Numerical evaluation of these averages has been performed by us in the case of the first four reactions of table 1. The values given in

table 2 below are taken from ref. ⁴⁾, and are believed and hoped to be accurate to within a few per cent.

TABLE 2

The reaction rate parameters, $\langle v\sigma \rangle$ (cm³/sec), averaged over Maxwellian distributions with temperature T

kT (keV)	$\langle v\sigma \rangle \times 10^{17}$ (cm ³ /sec)			
	D(d,p)T	D(d,n)He ³	T(d,n)He ⁴	He ³ (d,p)He ⁴
5	0.00862	0.00898	1.30	0.000727
10	0.0575	0.0613	11.1	0.0223
25	0.351	0.400	55.8	0.776
50	0.981	1.17	85.4	5.68
100	2.26	2.76	84.5	17.85
150	3.42	4.20	70.4	24.6

The cross sections used in the computations are the ones reproduced below from ref. ³⁾. In the case of the $He^3(d,p)He^4$ reaction we have used the results of Freier and Holmgren, and Yarnell, Lovberg and Stratton around the resonance top. It is of minor importance that we have used the corrected results of Bame and Perry⁵⁾ for the higher energies in the case of the $T(d,n)He^4$ reaction.

Computations of the reaction rate parameters have been published in several places lately. Since many misprints appear in these publications a few remarks on each might be of value to the user of the data.

The first publication of reaction rates of which we are aware is the one by Post⁶⁾. A graphical presentation of the total D-D and of the T-D reaction rates are given. Complete agreement is found with the T-D reaction rates, but the total D-D reaction rate seems definitely to be too low by a factor of about 1.6, the figure can, however, not be read off with an accuracy better than a few per cent.

A reference where a great number of reaction rate parameters can be found is the article by

³⁾ N. Jarmie, J. D. Seagrave and H. V. Argo, Editors, AEC Publication LA-2014 (1956).

⁴⁾ T. Hesselberg Jensen, O. Kofoed-Hansen, A. H. Sillesen and C. F. Wandel, Danish AEC Publication, Risø Report No. 2 (1958).

⁵⁾ S. J. Bame and J. E. Perry, Jr., Phys. Rev. **107** (1957) 1616.

⁶⁾ R. F. Post, Revs. Mod. Phys. **28** (1956) 338.

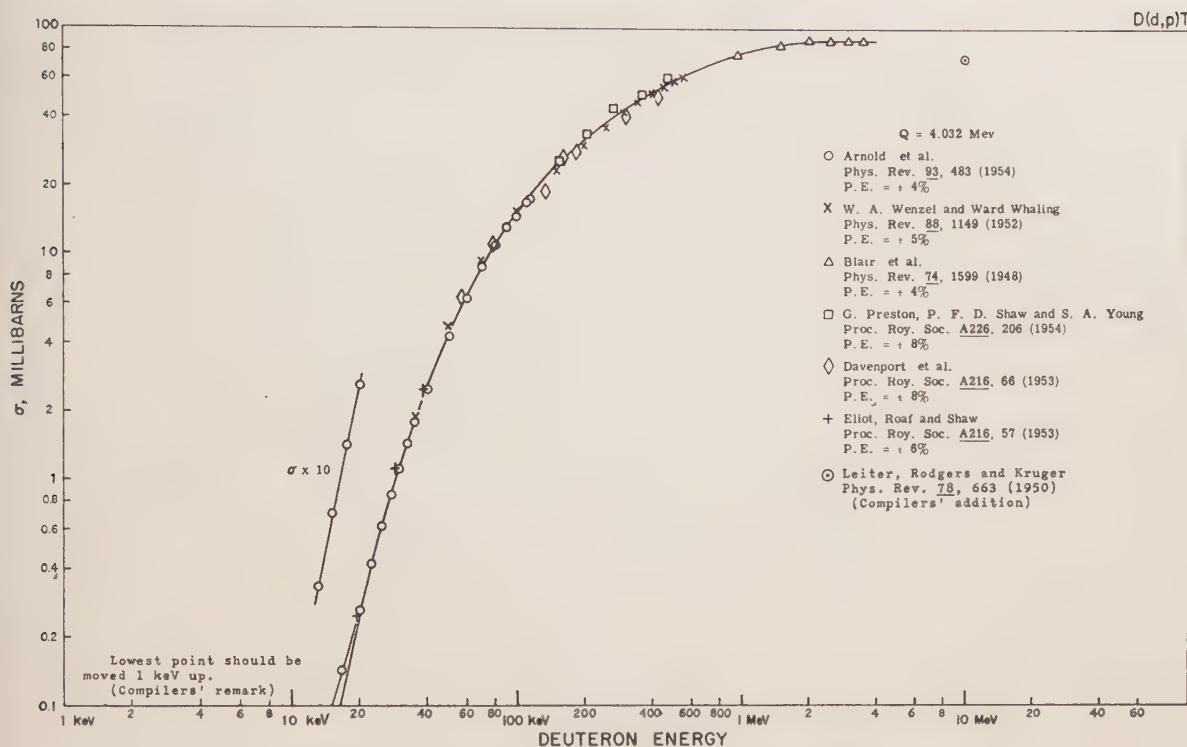
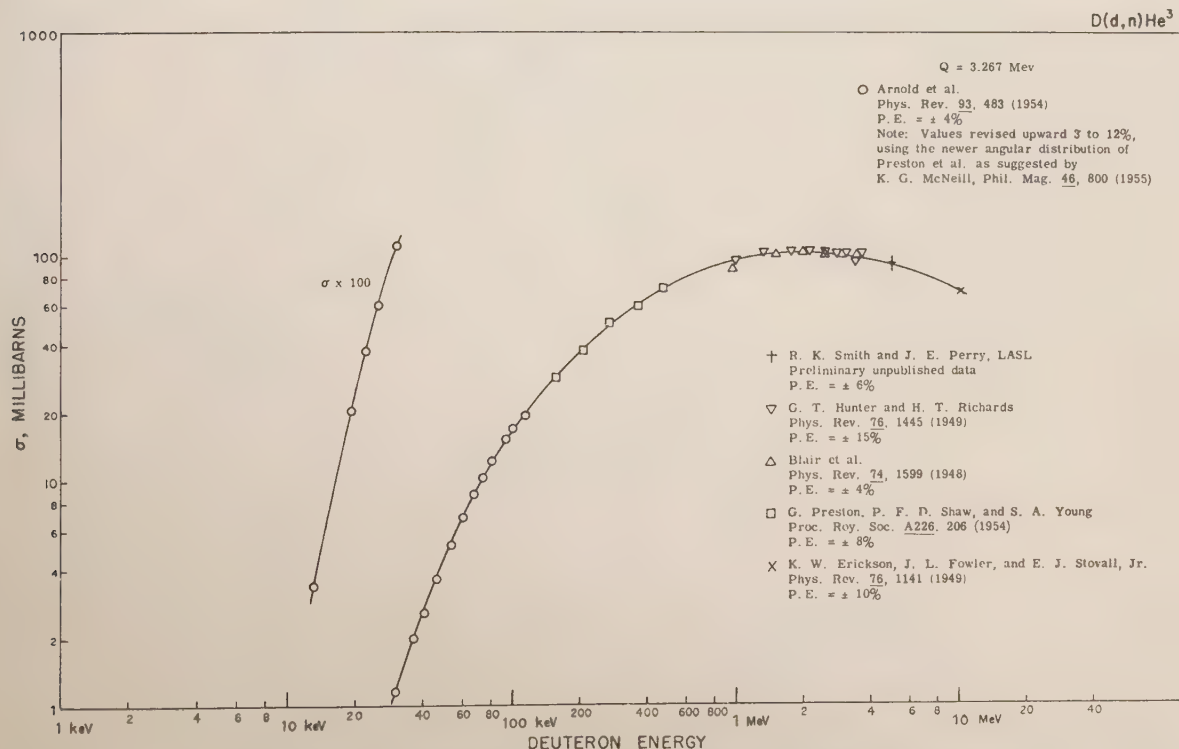
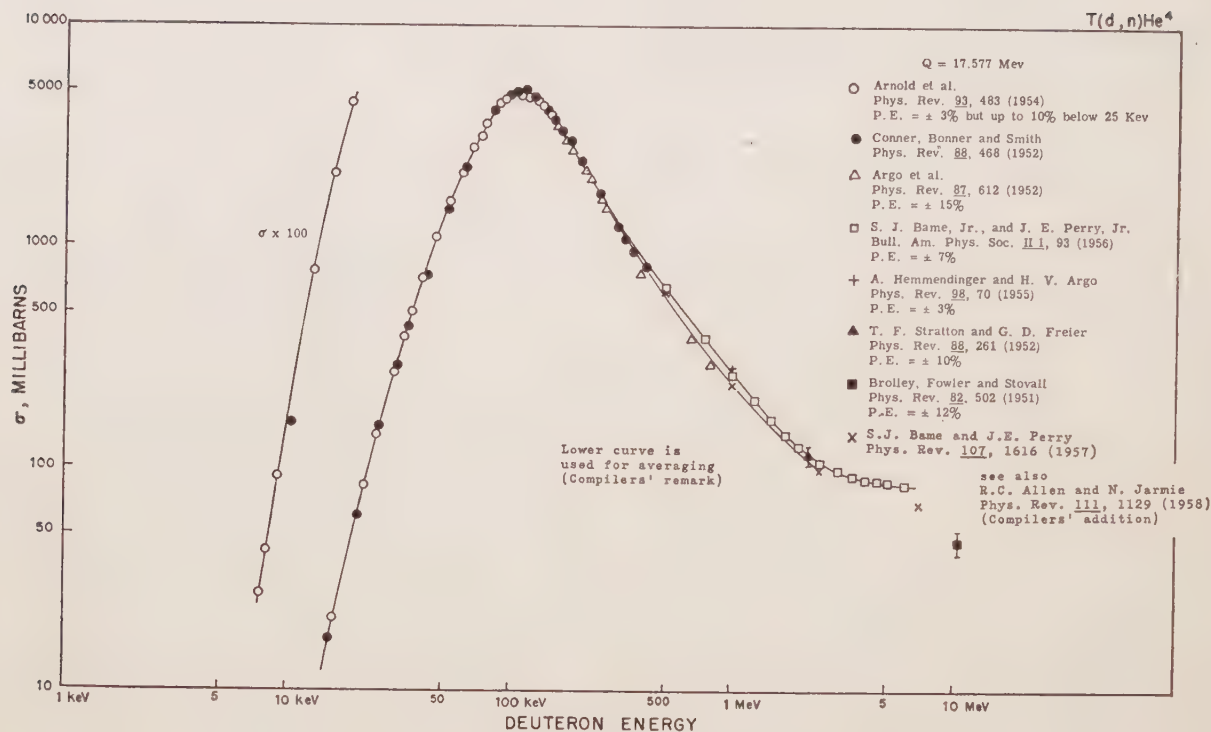
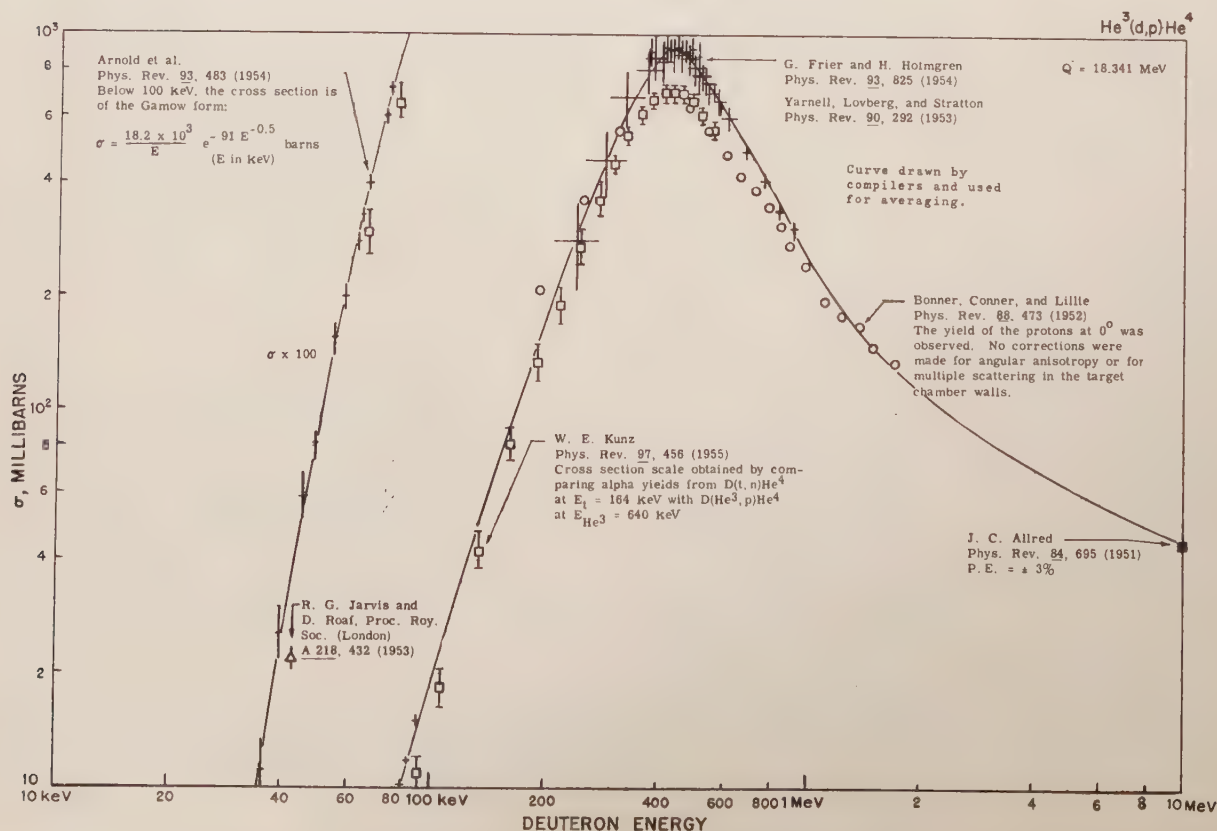
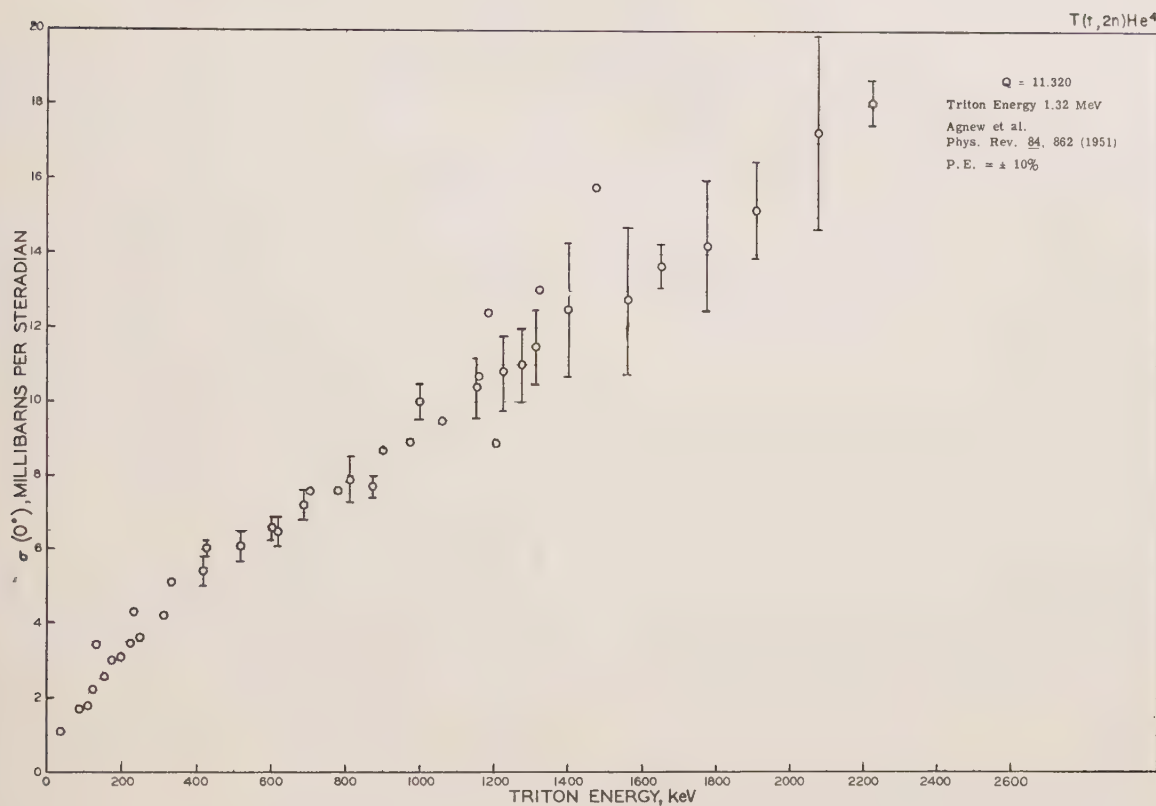
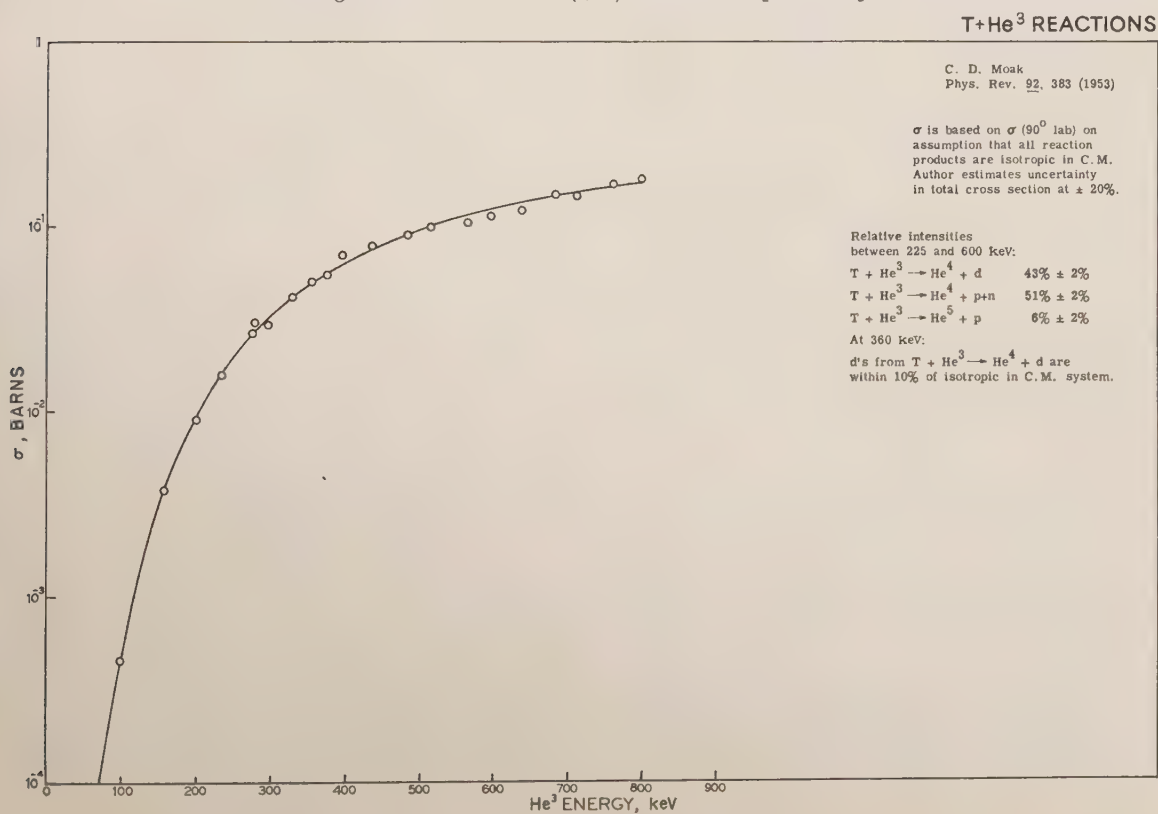


Fig. 1. Cross section for D(d,p)T reaction. [LA-2014].

Fig. 2. Cross section for D(d,n)He³ reaction. [LA-2014].

Fig. 3. Cross section for $T(d,n)He^4$ reaction. [LA-2014].Fig. 4. Cross section for $He^3(d,p)He^4$ reaction. [LA-2014].

Fig. 5. Cross section for $T(t, 2n)He^4$ reaction. [LA-2014].Fig. 6. Cross section for $T + He^3$ reactions. [LA-2014].

W. B. Thompson⁷⁾). Unfortunately, this article contains several misprints. The value given for the T-D reaction rate at 100 keV using the cross sections of Arnold *et al.* must be one of them. Apart from ref. ⁴⁾, ref. ⁷⁾ is the only one of which we are aware, that includes the important He³-D reaction rates. Unfortunately, the only two values lying within the range 5-150 keV disagree with the ones calculated in ref. ⁴⁾, which is used in this compilation, the one at 10 keV being a factor 3 too small and the one at 100 keV a factor 2.5 too big compared with the values given here.

A calculation of the reaction rate parameters for the reactions D(d,p)T, D(d,n)He³ and T(d,n)He⁴ in the range from 10 to 100 keV has been performed by Boulegue *et al.*⁸⁾, they all agree reasonably well with the values given here, if the values given for the T-D reaction are all multiplied by 100 to compensate for a trivial misprint.

Also a Russian calculation by Zubarev and Klimov⁹⁾ has come to our knowledge. It contains values for the D(d,n)He³ and T(d,n)He⁴ reactions in the range 0.1 to 200 keV based on calculations using the analytical Gamow expression for the fundamental cross sections. The agreement with the values given in this compilation is well within the limits that could be expected when use has been made of the analytical fit to the experimental data. The Russian data show, however, a peculiar behavior above 100 keV in the case of the T-D reaction. Above this temperature their reaction rate parameter shows an increase instead of a decrease; we have not been able to find the reason for this discrepancy.

Recently a calculation has been made by Eder and Motz¹⁰⁾ in the case of the total D-D and the T-D reactions. Agreement with the T-D reaction rate parameter is perfect while due to a numerical slip the values given for the total D-D reaction rate parameters are a factor $\sqrt{2}$ too small

After this, we can only hope that we will have luck enough to get our values through the publication process without misprints.

3. Coulomb Cross Sections

Since in a fully ionized plasma the only long range force between particles is the Coulomb force a section should be devoted to this form of interaction.

With the kind of particles considered here this type of collisions are always elastic ones. Nevertheless they play an important role in the slowing down of fast particles in a fully ionized plasma and for the different kinds of relaxation phenomena in non-equilibrium situations. Among these we shall mention the energy exchange between Maxwellian particle mixtures and the deflection of a stream of particles by multiple Coulomb scattering.

As it is well known the total scattering cross section in a pure Coulomb field is a divergent quantity due to the very large contribution from the small angle scattering corresponding to distant encounters. In the actual physical problems a cut-off distance will always limit the contributions from the distant encounters and we shall thus in the following give both the differential scattering in a pure Coulomb field and the cut-off distance in a fully ionized plasma.

3.1. DIFFERENTIAL CROSS SECTIONS

The cross section for scattering into the solid angle element, $d\omega$, forming a scattering angle, θ , with respect to the incident particle direction, is given by $\sigma(E, \theta)d\omega$.

3.1.1. Rutherford, Darwin and Mott Scattering

If the masses of the two particles are m_1 and m_2 and their charges are Z_1e and Z_2e respectively then the differential cross section for scattering through the angle, θ , in the centre of mass system (CM system) is given by the well known Rutherford scattering formula

$$\sigma(E, \theta) = \left(\frac{Z_1 Z_2 e^2}{4E} \right)^2 \frac{1}{\sin^4 \frac{1}{2}\theta} \quad (5)$$

⁷⁾ W. B. Thompson, Proc. Phys. Soc. (London) B 70 (1957) 1.

⁸⁾ G. Boulègue, P. Chanson, R. Combe, M. Feix and P. Strasman, Compt. Rend. 247 (1958) 284.

⁹⁾ D. N. Zubarev and V. N. Klimov, Plasma Physics and the Problem of Controlled Thermonuclear Reactions, Vol. 1, p. 249, (USSR Academy of Sciences, 1958) (in Russian).

¹⁰⁾ G. Eder and H. Motz, Nature 182 (1958) 1140.

where $E = \frac{1}{2}mv^2$ is the energy available in the CM system, since

$$m = m_1 m_2 / (m_1 + m_2) \text{ and}$$

v = relative velocity.

Of course, this formula will also describe the scattering in the laboratory system if the mass, m_1 , of the incident particle is much smaller than the mass, m_2 , of the target particle, the energy E should then be interpreted as the energy, E_1 , of the incident particle. In a plasma at high temperature, collisions between electrons and ions can be interpreted in this way if the energy of the ion is not very much higher than the electron energy.

If m_1 and m_2 are of the same order of magnitude formula (5) is of course still valid in the CM system. But in the system in which particle 2 is at rest before the collision the scattering cross section is given by the formula of Darwin

$$\sigma(E, \vartheta_1) = \left(\frac{Z_1 Z_2 e^2}{2E_1} \right)^2 \frac{1}{\sin^4 \vartheta_1} \frac{[\cos \vartheta_1 \pm \sqrt{1 - \left(\frac{m_1}{m_2} \right)^2 \sin^2 \vartheta_1}]^2}{\sqrt{1 - \left(\frac{m_1}{m_2} \right)^2 \sin^2 \vartheta_1}}. \quad (6)$$

For $m_1 < m_2$ only the positive sign before the square root should be used. For $m_1 > m_2$ both the positive and the negative signs should be used and the results added.

As it is well known both classical and quantum mechanical calculations lead to the same expression for the Coulomb scattering cross section except in the case of the scattering of identical particles where the formula developed by Mott applies (ref. ¹¹)

$$\sigma(E, \vartheta) = \left(\frac{Z^2 e^2}{4E} \right)^2 \left[\frac{1}{\sin^4 \frac{1}{2} \vartheta} + \frac{1}{\cos^4 \frac{1}{2} \vartheta} + 2 \left\{ \frac{1}{\cos^2 \frac{1}{2} \vartheta \sin^2 \frac{1}{2} \vartheta} \cos \left(\frac{Z^2 e^2}{\hbar v} \ln \tan^2 \frac{1}{2} \vartheta \right) \right\} \right]. \quad (7)$$

This formula corresponds to formula (5) above i.e. it expresses the scattering cross section in the CM system. The symbols used are also defined like the symbols in eq. (5). In the case of particles with spin 0 (α -particles) one should use the +2 while for particles

with spin $\frac{1}{2}$ (protons and electrons) one should use the -1.

The transition to a system in which one of the particles is at rest before the collision (often called the laboratory system, although it has not the same obvious meaning in a plasma as in ordinary scattering experiments) is effected by the following substitutions:

$$E \rightarrow \frac{1}{2} E_{\text{lab}}; \quad \vartheta \rightarrow 2\vartheta_{\text{lab}}; \quad v \rightarrow v_{\text{lab}}$$

and

$$\sigma \rightarrow \frac{\sigma_{\text{lab}}}{4 \cos \vartheta_{\text{lab}}}.$$

The Mott scattering deviations are not of any great importance for most of the phenomena in fully ionized plasmas. Since the small angle scattering is the dominating factor in both the energy exchange and the multiple scattering the non-classical term will always be small in these cases.

3.1.2. Shielding Distances

Because the small angle scattering dominates in the Coulomb interaction between particles in a plasma, it is a very essential problem to determine at what distance between particles deviations from the pure Coulomb force can be expected. While for non-ionized nuclei this distance is of the order of the Bohr radius, in a fully ionized plasma it is generally assumed to be given by the Debye shielding distance defined by

$$D = \sqrt{\frac{kT}{4\pi e^2 n_e}}. \quad (8)$$

(For a discussion of the problems involved in the determination of the shielding distance see ref. ¹²) where further references can be found.)

3.2. RELAXATION TIMES

A very useful concept, when discussing the rate at which changes in the velocity distributions of the particles in a plasma are effected by Coulomb collisions, is the relaxation time with respect to the change in question.

¹¹) N. F. Mott and H. S. W. Massey, *The Theory of Atomic Collisions*, 2nd Ed. (Clarendon Press, Oxford, 1952).

¹²) L. Spitzer, Jr., *Physics of Fully Ionized Gases* (Interscience Publishers, Inc., New York, 1956).

Here, we shall only consider three different types of relaxation times as given by Spitzer in ref. ¹²).

A particle, the test particle, with mass m_1 , charge Z_1e and velocity v_1 moves through a gas with a number density, n_2 , of fully ionized particles, called the field particles, with mass m_2 , charge Z_2e and with a Maxwellian distribution in velocity space

$$f(v_2) = \frac{l_2^3}{\pi^{\frac{3}{2}}} \exp(-l_2^2 v_2^2) \quad (9)$$

where

$$l_2^2 = \frac{m_2}{2kT_2}.$$

The two first relaxation times t_D and t_E give the times it takes the test particle to acquire a spread, in the direction and energy distribution respectively, so large that it can be said to be random in these respects. The time t_D is thus a measure of the time it takes before a directionally ordered motion of particles turn into an isotropic distribution while t_E gives the time for Maxwellization of a group of monoenergetic particles in the gas.

and

$$G(x) = \frac{\Phi(x) - x\Phi'(x)}{2x^2}.$$

The values of the functions $G(x)$ and $\Phi(x) - G(x)$ are given in table 3.

The constant A_D is given by

$$A_D = \frac{8\pi n_2 Z_1^2 Z_2^2 e^4 \ln A}{m_1}$$

where $\ln A$ is a slowly varying function of the temperature and density of the field particles only, if the test particle velocity is not too big compared with the average field particle velocity. Values of $\ln A$ are given in table 4 below which is also taken from ref. ¹²). It is noticed that in a wide range of densities and temperatures considered in plasma physics it can be set approximately equal to 15.

A quantum mechanical correction in $\ln A$ has to be made whenever $e^2/\hbar v \ll 1$. In the temperature range considered this happens mainly when electrons are involved and this correction has been included in table 4.

We shall finally give the relaxation time for energy exchange through Coulomb collisions,

TABLE 3
Values of $G(x)$ and $\Phi(x) - G(x)$

x	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
$G(x)$	0.0	0.037	0.073	0.107	0.137	0.137	0.183	0.198	0.208	0.213
$\Phi(x) - G(x)$	0.0	0.075	0.149	0.221	0.292	0.358	0.421	0.480	0.534	0.584
x	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9
$G(x)$	0.214	0.211	0.205	0.196	0.186	0.175	0.163	0.152	0.140	0.129
$\Phi(x) - G(x)$	0.629	0.669	0.706	0.738	0.766	0.791	0.813	0.832	0.849	0.863
x	2.0	2.5	3.0	3.5	4.0	5.0	6.0	7.0	8.0	10.0
$G(x)$	0.119	0.080	0.056	0.041	0.031	0.020	0.014	0.010	0.008	0.005
$\Phi(x) - G(x)$	0.876	0.920	0.944	0.959	0.969	0.980	0.986	0.990	0.992	0.995

[L. Spitzer, Jr., Physics of Fully Ionized Gases (Interscience Publishers, Inc., New York, 1956).]

The two times are given by the expressions

$$t_D = \frac{v_1^3}{A_D [\Phi(l_2 v_1) - G(l_2 v_1)]}$$

and

$$t_E = \frac{v_1^3}{4A_D G(l_2 v_1)}$$

where the two functions Φ and G are given by

$$\Phi(x) = \text{Erf } x = \frac{2}{\pi^{\frac{1}{2}}} \int_0^x \exp(-y^2) dy$$

between two Maxwellian gases with temperatures T_1 and T_2 . If this relaxation time, τ , is defined according to ref. ¹³) one gets

$$\frac{d(T_1 - T_2)}{dt} = - \frac{T_1 - T_2}{\tau}$$

¹³) C. F. Wandel, Energy Exchange by Coulomb Encounters for Maxwellian Ion Mixtures in the Plasma State, Risø Report No. 4 (1959).

TABLE 4
Values of $\ln A$ for an electron-proton gas

T (°K)	Electron Density n_e (cm^{-3})								
	1	10^3	10^6	10^9	10^{12}	10^{15}	10^{18}	10^{21}	10^{24}
10^2	16.3	12.8	9.43	5.97					
10^3	19.7	16.3	12.8	9.43	5.97				
10^4	23.2	19.7	16.3	12.8	9.43	5.97			
10^5	26.7	23.2	19.7	16.3	12.8	9.43	5.97		
10^6	29.7	26.3	22.8	19.3	15.9	12.4	8.96	5.54	
10^7	32.0	28.5	25.1	21.6	18.1	14.7	11.2	7.85	4.39
10^8	34.3	30.9	27.4	24.0	20.5	17.0	13.6	10.1	6.69

[L. Spitzer, Jr., Physics of Fully Ionized Gases (Interscience Publishers, Inc., New York, 1956).]

where τ is given by the expression

$$\tau = 2.325 \times 10^{11} \frac{(A_1 k T_2 + A_2 k T_1)^{\frac{3}{2}}}{(n_1 + n_2) Z_1^2 Z_2^2 (A_1 A_2)^{\frac{1}{2}} \ln A} \quad (10)$$

where A_1 and A_2 are the masses of the particles in atomic mass units, kT_1 and kT_2 are the respective temperatures in keV, Z_1 and Z_2 are the particle charges in electron charge units and n_1 and n_2 the densities in particles per cm^3 .

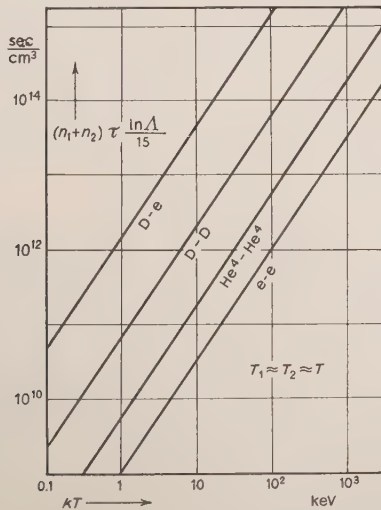


Fig. 7. Product of particle density and relaxation time for energy exchange between two Maxwellian gases with small temperature difference as a function of their temperature.

It is observed that energy exchange between light and heavy particles happens at a much slower rate than between particles of equal mass. The energy exchange between equal particles of small mass being again faster than between particles of large mass.

This is illustrated in fig. 7 where the product $\tau(n_1 + n_2) \ln A/15$ has been plotted for a number of particle combinations under the assumption that $T_1 \simeq T_2 \simeq T$.

Fig. 8 gives an altitude chart for the function $\tau(n_1 + n_2) \ln A/15$ with the temperatures T_1 and T_2 as independent variables. Curves are given both in the case of interaction between He^4 - and D-ions and of He^4 -ions with electrons.

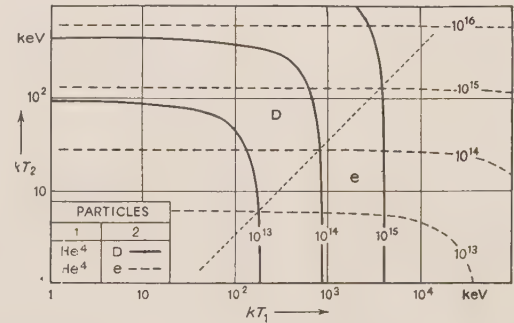


Fig. 8. Contour curves for the quantity $\frac{1}{15} (n_1 + n_2) \tau \ln A$ in the (kT_1, kT_2) -plane for He^4 as particles 1 and either deuterons or electrons as particles 2. He^4 gives off energy mainly to the electrons in the region marked "e" and to the deuterons in the region "D".

4. Bremsstrahlung Cross Sections

In a fully ionized plasma, energy will be emitted as bremsstrahlung. In this process the energy loss will mainly be suffered by the electrons since their smaller mass compared to the ion-mass will subject them to larger accelerations. In most calculations the plasma is considered transparent to bremsstrahlung. We shall in this section reproduce results both on electron-ion and electron-electron bremsstrahlung.

4.1. BREMSSTRAHLUNG FROM ELECTRON-ION COLLISIONS

The basic formulae for calculation of energy loss by bremsstrahlung in electron-ion collisions are the Sommerfeld and the Bethe-Heitler expressions for the spectral radiation intensity for one electron per sec per cm².

Both formulae are subject to limitations due to the approximations made in their derivations. The Sommerfeld formula (see ref. ¹⁴) is non-relativistic, but takes properly into account the influence of the Coulomb field on the wave functions of the incoming and outgoing electron. It is thus valid when $v_0 \ll c$ where v_0 is the velocity of the incoming electron and the spectral energy distribution $I(\omega)$ has the following form (ref. ¹⁵)

$$I(\omega)d\omega = \frac{3}{8} \pi \frac{\hbar^2 e^2}{c^3 m^2} \frac{\pi^2 x}{[\exp(2\pi\eta_0) - 1] [1 - \exp(-2\pi\eta)]} \frac{d}{dx} |F(i\eta_0, i\eta, 1, x)|^2 d\omega$$

where

$$\eta_0 = \frac{Ze^2}{\hbar v_0}, \quad \eta = \frac{Ze^2}{\hbar v}, \quad \frac{1}{2}m(v_0^2 - v^2) = \hbar\omega \quad (11)$$

and

$$x = -\frac{4\eta_0\eta}{(\eta - \eta_0)^2}.$$

Here v_0 and v are the initial and final velocity of the electron respectively, and F is the hypergeometric function with three parameters and one variable.

In the low velocity limit for $v < v_0 \ll Z\alpha c$ ($\alpha = e^2/\hbar c$ is the finestructure constant) formula (11) goes over into the semiclassical result obtained by using a classical orbit for the electron (see ref. ¹⁶).

The Bethe-Heitler formula is relativistic and based on the Born approximation, thus it does not take into account the distortion of the electron wave functions due to the Coulomb field and valid results can only be expected when $v_0 > v \gg Z\alpha c$, that is just the opposite of the classical limit.

For the explicit form we refer to Heitler's book, formula (16) p. 245 (ref. ¹⁷), which gives the corresponding cross section, this formula

should be multiplied by the energy of the emitted quantum to get the spectral intensity.

To get the power density emitted from a Maxwellian plasma by bremsstrahlung radiation, the above mentioned spectral intensities must be integrated over all frequencies and over the Maxwell distribution of electrons. This has mainly been done only after further approximations.

The most common procedure has been to use the non-relativistic limit of the Bethe-Heitler formula integrated with a non-relativistic Maxwell distribution. The result which is given in many references (refs. ^{6,18,19}) is

$$W_{\text{Born,N.R.}} = \frac{3}{8} \sqrt{\frac{2}{\pi}} Z^2 n_i n_e \frac{e^2}{\hbar c} \left(\frac{e^2}{mc^2} \right)^2 mc^3 \sqrt{\frac{kT}{mc^2}} \\ = 5.35 \times 10^{-24} Z n_e^2 \sqrt{kT} \frac{\text{erg}}{\text{cm}^3 \text{sec}} \quad (12)$$

where n_i and n_e are the number densities of the ions and electrons respectively in particles per cm³, while kT is the temperature in units of keV.

In the following we shall use this formula as a reference for comparison with other radiation power density calculations. In fig. 9, giving the calculated power densities, relative to $W_{\text{Born,N.R.}}$, as a function of the temperature kT , the reference formula (12) appears as a straight line marked B through the ordinate 1.

Due to the restriction $v_0 > v \gg Z\alpha c$ on the validity of (12) and the further non-relativistic approximation, valid when $v < v_0 \ll c$, the non-relativistic Born approximation result (12) can only be assumed to be valid in a region around $kT = 10$ keV for a homogeneous plasma.

For lower energies a formula derived by a semi-classical calculation and corresponding to the assumption $v < v_0 \ll Z\alpha c$ has been used and is often quoted in the literature (refs. ^{12,15,19}).

¹⁴ A. Sommerfeld, *Atombau und Spektrallinien*, II. Band (Braunschweig, Vieweg & Sohn, 1939).

¹⁵ G. Hettner, *Z. f. Physik* **150** (1958) 182-197.

¹⁶ L. Landau and E. Lifshitz, *The Classical Theory of Fields* (Addison-Wesley, Cambridge, 1951).

¹⁷ W. Heitler, *The Quantum Theory of Radiation*, 3rd edition (Oxford, Clarendon Press, 1954).

¹⁸ W. B. Thompson, Harwell Report, A.E.R.E. T/M 73 (1957).

¹⁹ V. I. Kogan and A. B. Migdal, p. 172, Vol. 1 in ref. ⁹.

It is given by an expression similar to (12), but with another numerical constant and can be written as

$$W_c = \frac{\pi}{2\sqrt{3}} W_{\text{Born, N.R.}} \quad (13)$$

The value of the numerical factor is $\pi/2\sqrt{3} = 0.907$ and is drawn as another straight line marked C in fig. 9. However, (13) can only be expected to be valid for temperatures below 1 eV which is the region outside the figure to the left.

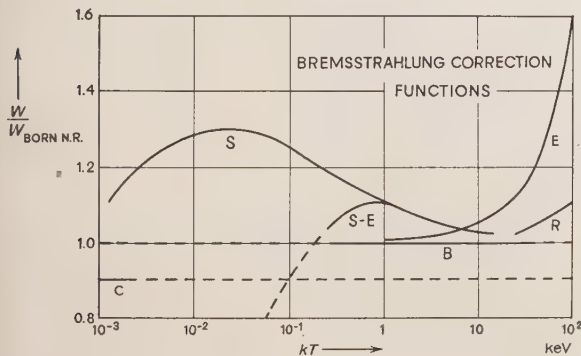


Fig. 9. Total bremsstrahlung power emission from a Maxwellian plasma calculated under various assumptions, relative to the value calculated from the Born approximation in the non-relativistic limit. C is the semi-classical result. S the result using the exact non-relativistic Sommerfeld cross section. S-E is obtained using the Sommerfeld-Elwert factor. R includes the relativistic electron-ion contributions. E takes the electron-electron bremsstrahlung into account.

Fortunately, the large region around 10 eV where neither the Born nor the classical approximation is valid has recently been covered in a report by Karzas and Latter²⁰). Their numerical results using the Sommerfeld formula has been drawn in the figure and marked S. At high energies it approaches the Born approximation result as it should, while at low energies it must be expected to join the semi-classical result of Hettner¹⁵) and of Kogan and Migdal¹⁹). A calculation similar to the one by Karzas and Latter has also been made by Hettner¹⁵). It is often claimed that the Born approximation

result for the cross section can be extended to the low energy region by use of the Sommerfeld-Elwert factor which, according to the 2nd edition of Heitler's book (ref. 17)) has the form

$$f(\eta, \eta_0) = \frac{2\pi\eta \, 2\pi\eta_0}{[1 - \exp(-2\pi\eta)][\exp(2\pi\eta_0) - 1]} \quad (14)$$

A numerical calculation of the power density including this correction factor in the integration over frequencies and electron-velocities has been performed by W. B. Thompson (ref. 18)) and has also been drawn in fig. 9. It is seen to extend the region of validity of the Born approximation result down to 1 keV, but deviates violently from the correct result at lower temperatures (curve marked S-E).

In the 3rd edition of ref. 17) another form of the Sommerfeld factor is given which remains larger than one for all values of η and η_0 and it is a reasonable guess that a calculation with this improved factor would approximate the correct result to lower temperatures†.

For temperatures above 20 keV relativistic corrections start to be significant, and the relativistic Born approximation result for the bremsstrahlung cross section should be used together with a relativistic Maxwell distribution for the electron velocities. The bremsstrahlung spectrum from a Maxwellian plasma at different temperatures in the relativistic region has been calculated in a report by Gluckstern *et al.*²¹). If one integrates their results over frequencies it is possible to get a lower limit for the total power density emitted by bremsstrahlung from electron-ion collisions in the temperature region from 10 to 100 keV. The result is drawn in fig. 9 and is marked R, the correct values are not believed to differ very much from this curve.

4.2. BREMSSTRAHLUNG FROM ELECTRON-ELECTRON COLLISIONS

The contribution from this type of radiation is generally assumed to be small due to the

²⁰) W. J. Karzas and R. Latter, U.S. AEC Report, AECU-3703 (Rev.) (1957).

²¹) R. L. Gluckstern, M. H. Hull, Jr. and G. Breit, U.S. AEC Report, AECU-4246 (1953).

† Note added in proof: A calculation using the "new" Sommerfeld-Elwert factor from 3rd ed. of Heitler's book¹⁷) gives an asymptotic value in the limit for small kT of $W/W_{\text{Born N.R.}} = 2 \ln 2 = 1.39$ in support of the guess.

fact that the dipole radiation term vanishes in this case. The main contribution comes then from the quadrupole radiation. From general arguments it can thus be expected that the ratio of electron-electron bremsstrahlung to electron-ion bremsstrahlung for $kT < mc^2$ is of the order of

$$\frac{W_{ee}}{W_{ei}} \simeq \frac{kT}{mc^2}. \quad (15)$$

$$W_E = 10^{-22} Z n_e^2 \frac{1 + 0.00624 kT}{1 + 0.00208 kT} F_0(kT) \quad (16)$$

where for $kT < 30$ keV

$$F_0(kT) = 0.0532 \sqrt{kT} (1 + 0.0009 kT) \quad (17)$$

while for $kT > 30$ keV, $F_0(kT)$ is given in table 5.

The result has been plotted in fig. 9 up to 100 keV and is marked E.

TABLE 5
 F_0 as a function of kT

$kT(\text{keV})$ $F_0(kT)$	50	80	100	150	200	250	300	400	500
	0.401	0.541	0.632	0.872	1.135	1.432	1.739	2.44	3.20

A calculation of W_{ee} has been made by Thompson¹⁸⁾ based on the work of Redhead²²⁾ and of Katzenstein²³⁾. Recently, however, some criticism of the results of these latter works has been put forward by Joseph and Rohrlich²⁴⁾. We have thus preferred to reproduce the result used by Zubarev and Klimov⁹⁾, although we have not been able to find the basis on which these results are obtained. It is for instance not clear whether the relativistic electron-ion correction has been included.

The formula used by Zubarev and Klimov is in our notation and with kT in keV

In fig. 9 those parts of the curves have been dotted which are definitely outside the range of validity of the corresponding approximations. It is interesting to note that even down to 1 eV the non-relativistic Born approximation result is closer to the correct value than the result of the semi-classical approximation.

²²⁾ M. L. G. Redhead, Proc. Phys. Soc. (London) A **66** (1953) 196.

²³⁾ J. Katzenstein, Phys. Rev. **78** (1950) 161.

²⁴⁾ J. Joseph and F. Rohrlich, Revs. Mod. Phys. **30** (1958) 354.

3. REPORT ON LINEAR PINCH DEVICES

R. LATHAM and J. A. NATION

Imperial College of Science and Technology, London

This report deals with various aspects of the linear pinch as a possible source of thermonuclear power.

The first section gives a very approximate assessment of the unstabilised linear pinch, and points out the severe practical difficulties of obtaining sufficient temperature and duration on present information.

As many experiments have been done using the stabilised linear pinch, the stabilisation problem is discussed

and various proposed geometrical arrangements are briefly described.

A summary is given of the published information on stability, mechanism of neutron production, temperatures achieved, and energy losses.

Brief conclusions are drawn as to the role of the linear pinch in thermonuclear work.

1. Dynamics of the Discharge During Constriction and Thermonuclear Possibilities

1.1. DYNAMICS OF THE DISCHARGE

The Snowplough Model^{1,2}, which need not be described in detail, applies where the current shell is thin and its velocity greater than the velocity of sound.

The equation of motion of the gas is given in terms of $y = r/R_0$ and

$$\tau = \sqrt[4]{\frac{V_0^2}{100 \pi \rho_0 R_0^4 L_0^2}} t, \quad \frac{d}{d\tau} \left[(1 - y^2) \frac{dy}{d\tau} \right] = \frac{-\tau^2}{y}. \quad (1)$$

It is assumed that the condenser voltage V_0 is substantially constant over the time to pinch t_p and that L_0 , the total inductance, lies mainly outside the tube.

If the variable tube inductance (i.e. the inductance of the discharge) is taken into account the equation becomes:

$$\frac{d}{d\tau} \left[(1 - y^2) \frac{dy}{d\tau} \right] = -\frac{\tau^2}{y(1 - \alpha \log_e y^2)^2} \quad (2)$$

where

$$\alpha = \frac{l}{L_0} = \frac{\text{Length of the discharge tube in cm}}{\text{External inductance} + \text{Discharge tube inductance (with the current sheet at the tube walls) in e.m.u.}}$$

The time to the first pinch is obtained by integrating eq. (2). This gives:

$$t_p = \lambda \sqrt[4]{\frac{100 \pi \rho_0 R_0^4 L_0^2}{V_0^2}} \quad (3)$$

where λ is a slowly varying function of α ($\lambda \approx 1.5$ for $\alpha = 0$ and $\lambda \approx 2$ for $\alpha = 1$).

In various laboratories^{2,3,4} these or similar equations have been shown to represent the gas dynamics fairly well as far as the velocity of collapse and the time to the first pinch is concerned.

It has been shown that the discharge retains its axial symmetry up to and during the first constriction and that irregularities begin to appear between the first and second "bounce". An examination of the thermonuclear possibilities of the first constriction is therefore relevant, as stability difficulties would not be expected to be present in this phase.

The integration of eq. (2) is normally used to give the temperature in a deuterium plasma by equating $\frac{1}{2} m_1 v^2$ to $\frac{3}{2} k (T_e + T_1)$. This gives:

$$T_e + T_1 \approx \frac{m_1}{3k} \frac{V_0}{(100 \pi \rho_0)^{\frac{1}{2}} L_0 R_0^2} \left(\frac{dy}{d\tau} \right)^2 \quad (4)$$

where $dy/d\tau$ is to be taken at the value of y corresponding to the minimum diameter of the discharge.

¹) M. Rosenbluth, Los Alamos Report, L.A. 1850.

²) O. A. Anderson *et al.*, Proc. Ven. Conf. (1957) p. 62.

³) P. Ohlin and K. Siegbahn, Proc. Ven. Conf. (1957) p. 808;

P. Ohlin and K. Siegbahn *et al.*, Nuclear Instruments 3 (1958) 237.

⁴) S. C. Curran *et al.*, Geneva Conf. (1958) P/1460.

The duration of the first constriction can be estimated, as an order of magnitude, at twice the time to bring the ions to rest with a constant deceleration

$$t_b \approx \frac{4(100\pi\rho_0)^{\frac{1}{4}}R_0}{(V_0/L_0)^{\frac{1}{2}}} \frac{(y)}{(dy/d\tau)} = \frac{4}{\lambda} \left\{ \frac{y}{dy/d\tau} \right\} t_p. \quad (5)$$

We could then utilise the value of $T_e + T_i$ and t_b to find the requirements which are imposed on a linear pinch if it is to produce appreciable thermonuclear power at the first constriction.

However, the minimum pinch diameter as defined by $K = y_{min}$ should perhaps be regarded as being in some degree controllable and allowed to take very small values. For small values of y_{min} the above expressions are not very convenient as $dy/d\tau$ varies rapidly as y tends to zero.

1.2. QUALITATIVE ACCOUNT OF THERMONUCLEAR POSSIBILITIES

An alternative qualitative approach can be made as follows:

1. The energy input up to the time of the first pinch can be written as

$$W = \int_0^{t_p} V I dt.$$

The integral scales as $V_0 \dot{I}_0 t_p^2$, where $\dot{I}_0$ is the initial rate of change of current with respect to time, and is equal to a numerical factor (varying to some extent with the experimental conditions) multiplied by $V_0^2 t_p^2 / L_0$. The numerical factor will be close to one half for the case of L_0 large and by estimation in typical discharges is roughly a quarter. Taking this latter figure

$$W \approx \frac{V_0^2 t_p^2}{4L_0}.$$

Substituting for t_p using eq. (3) and using $\lambda = 1.5$ we obtain

$$W \approx 0.55 \sqrt{100\pi\rho_0} R_0^2 V_0.$$

The energy per unit length of the discharge tube is therefore given by

$$\mathcal{E}_0 \approx 12 \sqrt{\rho_0} R_0^2 E_0 \text{ joules per cm.}$$

where E_0 is the volts per cm applied to the discharge tube.

W is of course necessarily less than $\frac{1}{2} CV_0^2$ though not much less or the condensers are not being fully utilised. The relation

$$\frac{1}{2} CV_0^2 = 12 \sqrt{\rho_0} R_0^2 V_0$$

then gives a connection between C and V_0 for reasonable utilisation of the condenser energy.

$$C \gtrsim \frac{4 \sqrt{\rho_0} R_0^2}{V_0} \text{ farads.}$$

2. The energy input up to the first bounce is distributed at the bounce between magnetic energy within the discharge tube and thermal energy of the gas. We assume with some experimental justification¹²⁾ that the energy input is roughly equally shared between the two. As we may be considering ultimately rather different conditions to the ones normally used the validity of this assumption may have to be re-examined later.

The energy used in heating the plasma is then

$$\mathcal{E}_p \approx 6 \sqrt{\rho_0} R_0^2 E_0 \text{ joules per cm.}$$

and if $T_i = T_e$ we can equate this to

$$3\pi r_p^2 n_p k T$$

where n_p is the density of ions at the pinch ($y = y_{min}$).

After simplification and use of the relation

$$\rho_0 = n_0 m_i = n_p r_p^2 m_i / R_0^2$$

we finally obtain for a D.T. plasma the relation

$$T \approx 10^{11} \frac{E_0}{K \sqrt{n_p}} \text{ } ^\circ\text{K} \quad (6)$$

3. The magnetic energy which is given by the relation:

$$W_m \approx 6 \sqrt{\rho_0} R_0^2 V_0$$

allows us to obtain a subsidiary relation between K , L_0 , and l .

$$\text{Using } \frac{d}{dt}(LI) = V$$

we have

$$L_p I_p = \int_0^{t_p} V dt.$$

The integral we will write as $f V_0 t_p$. The value of f will depend on how close the capacity

is to the limiting value for full utilisation i.e. on the voltage drop occurring between $t = 0$ and $t = t_p$.

A direct substitution of $I_p = fV_0 t_p / L_p$ into the expression $\frac{1}{2} L_p I_p^2$ for the stored energy and equating this to W_m gives

$$\log_e \frac{1}{K} \approx 1.6 f^2 L_0 / l$$

where L_0 is now the initial inductance in e.m.u. and l the length of the tube in cm. For f estimated as 0.6 this becomes

$$\log_e \frac{1}{K} \approx 0.6 L_0 / l. \quad (7)$$

4. The duration of the bounce t_b can be estimated as was done in the first section, but in this case deducing the velocity from

$$\frac{1}{2} m_i v^2 \approx \mathcal{E}_p$$

to make the expressions self consistent. The method may be entirely erroneous, but the form of the earlier result ($t_b = \text{const. } t_p$) seems to agree roughly with experiment.

Using this method we obtain

$$n_p t_b \approx 2.5 \times 10^{-4} \frac{n_p R_0 K}{\sqrt{T}} \text{ sec} \cdot \text{cm}^{-3}. \quad (8)$$

1.3. SUMMARY OF EQUATIONS GOVERNING DISCHARGE CHARACTERISTICS

The following three equations are the ones derived in the last section which we will use in the following sections

$$T \approx 10^{11} \frac{E_0}{K \sqrt{n_p}} \text{ } ^\circ\text{K} \quad (6)$$

$$n_p t_b \approx 2.5 \times 10^{-4} \frac{n_p R_0 K}{\sqrt{T}} \text{ sec} \cdot \text{cm}^{-3} \quad (8)$$

and

$$\log_e \frac{1}{K} \approx 0.6 \frac{L_0}{l}. \quad (7)$$

To find the requirements for a power balance reactor we use the results of Lawson⁵). For a D.T. mixture and a conversion efficiency of 30% these are

$$T \approx 1.2 \times 10^8 \text{ } ^\circ\text{K}$$

and

$$n_p t_b \approx 10^{14} \text{ sec} \cdot \text{cm}^{-3}.$$

Insertion of these into eqs. (6) and (8) yields on elimination of n_p

$$\frac{E_0^2 R_0}{K} \approx 6 \times 10^{15} \quad (9)$$

and the corresponding value of n_p satisfies the equation

$$n_p \approx 7 \times 10^5 \frac{E_0^2}{K^2}. \quad (10)$$

1.4. RELAXATION TIMES

Before this analysis is of value it must be shown that the duration of the bounce t_b is much greater than the ionic relaxation time t_i . We use the expression for t_i given by Spitzer⁶), eq. (5-26),

$$t_i = 1.6 \frac{T^{\frac{3}{2}}}{n_p}.$$

Eq. (8) gives the value of t_b , and the condition $t_b \gg t_i$ becomes:

$$\frac{E_0^2 R_0}{K} \gg 7 \times 10^{11}. \quad (11)$$

It will be seen, therefore, by comparison with (9) that the variables enter the expression above in the same way as in the condition for power balance and the condition for relaxation is automatically well satisfied. Indeed there is also time for electron-ion relaxation, (requiring $\approx 40 t_i$) which is some justification for our earlier assumption of $T_i = T_e$, and also possibly for build up of the high energy tail of the velocity distribution of the ions.

The mean free path for nuclear processes is, of course, much longer than that for ion-ion collisions. This mean free path need not be related to the pinch diameter as the ions are confined within the pinch for a time which satisfies $n_p t_b \approx 10^{14}$.

1.5. AXIAL TEMPERATURE LOSSES

Axial temperature losses can be estimated by introducing a characteristic time t_H for changes in temperature due to thermal conduction along the axis.

t_H is given by

$$t_H \approx 1.6 \times 10^{-10} \frac{n_p l^2}{T^{\frac{5}{2}}}.$$

⁵) J. D. Lawson, Proc. Phys. Soc. B **70** (1957) 6.

⁶) L. Spitzer, The Physics of Fully Ionized Gases (Interscience Publishers, New York, 1956).

Using again the equation for t_b and the condition $t_b \ll t_H$ we obtain the following expression for a minimum length of discharge tube

$$l^2 \gg 2 \times 10^6 \frac{T^2 R_0 K^3}{n_0}. \quad (12)$$

The values of l which are derived from this are of the order of magnitude of a few centimetres, so that for a tube of length a few tens of cms the axial heat conduction losses should not be prohibitive.

1.6. TABLE OF TYPICAL VALUES

The following table shows two typical sets of values of the appropriate parameters which satisfy the requirements outlined previously

E_0 volts per cm	3×10^5	5×10^5
R_0 cm	300	300
K	1/200	1/100
L_0/l	8.7	7.8
n_0 per cm ³	7×10^{17}	2×10^{18}
n_p per cm ³	3×10^{22}	2×10^{22}
Capacity (μ F) per cm of tube	3000	2000
Stored energy per cm of tube (MJ)	150	250
Minimum tube length (cm)	2	6

1.7. CONCLUDING REMARKS ON UNSTABILISED LINEAR PINCH

It must be emphasised that the preceding calculations are only a first attempt at deducing orders of magnitude. The treatment may well be wrong in its assumption that the initial energy is equally shared between thermal and magnetic energies and also wrong in treating K as a quantity whose value is decided on the basis of energy sharing. It may well be that K is decided by some process involving shock waves, in which case E_0 , R_0 and n_0 would be more closely specified.

On the basis of these orders of magnitude it is clear that there are severe difficulties in the use of an unstabilised linear pinch as a thermonuclear power source. The experiments so far done do not form a very reliable basis for extrapolation, and as the difficulties are rather sensitive to the initial conditions assumed it is perhaps unwise to generalise too far at the present stage.

In most laboratories, however, the major effort has been concerned with stabilised discharges for which the pulsed conditions are relaxed considerably, but it is then essential to produce a fully stabilised plasma. This is a problem which has already shown itself to be of the utmost complexity.

2. Stability of a Constricted Discharge

Most work so far has been done using axial magnetic fields present before the discharge starts and becoming trapped and carried in by the discharge as it contracts. The effects of such a field depend very much on its magnitude and it is convenient to consider three cases.

2.1. VERY LOW FIELDS

The main effect is to add an internal magnetic pressure $(B_{z0})^2/8\pi$, which becomes $((B_{z0})^2/8\pi)/(R_0^2/r^2)$ on constriction to radius r . This has little effect on the pinch dynamics apart from reducing the temperature of the first bounce. It will, however, have a serious effect on any very narrow "necks" set up as a result of the $m = 0$ perturbation.

2.2. MEDIUM FIELDS

As the magnetic field is inertialess the discharge will gradually change over as the field increases to a state where its diameter is set by the balance of the pinch pressure and that caused by compression of the initial B_{z0} . The inertia of the gas causes oscillations of the actual diameter about this transient equilibrium. The equilibrium diameter is set by

$$p_0 \left(\frac{1}{y} \right)^{2\gamma} + \frac{(B_{z0})^2}{8\pi} \left(\frac{1}{y} \right)^4 = \frac{I^2}{2\pi R_0^2} \left(\frac{1}{y} \right)^2 \quad (13)$$

which, for $\gamma \approx 2$ reduces to

$$y = \frac{R_0}{I} \sqrt{2\pi \left(p_0 + \frac{(B_{z0})^2}{8\pi} \right)}. \quad (14)$$

Defining a quantity β_z equal to

$$\frac{p_0}{(B_{z0})^2/8\pi}$$

the expression (14) reduces to

$$\left(\frac{B_z}{B_\theta}\right)_r = \frac{1}{\sqrt{1 + \beta_z}}. \quad (15)$$

For complete stability against $m = 0$ perturbations (Tayler⁷) assuming B_z external to the discharge can be neglected we require

$$B_z/B_\theta \geq 1/\sqrt{2},$$

or

$$\beta_z \leq 1$$

so that where the magnetic pressure due to B_{z0} exceeds the initial gas pressure the discharge becomes stable against $m = 0$ perturbations if the current remains in a thin shell.

2.3. LARGE FIELDS

For stability against $m = 1$ as well as $m = 0$, it is necessary to have⁷) $\gamma > 0.20$ and a conducting cylinder as an external return conductor. Usually p_0 is negligible in this case so we can simplify (14) to

$$\gamma = \frac{R_0 B_{z0}}{I^2 \frac{2}{2}}$$

or

$$B_{z0} > 0.2 \left(\frac{2I}{R_0}\right). \quad (16)$$

The condition for complete stability against $m = 0$ and $m = 1$ is made more difficult to achieve by an external B_z (Tayler⁷), Shafranov⁸), Colgate⁹). For example, if

$$\frac{B_z(\text{external})}{B_\theta(\text{at plasma surface})} = \frac{1}{2},$$

then only for a compression of 1.8:1 or less is the column stable. The region between the inner surface of the insulator and the conducting cylinder will act as a reservoir supplying some external B_z . It is advantageous therefore to have a thin insulating tube with a closely fitting conducting cylinder surrounding it.

It should be emphasised that once B_{z0} is enough to prevent the cylindrical sheet collapsing to the axis the mechanism for heating also becomes different. Instead of all the inward velocity due to the pinch pressure becoming randomised and used for heating part of the energy of compression is used to compress B_{z0} . The remainder compresses the gas adiabatically from its initial pressure and temperature to a pressure of the order of the pinch pressure $I^2/(2\pi r^2)$.

To obtain the temperature, B_θ and B_z must be measured as functions of r , and the gas pressure deduced from these measurements¹⁰). This pressure is set equal to $nk(T_1 + T_e)$ and if it is assumed that n is known because all the gas outside the equilibrium radius is swept in, then $(T_1 + T_e)$ can be found. $(T_1 + T_e)$ may still be quite large, but a serious part of the total energy available is used in compressing B_z . The stabilised pinch is more suited to the Ohmic heating of a gas sheet which is in equilibrium.

3. Experiments on the Mechanism of Neutron Production

3.1. COLUMBUS II^{10,11})

The neutron yield and the neutron energy asymmetry have been measured as a function of the axial magnetic field. The energy asymmetry is the difference between the actual energy of neutrons emitted in the anode $\rightarrow$ cathode direction and that expected for a hypothetical target of D^2 bombarded with very low energy deuterons.

It will be seen (fig. 1) that the change in the yield curve occurs (perhaps rather imaginatively) near to the value of B_{z0} (≈ 80 gauss) where the $m = 0$ perturbation becomes stable. It should be remembered that not the whole of the axial field will be trapped so that 80 gauss is an under-estimate. At the field required to stabilise $m = 1$ (4000 gauss), the yield curve is varying smoothly and neutrons are emitted at 4500 gauss.

The curve of energy asymmetry (fig. 2) also drops as B_{z0} increases. From this curve, under the assumption that the neutrons have their origin in a small number of deuterons accelerated to a high velocity which interact with the remainder at rest, the deuteron velocities can be calculated. Knowing (a) these velocities, (b) the cross-section as a function of velocity, and (c)

⁷) R. J. Tayler, Proc. Phys. Soc. B **70** (1957) 31, 1049.

⁸) V. D. Shafranov, J. Nucl. Energy **5** (1957) 86.

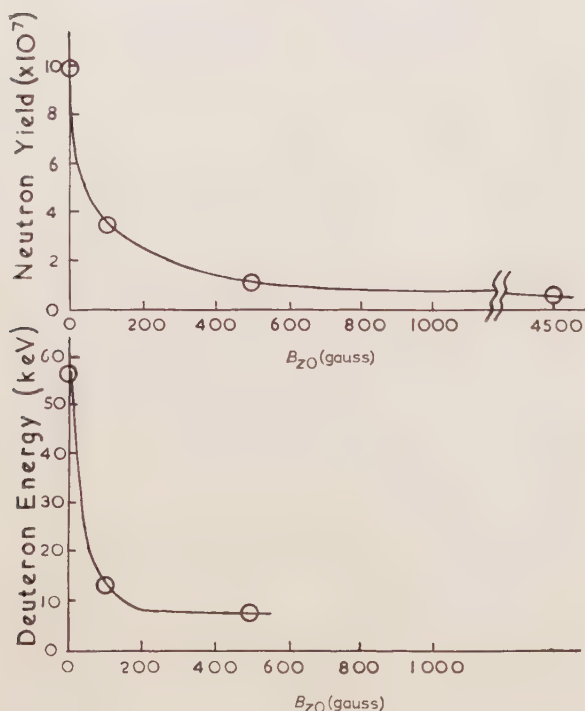
⁹) S. A. Colgate, T. Ferguson and H. P. Furth, Geneva Conf. (1958) P/369.

¹⁰) J. L. Tuck, Geneva Conf. (1958) P/1860.

¹¹) J. W. Mather and A. H. Williams, Geneva Conf. (1958) P/2394.

the total yield, the current of high velocity deuterons can readily be found.

This turns out to be a reasonable current for $B_{z0} = 0$ and a wholly unreasonable current



Figs. 1 and 2. Columbus II. Neutron yields and energy anisotropy as a function of axial magnetic field B . (J. W. Mather and A. H. Williams, Los Alamos.)

($\approx$ total tube current) for $B_{z0} = 500$ G. Thus it may be concluded that in the high B_{z0} region the mechanism is not one in which part of the deuterons are accelerated axially and interact with a target at rest.

3.2. MASS SPECTROMETER WORK

Work by Andrianov *et al.*¹²) has led to direct observation of the axially accelerated deuterons in the absence of a magnetic field. In this work, done in a tube 80 cm long by 17.2 cm diameter, deuterons of up to five times the tube voltage were observed and identified by simple mass spectrometry. They emerged mainly through the cathode, with very few either moving radially or through the anode.

3.3. EXPERIMENTS AT AWRE

Experiments at AWRE⁴⁾ have shown that the effects of inductance external to the discharge tube are extremely important in this work. With $L_0 \approx 200$ milli μ H the results are very similar to Anderson²⁾ and to a lesser degree Columbus II¹¹⁾. The neutron yield drops rapidly with B_{z0} and in their case by the time the B_{z0} has reached ≈ 100 gauss the yield has dropped by a factor of 10^3 . Above 100 gauss the rate of change of yield is not apparently so rapid.

On the other hand, for lower values of L_0 the situation is not so clear. The yield at $B_{z0} = 0$ is slightly reduced and the rate of change with B_{z0} is slow without evidence of a drastic change at the field where $m = 0$ becomes stable (≈ 30 gauss). It appears that the particular mechanism which produces the yield at low B_{z0} is not operative when the external inductance is low.

3.4. COLUMBUS S-4¹⁰⁾

Much detailed and interesting work has been done on this apparatus under clean and reproducible conditions, leading to the following conclusions:

(1) A well-developed current sheet is shown for the first few μ secs and the profiles of B_θ , B_z , i_θ and i_z with r have been found by probe work.

(2) For medium B_{z0} , enough to suppress $m = 0$ but not $m = 1$, the axial symmetry is lost after 3 to 4 μ secs in a manner suggesting the build-up of $m = 1$ instability.

(3) For high B_{z0} this helical break-up is suppressed and the discharge appears to possess no large-scale motion. However, azimuthal symmetry is again lost after a similar time by a different mechanism. The plasma boundary begins to show "flutter", i.e. a turbulent motion in which related motion is limited to regions of about 1 to 2 cm. This has been tested by observing correlation between signals from magnetic probes with different spacing.

(4) Neutron emission in the case above, where B_{z0} is high enough to inhibit both $m = 0$

¹²⁾ A. M. Andrianov *et al.*, Geneva Conf. (1958) P/2301.

and $m = 1$ instabilities changes its character. The neutron pulse is delayed, occurs over a longer time and begins at the same time as the onset of the "flutter", as observed on a probe trace. We shall refer to a possible origin of this instability later.

(5) Under the conditions of high B_{z0} the gas inertia plays little direct part but it does play an indirect part in causing oscillations of the diameter of the conducting sheet about the equilibrium value settled by B_{z0} and I . The frequency f is readily found to be

$$f = \frac{1}{2\pi} \sqrt{\frac{(B_{z0})^2 R_0^4 / r^4}{4\pi r \rho}} \quad (17)$$

where ρ is the density per unit area at the current sheet. These oscillations have been observed by the use of a probe to measure B_z at the axis. The magnitude of B_z oscillates with the shell diameter as flux is conserved, and forms a suitable method of detection.

The frequency varies in the right way with initial gas pressure, and it has been found that:

- (a) all the gas swept in remains on the sheet,
- (b) there are no effects due to gas evolution or absorption by the walls.

4. Suydam Stability Criterion

Two methods of calculating the degree of stability of a pinch have been developed. The "normal mode" method uses a small perturbation of an initial state (Tayler⁷, Shafranov⁸)) and the energy method (Kruskal¹⁴, Bernstein *et al.*¹⁵)) uses an effective potential energy for the plasma.

The second of these methods has been applied by Suydam¹³) to the case where diffuse boundaries exist between the axial stabilising field B_z and the confining field B_θ . The analysis yields the following condition for stability

$$\frac{8\pi}{B_z^2} \left(\frac{dp}{dr} \right) + \frac{1}{4} r \left(\frac{1}{\mu} \frac{d\mu}{dr} \right)^2 > 0$$

where p = plasma pressure and $\mu = B_\theta / r B_z$. It is shown that this is a necessary condition but not, so far, that it is a sufficient condition.

The inequality imposes a restriction on the radial gradient of pressure in a pinched discharge

and makes it seem unlikely that a fully stable pinch exists with uni-directional B_z and B_θ . With regard to reversals of B_θ and B_z external to the discharge Colgate, Ferguson and Furth⁹) point out that there exists in this region a plasma of relatively high conductivity and low density. The conductivity may well be high enough to make it difficult to obtain this reversal outside the constricted column.

A detailed application of the Suydam principle has been made on Columbus S-4 and is shown in fig. 3. From this it is deduced that the stability of the discharge is suspect for radii greater than 2.5 cm. For high B_z the "fluttering" motion occurs at a radius of 4 cm and this is accompanied by a long neutron pulse. It is

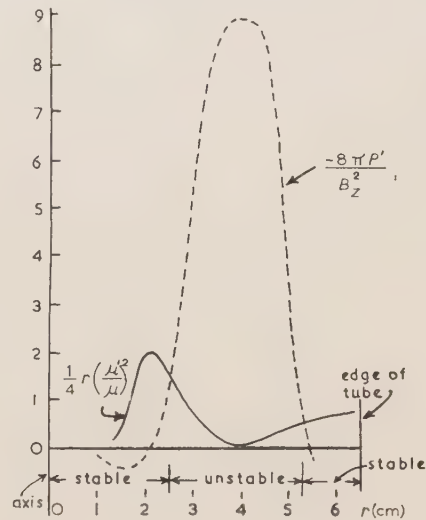


Fig. 3.

tempting to attribute both to a boundary layer instability arising when the Suydam criterion is violated.

Probe measurements on Columbus S-4 in the latter part of the first half-period of the current waveform have shown the existence of closed current loops within the tube. At the time when the total current through the tube has fallen to zero it has been found that about one-third of

¹³) B. R. Suydam, Geneva Conf. (1958) P/354.

¹⁴) M. Kruskal, Proc. Ven. Conf. (1957) p. 562.

¹⁵) I. B. Bernstein *et al.*, Princeton Univ. Rep. PM-5-25; I. B. Bernstein *et al.*, Proc. Roy. Soc. A **244** (1958) 17.

the original maximum current is still flowing along the axis. The current returns as a reverse current at larger radii. This reverse current may reverse the direction of B_θ and lead to modifications to the value of the second term of the inequality.

Further work on the mathematical side is necessary, to establish whether the criterion is sufficient and also to find out what kind of perturbation would arise if it is not satisfied. On the experimental side it would be useful to devise discharge configurations which satisfy the criterion and see what improvements, if any, are observed.

5. Other Proposals for Stabilisation

5.1. SHEET PINCH "TRIAx"^{16,17}

The stability difficulties are easier for a plane current sheet of infinite extent in that some

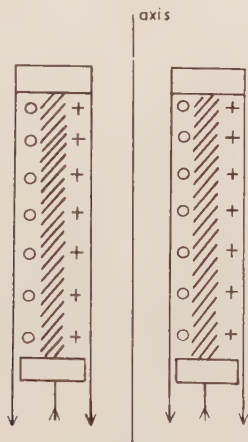


Fig. 4.

modes are stable and others have neutral stability. Experiments at Livermore are in progress with a cylindrical sheet (fig. 4) which approximates to the above case.

The various relevant features are as follows:

(1) The compression ratio is less than for the normal linear pinch as the compression is only taking place in one dimension, but no less than is allowable in a stabilised linear pinch.

(2) The axial loss is comparable to an unstabilised pinch and better than would be expected for a stabilised pinch. The stabilised

pinch, with some mixing of B_θ and B_z , allows fairly rapid loss to the electrodes along the magnetic lines of force. This reduction of axial loss has been tested by temperature measurements on the electrodes, and by varying the effective length of the tube by inserting dummy electrodes at several positions along the axis.

(3) Magnetic probe measurements in a low power discharge show reproducible repeated oscillations of the total thickness of the discharge. These oscillations are in full agreement with inductance measurements made from the voltage and current oscillograms. Up to 10 oscillations have been observed without any sign of instability.

In the attempts made to produce thermonuclear reactions with "Triax" a discharge similar to that referred to in (3) is used to preheat the gas to ≈ 10 eV and then a sudden further pinching is produced by applying a high power discharge with a rate of rise of current of the order of 10^{12} A/sec. This should be capable of producing an ion temperature of the order of several hundred electron volts. $10^4 \rightarrow 10^5$ neutrons are produced in a pulse lasting a few tenths of a microsecond, about one or two microseconds after the main bank is triggered in. If the yield is thermonuclear in origin, a temperature $\approx 300 \rightarrow 400$ eV would be needed. It should be noted, however, that the neutron burst is associated with a $2 \rightarrow 3$ kV voltage transient and impurity radiation also sets in at about the same time. The resistance of the discharge leads to an electron temperature of about 50 eV.

5.2. LINEAR PINCH WITH EXTERNAL B_z

Stability theory for an infinitely thin cylindrical shell shows that the $m = 0$ instability can be stabilised by a sufficient external B_z . The magnitude of B_z required is proportional to the wavelength of the perturbation. $m = 1$ perturbations are not stabilised by external B_z but grow much more slowly than $m = 0$ and so for

¹⁶ O. A. Anderson *et al.*, Geneva Conf. (1958) P/2349.

¹⁷ W. R. Baker and S. A. Colgate, Geneva Conf. (1958) P/1064.

a transient discharge may not reach sufficient amplitude to be troublesome. This partial stability is being examined by Colgate^{17,18)} using a geometry called the "Screw Dynamic Pinch" in which the external B_z is obtained by giving a helical shape to the return conductor. In the absence of the conducting plasma external to the discharge it is difficult and wasteful to provide the external field. The existence of the plasma external to the discharge helps in this instance by compressing and increasing the external B_z . The pitch of the helix necessary is therefore reduced. The series inductance introduced by the helix and its stored energy are also reduced.

5.3. "UNPINCH GEOMETRY"

R.J. Bickerton¹⁹⁾ has suggested an alternative linear arrangement similar to the "Triax" in which the field external to the discharge is in the z direction (fig. 5). It is produced by external coils and could be held within the discharge tube during the pulse by a conducting outer wall. The field within the discharge is in the θ

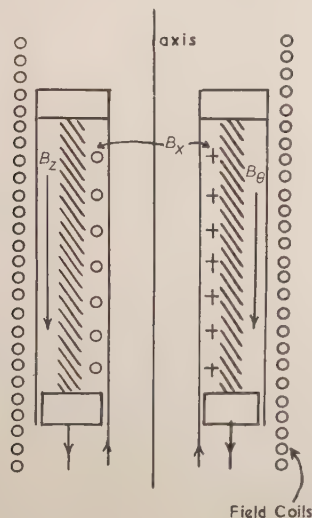


Fig. 5.

direction as in the "Triax". This is suggested as a means of testing whether hydromagnetic instabilities are responsible for the energy loss from the discharge. The loss phenomena for B_z outside the discharge should differ from the

case with B_θ outside. Lifetime measurements should show up this difference.

6. Temperatures obtained with Linear Pinches

The original work of the Livermore group on the unstabilised pinch gave $k(T_e + T_i)$ of the order of 100 eV. This was estimated using $\frac{1}{2} m_i v^2 = \frac{3}{2} k(T_e + T_i)$. Adrianov *et al.*¹²⁾ use a different though equivalent method of estimation. They measure the electrical power input as a function of time and also the magnetic stored energy by means of magnetic probes. The difference represents the energy used for gas heating and is equal to $\frac{3}{2} nk(T_e + T_i)$. This gives an ion temperature of 70 eV on the basis of $T_i = T_e$. Adrianov *et al.* have also estimated this temperature using a piezo-ceramic detector on the axis of the pinch. Assuming all the particles are swept into the pinch, the pressure recorded corresponds to a temperature of between 50 and 100 eV. The electron temperature is obtained by measuring the conductivity using the experimentally determined sheath thickness. The value is estimated from this to be not greater than 10 eV.

For the higher voltage bank used in the same work the value of $T_e + T_i$ has been estimated by calculating the electrical energy input to the discharge up to the time of first pinch, as before, and then allowing 35% of this (a theoretical estimate) for the magnetic stored energy. This was done because the field distribution was too difficult to measure at these high powers. The results gave, assuming $T_i = T_e$, an energy of 500 eV per particle.

Detailed experiments have been made by Fünfer *et al.*²⁰⁾ on an unstabilised linear discharge in deuterium, in which clear Kerr-cell photographs of the collapsing current sheet and the growth of surface irregularities are shown. Temperatures are estimated at between 300 and 500 eV.

For the linear stabilised pinch of Columbus

¹⁸⁾ S. A. Colgate *et al.*, Geneva Conf. (1958) P/372.

¹⁹⁾ R. J. Bickerton, CERN Meeting (25 Sept. 1958) Study Group on Fusion.

²⁰⁾ E. Fünfer *et al.*, Zeitschrift für Naturforschung, 13a (1958) 7, 524.

S-4 the temperatures can be estimated in more detail, as is discussed by Tuck¹⁰). During the contraction, at times of 1.4, 1.8 and 2.6 μsec the values of $T_e + T_i$ are 6, 13 and 33 eV. The temperatures estimated by the axial conductivity of the tube, obtained also in detail from the probe measurements, give 5, 12 and 4 eV respectively. There is good agreement at 1.4 and 1.8 μsec but between 1.8 and 2.6 μsec , though the energy input is rising, the electron temperature as found from the conductivity has decreased.

It is very plausible to consider that owing to the "flutter" which sets in at about this point the sheath becomes turbulent and the pressure instead of being nkT becomes rather $\frac{1}{2}nmv^2$, where v is the r.m.s. value of the velocity of the turbulent movement. This suggestion is qualitatively in agreement with the results and would mean that the estimate of 33 eV for kT at 2.6 μsec may well be an over-estimate.

7. Energy Losses from Discharge

7.1. LOSSES TO ELECTRODES

The loss of heat to the electrodes in a linear discharge must always be a limitation but one which differs in importance rather widely.

For the unstabilised pinch the electrode losses during the constriction are small, being of the same order as Joule losses in the discharge. The particle velocities as the shell contracts are mainly radial and until the first "bounce" do not affect the electrodes. At the first bounce serious end losses begin. Rough calculations (section 1.5, eq. (12)) show that even if the full conductivity is assumed, the first bounce has a short enough duration to render the losses not serious if the length of the tube is appropriately chosen. As the shell is of finite thickness, some B_θ will exist within this thickness. The losses will then be reduced by the reduction of the coefficient of thermal conductivity due to the transverse magnetic field. The same arguments would apply to "Triax".

For the stabilised pinch, on the other hand, where the duration expected is more of the order of the total pulse length than the "bounce

time", end effects are serious and Colgate¹⁹) and others have estimated that in the usual type of apparatus, 10^4 – 10^5 joules, 3–4" diameter, 20" long, conduction effects limit the central temperature to 50 eV or so, much as is obtained experimentally. However, in the case of similar sizes and energies in toroidal geometry little improvement is obtained, suggesting that perhaps other losses are larger than those due to the electrodes.

7.2. RADIAL ENERGY LOSSES

Anomalously high losses are now of fairly wide occurrence in both toroidal and linear devices. These are particularly well shown in toroidal geometry.

As mentioned earlier in connection with Columbus S-4, there appear to be other types of hydromagnetic instability than the well-known ones leading to "gross" motion of the plasma. The experiments in this case give evidence of the generation of fairly localised perturbations more in the nature of small-scale turbulence. This turbulence probably causes much more rapid diffusion than would be expected. It is suggested that such effects occur when the Suydam stability criterion is violated. If this turns out to be correct, then this form of instability may be quite widespread.

The experiments on prolonging the discharge in Perhapsatron S-4^{10,21}) demonstrate quite clearly that such radial losses occur. The discharge is maintained in a steady state for about 30 μsec , during which time about 5500 eV per electron is put in to the plasma. There is no evidence of rising electron temperatures, ion temperatures, or rate of neutron output during this phase, so one must conclude that the energy is being lost by some dissipative process. The discharge is apparently stable from the point of view of gross plasma motion.

7.3. RUNAWAY ELECTRON LOSSES

Electron runaway has been treated theoretically by Dreicer^{22,23}). He considers the problem

²¹) J. P. Conner, Geneva Conf. (1958) P/2488.

²²) H. Dreicer, Proc. Ven. Conf. (1957) p. 249.

²³) H. Dreicer, Geneva Conf. (1958) P/2292.

of electrons moving through an infinite gas of positive ions under the influence of a static uniform electric field of arbitrary strength. The analysis can be divided into two clear divisions:

1. "*The Strong Field Regime*" in which the interaction of the particles with the applied field is the dominating feature. Dreicer shows²³) that such an analysis leads to a critical electric field E_c and that for applied fields greater than this all electrons will try to run away.

2. "*The Weak Field Regime*" in which the interaction of the particles with each other is the major consideration. Even in this case runaway will still occur for electrons in the high energy tail of the initial Maxwellian distribution.

As the temperature of the gas increases the critical electric field drops. This field is proportional to the number of electrons per cc divided by the temperature of the gas

n_e/cm^3	T (eV)	$\bar{E}$ (V/cm)
10^{15}	100	≈ 8
10^{15}	1000	≈ 0.9
10^{16}	100	≈ 80
10^{16}	1000	≈ 9

The table shows clearly how rapidly the situation deteriorates as the temperature increases.

The process of electron runaway is a very serious factor in loss of energy from discharges. In a linear device all the electrons that diffuse into the runaway region must ultimately come into contact with an electrode. Hence their energy is lost from the discharge. It should also be emphasised that only a small percentage of runaway electrons are needed to account for very high gas currents.

In the toroidal device "Gamma"⁹), used at Livermore, the circulating current appears to consist almost entirely of runaway electrons which describe about half a turn before striking the walls. This has been studied by measuring the energy of the X-rays produced on a fluores-

cent target forming part of the wall of the torus.

Runaway electrons can cause as secondary effects two possible forms of energy loss:

(a) by upsetting the equality of n_i and n_e and therefore inducing plasma oscillations transverse to the axis of the tube;

(b) by causing longitudinal oscillations.

The effective dynamical friction of the gas rises to a maximum and then falls as the electron energy increases. When the electron is in the falling region, longitudinal oscillations are possible¹⁰), the falling frictional force causing negative damping. The oscillations can in theory reach large amplitudes in times of the order of a few μsec and will give rise to radial motion if some form of coupling exists. Thus radial energy loss might arise. This process has not yet been observed experimentally.

8. Summary and General Conclusions

8.1. UNSTABILISED DISCHARGE

The break-up of an unstabilised linear discharge is experimentally found to be quite rapid and the state of affairs during the first constriction is the important one. From the qualitative ideas of the first section it appears the first bounce of an unstabilised linear discharge is unlikely to be successful as a power producing device. However, in making such a qualitative evaluation we have been led to the requirement that the compression ratio $(1/K)$ should be very high and requires some degree of control in order to ensure that the electrical energy is correctly utilised. No really detailed experimental work has been done so far on the state of affairs at the first "bounce". For example, one would like to see how far the observed minimum diameter is influenced by putting on such heavy pre-ionisation as to reduce the electrical skin depth to a small value. One would also like to know how far shock wave propagation on the one hand, and gas diffusion on the other, affect the constriction. In the one-dimensional case discussed by Allen²⁴) the presence of shock fronts does in fact

²⁴) J. E. Allen, Proc. Phys. Soc. B 70 (1957) 24.

control the minimum thickness. If some reliable estimate could be made of the maximum compression ratio and how it varies with the parameters, the possibilities of the linear unstabilised pinch would be on a much firmer basis. An appreciable relaxation of the conditions could be made if the discharge were made quasi-stable for a time of the order of the time to pinch.

The wall effects referred to by Andrianov *et al.*¹²⁾ in their very high power discharges will undoubtedly form a serious limitation and any ideas on metal walled systems would be worth pursuing.

Runaway particles as mentioned earlier are a serious disadvantage in most pinch work and the critical field is never much above the fields required. A careful experimental study of the runaway process, to test and extend the various theoretical studies, would be most desirable at the present stage.

With regard to the break-up of the linear unstabilised discharge we have found that accidental irregularities in the insulating tube play quite a large part in the growth of the perturbations. As the main advantage of the linear system is its symmetry, it would be interesting to find out how far one can go towards improving stability by ensuring complete electrical and mechanical uniformity.

It will be seen that the linear unstabilised discharge is at a state where its prospects are rather sensitive to the findings of a more thorough study of the details of the constriction, and to any even minor improvements in the

length of time the compressed state can be made to last.

8.2. STABILISED DISCHARGES

It is very likely in this case the linear pinch is not the most satisfactory approach. If true stabilisation can be achieved, the time scale can be lengthened and ultimately the cooling effect of the electrodes becomes a limiting factor. A toroidal system is the obvious answer, though there is of course an unavoidable asymmetry in the torus. Torodial discharges therefore require positive stability and the problem of stabilisation is made more difficult. With a toroidal discharge there is also the possibility of containing the runaway particles within the torus and perhaps even using their momentum to assist in keeping the discharge stable.

The existence of linear stabilised pinches in which the electrode loss is expected to be small, such as "Triax", is also very interesting and might form a useful compromise.

The stabilisation so far envisaged assumes an initial steady state and also that the perturbations are small enough to allow the equations to be linearised. It should not be ruled out that other types of stabilisation may occur using rapidly varying magnetic fields or dependent on the existence of non-linear terms in the equations.

It would be fair to say at present, however, that there is no evidence of complete stability in any of the stabilised pinched discharges and it may be a long step between the absence of large-scale movements of the gas and the achievement of a fully quiescent discharge.

4. BRIEF REVIEW OF THE TOROIDAL STABILISED PINCH

R. J. BICKERTON

Atomic Energy Research Establishments, Harwell, Didcot, Berks.

This report gives a short discussion of the stabilised toroidal pinch approach. Stability conditions and heating

mechanisms are briefly summarized. A survey is given of experimental setups and results obtained with these.

1. Introduction

The pinched discharge is distinguished from other approaches to fusion in that the heating and confinement of the hot plasma is achieved entirely by currents flowing in the gas. This is both an advantage and a disadvantage in that the external apparatus is basically simple while inside the discharge the interdependence of different processes makes for complexity. More experimental work has been done on pinches by laboratories throughout the world than on other fusion systems. At each stage of the work new difficulties have arisen. The nature of the latest difficulty is at present not understood, but this does not mean that the pinch approach should at once be abandoned in favour of other less-developed and therefore apparently more promising systems. At the present stage of fusion research there is insufficient evidence on which to choose between systems. Given unlimited resources a vigorous pursuit of them all would clearly be advisable.

2. Conditions for Hydromagnetic Stability

Since the point of going from simple linear pinches to toroidal discharges is to eliminate the loss of heat to the electrodes, and since such a loss is only important for times greater than that required for the development of instability, then we are interested only in stable toroidal discharges. It was shown by several authors^{1,2,3}) that if an axial magnetic field were trapped inside the discharge plasma then with certain limitations on the plasma pressure, the pinch radius, and the axial field strength outside the pinch, stability could be achieved. The currents

were assumed to flow in an infinitely thin skin on the surface of the plasma, and the pinch to be enclosed in an infinitely long and infinitely conducting straight tube. There is as yet no theory of discharge stability using toroidal geometry. More recent work⁴) in which the current was considered as flowing in a surface layer of finite thickness has revealed a singularity in this layer. The result is that only when the axial field outside the pinch is in the reverse direction to that inside can a stable layer exist. The importance of the rather fine scale instabilities discussed in this treatment is questionable. They may well be damped by the viscosity of a real plasma. A further treatment⁵) of the same geometry but with diffuse boundaries between the stabilising axial field and the confining self-magnetic field uses the energy principle⁶) to treat the most dangerous modes of instability. By this means a criterion is obtained which is necessary, but has not been shown to be sufficient for stability. It is

$$\frac{p'}{(B_z^2/8\pi)} + \frac{1}{4} r \left(\frac{\mu'}{\mu} \right)^2 > 0 \quad (1)$$

where p is the plasma pressure, B_z is the axial component of magnetic field, B_θ the azimuthal component and $\mu = B_\theta/rB_z$. The prime denotes differentiation with respect to the radius r . This criterion is most powerful in the sense that it

¹) V. D. Shafranov, *Atomnaya Energiya* 1 (1956) 33.

²) M. Rosenbluth, Proc. 3rd Int. Conf. on Ionisation Phenomena in Gases (Venice, 1957).

³) R. J. Tayler, Proc. Phys. Soc. B 70 (1957) 31.

⁴) M. Rosenbluth, Geneva Conference (1958) Paper 347.

⁵) B. R. Suydam, Geneva Conference (1958) Paper 354.

⁶) I. B. Bernstein, E. A. Frieman, M. D. Kruskal and R. M. Kulsrud, Report N.Y.O. 7315.

can be directly applied to experimentally determined magnetic field distributions to give a clear indication of unstable regions in the discharge.

Finally by imposing the strict condition that every term in the energy integral should be positive, distributions of magnetic field that certainly lead to the stable confinement of plasma have been derived⁷⁾. The resulting discharge contains a central, intermediate and outer region. The axial current flows in opposite directions in the inner and intermediate regions, and the net current is zero. In these two regions the plasma pressure increases outwards, while in the outer region no axial current flows and the pressure falls to zero. A distribution of currents qualitatively similar to this has been observed in linear pinches⁸⁾, at the end of the first half cycle when the total current falls to zero. This has been explained as due to the presence of conducting plasma outside the pinch and the normal skin effect.

In the above theories the stability of an equilibrium state is considered. To arrive at these states with trapped axial field in the plasma it is necessary to collapse the pinch from the state in which it fills the tube to its final radius quite rapidly. Thus accelerations of the plasma are involved, it is clear that acceleration inwards constitutes a destabilising influence⁹⁾ and acceleration outwards a stabilising one. From a consideration of the scaling laws for field trapping¹⁰⁾ it can be seen that these effects are more severe in small bore discharge tubes.

3. The Heating of the Plasma

The plasma may be heated by any one of several methods, but outstanding in its simplicity is that of Joule heating by the currents producing the confining field. The stabilised pinch with spatially separated axial and aximutal fields is not a true equilibrium when interparticle collisions are considered. Such collisions imply a finite conductivity of the plasma and ohmic heating will occur. Qualitatively the initial crossed field configuration represents a higher energy state than that

prevailing after diffusion has occurred and the components of the magnetic field are mixed. In the absence of radiation and other energy losses from the system this energy difference goes into heating the plasma. A quantitative analysis of this problem has been made⁴⁾ for the case of plane geometry and initial sharp separation between the orthogonal magnetic fields. There are strong temperature and density gradients in the region of the current sheath but as a rough generalisation the plasma in the sheath is heated to about one half of the Bennett temperature T_B , where this is defined by

$$2n_0kT_B = \frac{B_0^2}{8\pi} \quad (2)$$

n_0 and B_0 are the initial ion density and magnetic field strength respectively. It is assumed that the ion density is such that rapid equilibrium between the electron and ion temperatures is established. Assuming that the plasma is heated in this way, and that it is stable until the thickness of the current sheath has broadened out to equal the pinch radius, it can be shown⁴⁾ that a net power gain is obtainable from a D-T pinch passing a current of the order 3×10^6 amps. This current is independent of the tube size^{4,10)}, that size being determined by other considerations such as the heat loading on the walls and the available techniques of energy storage and switching.

The pinch can also be heated by making the initial collapse velocity very high so that irreversible work is done on the gas by the pinch forces (cf. linear dynamic pinches). In this connection the existence, structure and effect of so-called "collision free" shock waves is of great interest¹¹⁻¹⁶⁾. However, the establish-

⁷⁾ E. W. Laing, A.E.R.E. Report T/M.161.

⁸⁾ L. C. Burkhardt and R. M. Lovberg, Geneva Conference (1958) Paper 2395.

⁹⁾ J. Berkowitz, H. Grad and H. Rubin, Geneva Conference (1958) Paper 376.

¹⁰⁾ R. J. Bickerton and H. London, Proc. Phys. Soc. **72** (1958) 116.

¹¹⁾ C. S. Gardner *et al.*, Geneva Conference (1958) Paper 374.

¹²⁾ J. Adlam and J. E. Allen, Geneva Conference (1958) Paper 1.

¹³⁾ L. Biermann, Geneva Conference (1958) Paper 1056.

ment of the very large electric fields required in a toroidal apparatus is at present a matter of considerable technical difficulty and no experimental work on shock heated toroidal pinches has yet been reported.

Finally most of the heating methods discussed in connection with the Stellarator can in principle be applied to the stabilised pinch.

4. Experimental Results

4.1. SIMILARITY

Table 1 contains the main dimensions and characteristic parameters of the toroidal pinches

t/R^2 are similar. R is the radius of the containing tube and t the time to peak current. Under these conditions the total number of nuclear reactions, Joule heating, losses in bremsstrahlung and cyclotron radiation, the kinetic energy of the plasma and the magnetic energy per unit length are all independent of the tube radius. Among parameters which do not scale in this way are the energy deposited on unit area of the wall, the ratio of collapse velocity to Alfvén velocity in the plasma, the radial acceleration of the plasma and the ratio of the discharge lifetime to the growth time of instabilities. As can be

TABLE 1

Device	Location	Torus bore (2R) (cm)	Torus diam (cm)	Gas current (Amps)	Time to peak current τ (sec)	τ/R^2 (sec·cm ⁻²)
Uppsala torus	Sweden	28	130	†	1.7×10^{-4} (variable)	$0.5-4 \times 10^{-6}$
Saclay torus	France	8	78	5×10^4	10^{-5}	6×10^{-7}
Alpha	U.S.S.R.	100	300	2×10^5	10^{-3}	4×10^{-7}
Zeta	U.K.	100	300	2×10^5	10^{-3}	4×10^{-7}
Perhapsatron S.4.	U.S.	14	70	2×10^5	10^{-5}	2×10^{-7}
Gamma Pinch	U.S.	10	60	3×10^5		
Sceptre	U.K.	30	110	2×10^5	10^{-4}	5×10^{-7}
Moscow torus	U.S.S.R.	48	130	2×10^5	3×10^{-4}	5×10^{-7}

† No experimental data reported.

reported on at Geneva¹⁷⁻²⁶). As a consequence of the Bennett relation:

$$2NkT = \frac{1}{2}\beta I^2 \quad (3)$$

(N = no. of ions per cm length of discharge, I = discharge current in e.m.u., β is a constant (< 1) the value of which depends on the amount of axial field trapped in the plasma).

The most interesting results (high temperatures, neutron emissions etc.) are obtained with the line density N near the minimum value at which a high current discharge can be obtained. It is an experimental fact that this value is independent of tube size and has the value for deuterium gas of about 5×10^{16} ions/cm length. The reason for this is not understood. According to the similarity relations for stabilised pinched discharges^{4, 10, 27}), those with constant N , I and

¹⁴ S. A. Colgate and R. E. Wright, Geneva Conference (1958) Paper 368.

¹⁵ R. S. Sagdeyev *et al.*, Geneva Conference (1958) Paper 2214.

¹⁶ A. I. Akhiezer *et al.*, Geneva Conference (1958) Paper 2300.

¹⁷ P. C. Thonemann *et al.*, Geneva Conference (1958) Paper 1519.

¹⁸ J. L. Tuck, Geneva Conference (1958) Paper 1860.

¹⁹ R. Aymer *et al.*, Geneva Conference (1958) Paper 1181.

²⁰ K. Siegbahn and P. Ohlin, Geneva Conference (1958) Paper 147.

²¹ T. E. Allibone *et al.*, Geneva Conference (1958) Paper 3.

²² J. A. Phillips *et al.*, Geneva Conference (1958) Paper 2488.

²³ S. A. Colgate *et al.*, Geneva Conference (1958) Paper 1064.

²⁴ P. C. Thonemann *et al.*, Nature **181** (1958) 217.

²⁵ N. L. Allen *et al.*, Nature **181** (1958) 222.

²⁶ J. L. Honsaker *et al.*, Nature **181** (1958) 231.

²⁷ S. I. Braginsky and V. D. Shafranov, Geneva Conference (1958) Paper 2500.

seen from the table the discharge currents and the values of t/R^2 achieved in the experiments do not vary over a very wide range. It is not surprising then that essentially similar results have been obtained in these discharges, although the toruses vary in bore from 5 to 100 cm.

4.2. MAIN RESULTS

There is evidence from magnetic probes, streak pictures and current and voltage oscillograms that the discharges are stable in the gross sense. For example the magnetic probe signals are reproducible in their outline; but with smaller, high frequency fluctuations superimposed. In Zeta with deuterium gas and clean conditions large voltage transients and fluctuating probe signals do, however, indicate violent motions of the current "channel".

Electron temperatures have been deduced from the discharge resistance. In all cases these lie in the range 5–30 eV. Temperatures determined from 8 mm microwave noise experiments are in approximate agreement with those determined from the resistance¹⁷). Finally from the degree of ionisation and excitation of impurities an upper limit of about 100 eV can be placed on the electron temperature¹⁷).

Measurements of the electron density by microwave transmission give densities consistent with all the gas initially present being compressed into a current channel with the radius indicated by magnetic probe measurements¹⁷).

Doppler widths of impurity ions interpreted as kinetic temperatures lead to values in the range 100–600 eV^{17, 21}). As yet is not known to what extent mass motion contributes to these "temperatures".

Estimates of the sum of the electron and ion temperatures obtained from pressure balances using the results from magnetic probes^{18, 22, 26}) lead to figures in the range 100–600 eV.

Neutron pulses of the order 10^5 – 10^6 per discharge have been observed^{17, 21, 22}). Measurements of the neutron energies^{17, 22, 28}), and in one case proton energies²¹), have shown that the nuclear reactions take place with a centre of mass velocity of the order 2×10^7 cm/sec

in the direction of current flow. This can be interpreted as due to accelerated deuterons with energies of ≈ 10 –20 keV colliding with stationary ones. It has been suggested²⁹) that the energetic deuterons acquire their energy by simple runaway process and acceleration in the applied electric field. However, it can be shown³⁰) that the dynamical friction on the deuterons due to collisions with the electron gas increases with deuteron velocity until that velocity is greater than the mean electron speed. In Zeta, for example, this last condition is not satisfied until the ion energy exceeds 200 keV. It seems clear that the acceleration process is a more complicated phenomena, in all probability associated with instabilities.

X-rays with energies up to 500 keV²²) are observed. The runaway electron currents required to account for the higher energy (> 20 keV) X-ray intensities are quite small relative to the main discharge currents. However, in one case³¹) the intensity of soft X-rays in the range 2–5 keV was sufficient to account for the whole of the energy input into the discharge, and corresponded to the total gas current being carried by runaway electrons.

Observations³²) of the Doppler shift of impurity lines when viewed parallel and anti-parallel to the current direction show that the impurity ions are moving round the torus in the direction of current flow with a velocity $\approx 2 \times 10^6$ cm/sec. Interpreting this as a drift velocity of the plasma as a whole, it is in fair quantitative agreement with the transfer of momentum to the walls to be expected if the discharge is losing energy entirely by the collision of ≈ 2 keV runaway electrons with the tube wall³¹).

The magnetic field distributions measured with probes show that the initial stabilising axial field is enhanced on the axis of the discharge by a factor of the order ten. However, in no case do the currents flow in a thin skin,

²⁸) B. Rose *et al.*, *Nature* **181** (1958) 1630.

²⁹) K. A. George, *Nature* **182** (1958) 745.

³⁰) A. Gibson, *Nature* **183** (1959) 101.

³¹) S. A. Colgate, Unpublished.

³²) A. A. Ware, *Nature* **183** (1959) 7.

and the two field components are well mixed. These observed distributions remain largely unexplained. Application of the Suydam criterion shows that the discharges should be unstable.

In Zeta¹⁷⁾ by dividing the estimated plasma energy by the observed power input an energy "containment time" of 100 μ sec is obtained. Similar results, allowing for scaling, are obtained with the other toroidal pinches. In all cases a density-time product³³⁾ $nt \approx 10^{10}$ is obtained. In one case²²⁾ in which the gas current was sustained at a high value for a relatively long time through the use of a subsidiary condenser bank, the neutron emission continued steadily throughout the discharge lifetime, and despite a power input of 900 MW the plasma temperature remained unchanged.

5. Discussion of Results

It is clear that although these discharges appear to be grossly confined away from the tube walls there is a very high rate of energy loss from the plasma to the walls. Broadly this may be the result of any one or a combination of the following processes,

- (1) Radiation from impurity atoms
- (2) Hydromagnetic instability
- (3) Electrostatic instability, or "co-operative phenomena".

There is some evidence to discount the first as the primary energy loss²²⁾, but until accurate total radiation experiments have been performed it remains a possibility. We know from the magnetic probe measurements that in theory the discharge should be hydromagnetically unstable, and the neutron emission could be quoted as evidence that this is so. To improve this stability we need to know how to form a thin current sheet and so to obtain well-separated magnetic fields. Finally the so-called electrostatic instabilities have been much discussed recently³⁴⁾. It is proposed that space charge waves or plasma oscillations are amplified by the interaction of electron streams with the plasma. The process is analogous to slipping stream amplification^{35,36)}. It is further proposed that the fluctuating electric fields resulting from

these oscillations enable electrons to cross lines of magnetic force and so to escape confinement. This last part of the theory is little more than a qualitative argument at present. The required initial streaming of electrons can be provided by the normal runaway process. If it is true that the process is initiated by runaway electrons, then the instability can be stopped by operating pinches with higher line densities N . Simple calculations using the theory of electron runaway^{37,38)} show that substantial runaway is avoided if $N \gg 10^{18}/\text{cm}$. The problem then is that very large gas currents ($> 10^6$ Amps) are required (see eq. (3)) to obtain interesting plasma temperatures.

6. Enumeration of the Main Problems

The following is a list of the main questions and problems that arise in connection with the toroidal pinch. The design of experiments to answer these questions must be left to the ingenuity of individual physicists and engineers.

(1) In what form does the energy reach the wall in toroidal pinches?

(2) What conditions are necessary to ensure a thin current sheet separating the axial and azimuthal magnetic fields, and how do these conditions scale?

(3) What is the effect of reversing the external axial field on the stability of such a current sheet?

(4) The plasma heating produced by the inter-diffusion of the initially orthogonal magnetic fields.

(5) Experimental confirmation of the theory of runaway electrons.

(6) Experimental measurements on the interaction between beams of charged particles and plasma.

(7) More experimental evidence on the con-

³³⁾ J. D. Lawson, Proc. Phys. Soc. B **70** (1958) 6.

³⁴⁾ O. Bunemann, Phys. Rev. Letters **1** (1958) 8.

³⁵⁾ A. V. Haeff, Phys. Rev. **74** (1948) 1532.

³⁶⁾ J. R. Pierce, J. App. Phys. **20** (1949) 1060.

³⁷⁾ H. Dreicer, Third Int. Conf. on Ionis. Phenomena in Gases (Venice, 1957).

³⁸⁾ A. Gibson, Third Int. Conf. on Ionis. Phenomena in Gases (Venice 1957).

ditions under which the classical theory of diffusion of plasma across a magnetic field is valid.

(8) The shock-heating of plasma in the toroidal pinch by the rapid collapse of the plasma column.

(9) Theoretical and experimental work on the stability of plasma and magnetic field under conditions of high acceleration.

(10) What are the mechanisms of neutron and X-ray production in toroidal pinches?

(11) The identification of spectral lines from highly excited atoms. What information about the plasma can be obtained from impurity spectra?

(12) The behaviour of toroidal pinches carrying currents of the order 10^6 Amps operating with line densities $N > 10^{18}/\text{cm}$.

(13) The interaction between plasma and material walls.

(14) The development of wall materials to withstand the arduous conditions involved, i.e. high voltage gradients, heat fluxes, neutron bombardment, etc.

It is apparent that the solution of many of these problems has a direct bearing on other approaches to fusion involving plasma and magnetic confinement.

5. REMARKS ON THE STELLARATOR-SCHEME†

L. BIERMANN

Max-Planck-Institut für Physik und Astrophysik, München††

Some points of interest in connection with devices of the Stellarator type are briefly discussed. Especially some

difficulties are pointed out that may be caused by the use of longitudinal magnetic fields for Ohmic heating.

It is known that charged particles may be confined continuously by magnetic fields of toroidal connectivity if the magnetic lines of force have a helical structure around the toroidal axis; the needed "rotational transform" can be produced either by a suitable geometry of the externally produced magnetic field or by having electric currents flowing in the contained gas along the toroidal axis. The choice of the possibility first mentioned is the main distinctive feature of the stellarator scheme. The necessary geometry of the magnetic fields can be brought about by giving suitable shapes to the endless tube in which a solenoidal field is present (e.g. the figure eight shape), or by electric currents in additional helical windings around the tube, or by a combination of these possibilities.

It follows from this:

(1) The machine can in principle operate steadily over times long compared to the time $d^2\sigma_m$ (d smaller diameter of the plasma, σ_m electric conductivity (e.m.u.)), which is the time scale, in which longitudinal (pinch type) currents and the related (meridional) component of the magnetic field would decay. The decay time for the meridional (diamagnetic) currents in a stellarator and for the corresponding gradient of the magnetic field is of the order of $d^2\sigma_m/\beta$ ($\beta = 8\pi P_0/B^2$). This decay can, in principle, be completely overcome by injecting continuously new gas into the interior.

(2) The production of the magnetic field by currents in external conductors requires a considerably higher technical effort than the generation of pinch type currents, when devices of similar plasma volume or of similar axial

length are compared. Already the need for strong external currents forces the ratio of the two radii of the endless tube to be rather small.

(3) The pinch type sausage instability does not occur, since the confining magnetic field approximates a vacuum field as long as β is small, rather than a more general force-free magnetic field as it does in a pinch-type arrangement.

(4) In principle it should be possible to reach both high temperature and reasonable values of β in a stellarator. Technical considerations have, however, led to the conclusion, that in the range of dimensions of the stellarator C scheme not both aims can be attained. Preference has been given in that scheme to the possibility to reach high temperature (around 10 keV), since investigations on the different kinds of heating mechanisms were thought to be particularly important.

(5) It may be surmised that longitudinal electric fields stimulate cooperative phenomena, e.g. plasma oscillations, possibly by causing an excess of energetic particles above the tail of the Maxwell distribution (run-away electrons), and that such phenomena limit the time, over which the magnetic confinement is effective. To the extent, to which this is the only cause of cooperative phenomena, the absence of longitudinal electric fields (after the period of the initial heating, see below) is an advantage of the stellarator scheme.

At present, point (1) does not appear to carry great weight, the more since it is conceiv-

† References are collected at the end of this paper.

†† These remarks are based on discussions with G. v. Gierke, K. Hain, A. Schlüter and W. Stodiek.

able that $d^2\sigma_m$ can be made large enough to allow a considerable part of the heavy hydrogen to react. As a consequence, it appears that the design of experimental machines must not necessarily aim, at the present stage, for a truly continuous mode of operation. As to point (3), the pinch sausage instability, which in other schemes using endless tubes might arise from longitudinal electric currents, can there, in principle, be avoided too by a suitable geometry of the magnetic fields.

A rotational transform angle of magnitude i_k can be reached (point (4)) by a suitable shape of the tube (figure eight) in which case i_k is independent of r (the distance from the magnetic axis). In the case of l pairs of helical windings, i_k is $\propto r^{2l-4}$ ($l > 2$, $r \rightarrow 0$). Here again for $l = 2$, i_k is independent of r . The maximum of β resulting from equilibrium considerations, β_E , is $\propto (i_k/\text{length})^2$ and is largest for $l = 2$. On the other hand for the stability against interchange the limiting β ($= \beta_C$) depends on the shear of the magnetic lines of force and varies as $(di_k/dr)^2$. Thus β_C is zero for $l = 2$ and also for the figure eight configuration. Hydromagnetic stability can be obtained by using three pairs of helical windings. In stellarator C a combination of $l = 2$ with $l = 3$ is planned and makes both limiting values (β_E and β_C) roughly equal; the parameters chosen lead to values of the order of 0.1%.

Another condition to be fulfilled is given by the Kruskal limit. This states roughly that the sum of the rotational transforms caused by the longitudinal currents and by the helical windings must be kept between 0 and 2π ; it limits these currents during the phase of Ohmic heating.

The application of the general theory of hydromagnetic stability to the stellarator scheme is still approximate in several respects in spite of important successes. In particular, the curvature of the magnetic axis has not yet been taken into account, and the magnetic field due to the helical windings has been assumed to be a small fraction of the total field; the gas pressure has been assumed to be small compared to the magnetic pressure, and the gyration radii

are assumed to be small compared to the radius of the tube.

On the balance, devices with near-vacuum magnetic fields—like the stellarator and the mirror machine—appear to be inherently more stable than devices with confining magnetic fields largely produced by currents flowing in the plasma. This impression should be contrasted, however, with our still imperfect understanding of instabilities and of the losses of plasma ("pump-out"), which are observed to occur. Also it should be kept in mind, that the attempts to overcome the corresponding difficulties in devices with longitudinal currents may become successful.

The major difficulty which has appeared in the experiments with stellarators made hitherto seems to be the so-called pump-out, namely the fact, that the plasma is being lost—across the magnetic surfaces—much faster, by a factor of the order of 100–1000—than would be expected from the present theory. However, no experimental results are known as yet on a torus with helical windings and with the degree of purity of the gas attainable by "baked" components. Such results are clearly desirable.

Larger machines could be expected to be more successful with respect to the "pump out" phenomenon. On the other hand, what is probably required next, is an understanding of the phenomena observed in medium scale machines; it is not quite obvious, that this need must be answered by experiments with larger machines, the construction of which at the present stage must in any case allow for the possibility, that new discoveries affect the physical requirements embodied in the design.

The various methods of heating proposed for the stellarator are not so characteristic for this scheme itself, most of them having been discussed also in connection with other schemes. These methods, collectively called magnetic pumping, consist in enforced periodic oscillations of the magnetic field. According to the frequency employed, they are called collision time, transit time and ion-resonance heating.

In all stellarator experiments initial heating

by longitudinal electric fields via Ohmic losses has been used, the possible dangers of which were pointed out already. The possibility of the ion-resonance heating has been verified, the temperature range in which transit or collision time heating may become really effective has not yet been reached.

In connection with the necessity to maintain a high degree of purity of the plasma, one should mention the so-called divertor, in which a suitable local deformation of the magnetic field is used for diverting the outer shell of the plasma into a separate chamber. This and the highly developed vacuum techniques worked out at Princeton are obviously useful not only in connection with the stellarator scheme itself.

The most important experimental results published, are:

- (1) Confining properties of the stellarator fields for electrons.
- (2) Confirmation of the Kruskal limit.
- (3) Some measurements of run-away electrons.
- (4) The establishment of the efficiency of the ion-resonance heating by measuring the load on the generator. No indication of beginning efficiency of transit time heating.
- (5) The divertor has yielded a reduction of the impurity level by a factor of about 3 on the surface of the plasma. The temperature was thereby increased by a factor 5.
- (6) Pump out.

References

- A. S. Bishop, Project Sherwood, The U.S. Program in Controlled Fusion (Addison Wesley Publishing Company Inc., Reading, Mass. U.S.A.).
- L. Spitzer, The Stellarator (Scientific American, Oct. 1958).
- L. Spitzer, Orientation lectures, AEC Report No. NYO-6049 (PM-S-16) (1954).
- L. Spitzer, AEC Report No. NYO-993 (PM-S-1) (1951).
- L. Spitzer, AEC Report No. NYO-995 (PM-S-3) (1951).
- L. Spitzer, AEC Report No. NYO-997 (PM-S-4) (1952).
- M. D. Kruskal, AEC Report No. NYO-998 (PM-S-5) (1952).
- L. Spitzer and L. Witten, AEC Report No. NYO-999 (PM-S-6) (1953).
- M. D. Kruskal, AEC Report No. NYO-6045 (PM-S-12) (1954).
- J. M. Berger and W. A. Newcomb, AEC Report No. NYO-6046 (PM-S-13) (1954).

- J. M. Berger and E. A. Frieman, AEC Report No. NYO-6046 (PM-S-16) (1954).
- H. Koenig, AEC Report No. NYO-7310 (PM-S-20) (1956).
- J. M. Berger and L. M. Goldman, AEC Report No. NYO-7311 (PM-S-21) (1956).
- J. M. Berger, AEC Report No. NYO-7312 (PM-S-22) (1956).
- L. Spitzer, AEC Report No. NYO-7316 (PM-S-26) (1957).
- E. F. Johnson, AEC Report No. NYO-7900 (PM-S-30) (1957).
- W. Bernstein, M. A. Heald, A. Z. Kranz, AEC Report No. NYO-7901 (PM-S-31) (1957).
- M. D. Kruskal and R. M. Kulsrud, Equilibrium of a Magnetically Confined Plasma in a Toroid, Proceedings of the 1958 Geneva Conference. (Publ. in "The Physics of Fluids", Vol. 1, No. 4).
- J. L. Johnson, C. R. Oberman, R. M. Kulsrud and E. A. Frieman, Some Stable Hydromagnetic Equilibria, Proceedings of the 1958 Geneva Conference. (Publ. in "The Physics of Fluids", Vol. 1, No. 4).
- M. D. Kruskal, J. L. Johnson, M. B. Gottlieb and L. M. Goldman, Hydromagnetic Instability in a Stellarator, Proceedings of the 1958 Geneva Conference. (Publ. in "The Physics of Fluids", Vol. 1, No. 5).
- W. Bernstein, F. F. Chen, M. A. Heald and A. Z. Kranz, "Runaway" Electrons and Cooperative Phenomena in B-1 Stellarator Discharges, Proceedings of the 1958 Geneva Conference. (Publ. in "The Physics of Fluids", Vol. 1, No. 5).
- T. Coor, S. P. Cunningham, R. A. Ellis, M. A. Heald and A. Z. Kranz, Experiments on the Ohmic Heating and Confinement of Plasma in a Stellarator, Proceedings of the 1958 Geneva Conf. (Publ. in "The Physics of Fluids", Vol. 1, No. 5).
- J. M. Berger, I. B. Bernstein, E. A. Frieman and R. M. Kulsrud, On the Ionization and Ohmic Heating of a Helium Plasma, Proceedings of the 1958 Geneva Conference. (Publ. in "The Physics of Fluids", Vol. 1, No. 4).
- J. M. Berger, W. A. Newcomb, J. M. Dawson, E. A. Frieman, R. M. Kulsrud and A. Lenard, Heating on a Confined Plasma by Oscillating Electromagnetic Fields, Proceedings of the 1958 Geneva Conf. (Publ. in "The Physics of Fluids", Vol. 1, No. 4).
- T. H. Stix, Generation and Thermalization of Plasma Waves, Proceedings of the 1958 Geneva Conf. (Publ. in "The Physics of Fluids", Vol. 1, No. 4).
- T. H. Stix and R. W. Palladino, Experiments with Ion Cyclotron Resonance, Proc. of the 1958 Geneva Conf. (Publ. in "The Physics of Fluids", Vol. 1, No. 5).
- C. R. Burnett, D. J. Grove, R. W. Palladino, T. Stix and K. E. Wakefield, The Divertor, a Device for Reducing the Impurity Level in a Stellarator, Proceedings of the 1958 Geneva Conf. (Publ. in "The Physics of Fluids", Vol. 1, No. 5).
- Project Matterhorn, Princeton University, AEC Report No. NYO-6047 (1.8.1954) PM-S-14.
- Project Matterhorn, Princeton University, AEC Report No. NYO-7899 (28.9.1957).
- Quarterly Report covering the period:
(January 1-March 31, 1958) NYO-2193 (PM-Q-2).
(April 1-June 30, 1958) NYO-2194 (PM-Q-3).

6a. LES MACHINES À MIROIRS MAGNÉTIQUES

RÉSUMÉ DE L'ÉTAT DE LA QUESTION ET ESQUISSE D'UN PROGRAMME DE RECHERCHES

M. BINEAU, T. CONSOLI, P. HUBERT, F. PRÉVOT, P. RICATEAU et A. SAMAIN

Commissariat à l'Énergie Atomique, France

This report will give a brief summary of our present knowledge of magnetic mirror machines, both from a theoretical and experimental point of view. An attempt is also made

to point out some lines of approach along which progress could be expected.—A more extensive treatise on the same subject is given in report 6b, (p. 290).

Introduction

La présente note a pour but d'énoncer sous une forme très condensée l'état actuel de nos connaissances expérimentales et théoriques sur les engins à miroirs magnétiques. Un effort est tenté pour définir les directions dans lesquelles un progrès pourrait être espéré.

Bien que la plupart des considérations exposées soient d'un caractère général, nous n'envisageons ici leur application qu'à des systèmes en régime permanent ou en évolution moyennement rapides, écartant de nos préoccupations les systèmes très rapides.

Les diverses applications numériques concernent un plasma de deutérium.

l'on néglige toute interaction avec le gaz neutre, le gaz d'électrons et le gaz injecté lui-même.

Le gaz neutre conduira surtout à des pertes par échange de charge décrites par ailleurs. Les gaz du plasma, ions et électrons conduiront à deux catégories de pertes: des pertes par diffusion coulombienne et des pertes par interaction avec les champs propres au plasma (effets ambipolaires, phénomènes collectifs, instabilités).

On aura perte par diffusion lorsque les chocs déplaceront les ions sur une trajectoire traversant le plan des miroirs (pertes aux miroirs), ou recoupant les parois ou l'injecteur (pertes par diffusion latérale).

POINT DE VUE THEORIQUE

1. Principe du Confinement

Le système envisagé concerne un champ magnétique de révolution, utilisé dans une région où l'intensité du champ sur l'axe passe par un minimum.

Le principe du confinement des particules individuelles s'exprime par la loi approximative de conservation du moment magnétique $\mu = E_{\perp}/B$. La non validité de cette approximation peut conduire à un défaut de confinement des particules. Il a été montré que dans certaines circonstances, cet effet peut n'être pas trop grave.

2. Problème des Fuites

Une particule correctement injectée décrit une trajectoire confinée quasi indéfiniment si

2.1. PERTES AUX MIROIRS †

L'échelle de temps du mécanisme correspondant aux collisions des ions entre eux est donnée par l'expression:

$$t_D = \frac{2 \times 10^{10}}{n} E_1^{\frac{3}{2}}.$$

Une théorie plus perfectionnée fait apparaître un temps caractéristique:

$$t_p = t_D \times 3K \log_{10} R.$$

Cette formule montre le peu d'intérêt des très grands rapports de miroir dont le prix croît certainement plus vite que $\log_{10} R$. Comme d'autre part, pour des chocs entre particules différentes:

$$t_D \approx \frac{A^{\frac{1}{2}} T^{\frac{3}{2}}}{Z^2 Z'^2}$$

les fuites aux miroirs font disparaître de préférence les particules froides ou plusieurs

† Voir pour les symboles p. 289.

fois ionisées. Ce mécanisme contribue puissamment à la purification du plasma.

2.2. PERTES PAR DIFFUSION TRANSVERSALE

La théorie classique, justifiable seulement si le rayon de gyration des ions peut être négligé devant les dimensions de l'appareil, conduit à n'attribuer les fuites qu'à des chocs de particules d'espèces différentes.

Ces pertes sont faibles aux températures et champs des bouteilles magnétiques. L'échelle de temps du phénomène est donnée par :

$$\tau = t_D \frac{L^2}{r_1^2} \left(\frac{v_e}{v_1} \right)^3 \frac{m_e}{m_1}.$$

Cette expression n'est valable que si la densité du plasma est faible et si $v_e > v_1$.

Dans le cas du DCX où cette théorie ne tient pas, une évaluation grossière donne aux pertes sur l'injecteur et la paroi le même ordre de grandeur qu'aux pertes par les miroirs.

2.3. EFFETS AMBIPOLAIRES

Les fuites naturelles des électrons provoquées par leurs collisions entre eux, devraient être plus rapides que celles des ions suivant une échelle de temps :

$$t_e = t_D \left(\frac{v_e}{v_1} \right)^3 \left(\frac{m_e}{m_1} \right)^2.$$

On pense que la fuite des électrons sera ralentie par l'apparition d'un champ électrique, de sorte que le système évoluera plutôt avec un temps caractéristique voisin de t_D .

Dans le cas où $T_e \ll T_1$, cet effet modifie le rapport des miroirs suivant la loi :

$$R_{\text{eff}} = \frac{R}{1 + \gamma T_e/T_1}$$

où γ est un paramètre voisin de 1.

3. Problème de la Stabilité

L'étude de la stabilité d'une bouteille magnétique peut être abordée dans le cadre de l'approximation magnétohydrodynamique, en particulier, à l'aide du principe d'énergie.

Ce principe conduit à l'instabilité d'un plasma sans champ, maintenu par la pression magnétique, si en un seul point, la concavité est tournée vers l'intérieur. Mais il permet aussi de

montrer la stabilité de certaines bouteilles, à concavité vers le plasma, dans lesquelles les lignes de champ magnétique sont terminées par deux plaques métalliques. Ces bouteilles sont définies par les trois paramètres : rapport de miroir, rapport des pressions du plasma et du champ magnétique, rapport du diamètre et de la longueur du plasma. (Si les instabilités se manifestent ultérieurement de façon gênante, une solution de pis aller serait l'utilisation d'une configuration du type Picket-Fence.)

L'expérience semble indiquer que le critère employé est trop sévère.

Rosenbluth et Rostoker ont calculé la variation d'énergie totale du système pour une perturbation électromagnétique donnée.

Ce calcul est effectué :

(1) à l'ordre 0 par rapport à r_g/λ

r_g = rayon de gyration des particules

λ = échelle du plasma

(2) en négligeant la perturbation du champ électrique parallèle aux lignes de force.

Ce calcul est une généralisation de l'étude des instabilités en flûte.

Un résultat important de ce calcul est que l'approximation magnétohydrodynamique est pessimiste du point de vue de la stabilité. Par contre, l'approximation CGL est optimiste.

Un progrès fondamental dans l'étude de la stabilité consisterait à réussir ce calcul à l'ordre >0 par rapport à r_g/λ et en tenant compte du champ électrique longitudinal.

Mais d'autres types d'instabilité peuvent exister.

L'instabilité due à l'anisotropie de la pression ne serait dangereuse d'après une étude de Sagdeyev, que pour un rapport β entre l'énergie du plasma et l'énergie magnétique assez élevée $\beta \approx P_{\parallel}/P_{\perp}$.

L'existence locale de courants contraires d'ions pourrait être aussi une cause d'instabilité qui se produirait lorsque la vitesse du faisceau est supérieure à la vitesse thermique des ions.

4. Le Remplissage en Plasma

Les méthodes proposées jusqu'à présent sont les suivantes :

4.1. IONISATION DU GAZ RÉSIDUEL

Cette ionisation résulte d'une décharge provoquée soit par une source auxiliaire (continue ou H.F.) soit par le champ électrique résultant de l'application rapide du champ magnétique de confinement.

Cette ionisation conduit, en général, à un plasma dont la température ne dépasse pas quelques eV avec $T_e > T_i$.

4.2. INJECTION DE PARTICULES RAPIDES

Cette méthode crée le plasma à une température plus ou moins élevée suivant l'énergie des particules injectées. Dans le cas de l'injection d'ions, il importe d'assurer à chaque instant la neutralité électrique du milieu par un moyen approprié.

Le tableau No. 1 résume les divers procédés de capture qui sont énumérés ci-après :

(a) Montée rapide du champ magnétique pendant le transit. Cette méthode s'applique aux ions et au plasma. Elle est essentiellement limitée dans le temps.

(b) Variation de la position des miroirs. Cette méthode s'applique également aux ions et au plasma. Elle est limitée dans le temps mais susceptible d'être répétée avec accumulation. Elle peut servir au transfert du plasma.

(c) Interaction collective de nuages de plasma. Cette méthode est limitée dans le temps mais

permet de réaliser une intensité instantanée énorme, en outre, elle est susceptible d'être répétée avec accumulation.

(d) Augmentation de la vitesse transversale des particules par un champ électrique de fréquence égale à la fréquence cyclotron. S'applique à des ions (ou à des électrons). C'est un procédé continu dans le temps.

(e) Modification du rapport e/m des particules: dissociations d'ions moléculaires ou ionisation de molécules neutres. C'est un procédé continu.

(f) Collisions élastiques individuelles des particules injectées sur le plasma. Cette méthode peut s'appliquer, soit par injection continue de particules sur un plasma déjà existant, soit par injection brutale d'un plasma suffisamment dense pour interagir sur lui-même.

Concept de courant critique

L'existence d'une pression de gaz résiduelle au début et l'importance des phénomènes d'échange de charge imposent dans toute procédure d'injection la nécessité d'un courant minimum critique. Ce courant critique a été calculé dans 3 variétés du procédé de capture (e).

Les résultats sont :

$$I_{\text{dissocié}} = \frac{1}{4} e \frac{v_0 N_0 S}{\sigma_{+0} + \sigma_i}$$

TABLEAU 1
Méthodes de capture

	Variation de H	Miroirs mobiles	Interaction collective	Résonance cyclotronique	Variation l/m dissociation ou ionisation	Collisions élastiques individuelles
Limité dans le temps	i e p					
Limité dans le temps mais répétable avec accumulation		i e p	p			p
Continu				i ou e	i n	i e

i = ions; e = électrons; n = neutres; p = nuage de plasma.

dans le cas du DCX

$$I_{\text{moléculaire}} = \frac{1}{4} e \frac{V}{l^2} \frac{\sigma_{+0} v}{\sigma_d^2}$$

dans le cas de OGRA

$$I_{\text{neutre}} = \frac{N_0 v \sigma_{+0} v}{2 R \sigma_{0+}}$$

dans le cas de l'injection de neutres rapides

L'application numérique de ces formules révèle de sérieuses difficultés de réalisation pratiques qui augmentent dans l'ordre des cas cités.

5. Le Chauffage

Si le plasma formé après injection à une température insuffisante, les méthodes de chauffage proposées jusqu'à présent sont les suivantes:

(a) Injection de particules rapides cédant de l'énergie au plasma.

(b) Compression, soit par augmentation du champ (compression transversale), soit par rapprochement des miroirs (compression longitudinale).

Il existe des expressions simples montrant les conditions que la compression doit remplir pour rester adiabatique et pour que les fuites restent faibles au cours de son application.

Il serait souhaitable de réaliser finalement un plasma avec $T_i > T_e$. On ne peut aller très loin dans cette voie puisque le temps d'échange d'énergie entre les ions et les électrons peut s'évaluer par l'expression:

$$t_D \left(\frac{v_r}{v_i} \right)^3 \frac{m_e}{m_i}$$

Ce temps est égal à t_D si $T_i/T_e = 15$, de sorte qu'une valeur beaucoup plus grande de ce rapport, serait abaissée en un temps court vis-à-vis de la durée de confinement des ions dans la bouteille.

ETAT DES EXPERIENCES

6. Machines à Compression

Les principales machines connues sont énumérées dans le tableau No. 2, à l'exception de "Albedo" qui est abandonné et de "Felix" qui est encore dans un état préliminaire, la méthode d'injection adoptée utilise, dans tous les cas, des sources de plasma à titane, la capture étant

assurée par les effets de collision puis par la croissance du champ. Le chauffage est assuré par compression.

Dans ces conditions, on annonce des densités allant jusqu'à 10^{14} , et, sous certaines réserves, des températures électroniques de 20 keV et ioniques de 2 à 3 keV.

Le temps de confinement du plasma est un bon indice de l'absence d'instabilités.

La pureté est suffisante (dans des conditions non précisées), pour que les pertes par rayonnement, ne soient pas catastrophiques.

Les auteurs des expériences n'ont donné aucune raison expliquant pourquoi $T_i < T_e$. Nous pensons que cela résulte de l'état initial du plasma, de sorte que le chauffage par compression adiabatique s'applique à un plasma ayant dès l'origine $T_i < T_e$.

Il est remarquable de constater que le plasma obtenu soit propre, malgré l'usage de canons à titane. Il faut en conclure que les mécanismes de purification sont très efficaces.

7. Machines sans Compression

Les principales caractéristiques de ces machines sont rappelées dans le tableau No. 3.

Les deux machines à injection d'ions moléculaires en régime continu DCX et OGRA, sont encore trop récentes pour avoir donné des résultats expérimentaux définitifs.

Actuellement, le principal obstacle à surmonter consiste à injecter avec une intensité suffisante pour dominer les phénomènes de pertes dues au gaz résiduel.

De plus, la grande dimension des rayons de gyration aggrave dans ces machines le problème des pertes. On espère qu'il sera possible d'obtenir un plasma d'une densité atteignant 10^{12} à 10^{13} part/cm³ avec une température ionique pour laquelle les estimations s'échelonnent de 10 à 200 keV. Ces performances devraient pouvoir être grandement améliorées par une compression ultérieure.

8. Les Magnétrons à Ions

Ce sont des dispositifs à miroirs statiques utilisant un injecteur interne dont les propriétés

TABLE
L
Machines à Miroir réalisées

Nom	Références	Dimensions l = longueur en cm $\varnothing$ = diamètre du tube en cm	Champ maximum au miroir $h I'$	*Rapport de miroirs (B_1/B_0)	Source de courant	Constante de temp. du champ en seconde montée-décroissance
Albedo	G-381†	$l = 300$ $\varnothing = 15$	3		courant continu	∞
Cucumber	G-377	$\varnothing = 45$	6	300	courant continu	∞
TOY-TOP	Project Sherwood	$l = 30$ $\varnothing = 2.5$	200		50 kJ	
3 Stage	Project	$\varnothing = 45$	3			
TOY-TOP	Sherwood	à $\varnothing = 7.5$	à 100			
Table TOP	G-378	$\varnothing = 45$ $l = 110$	30	2	200 kJ	5×10^{-4} 10^{-2}
	G-381	$\varnothing = 5$ $l = 15$	114	2	320 kJ	5×10^{-5} 3×10^{-3}
		$\varnothing = 15$ $l = 80$	44	1.5	1 MJ	5×10^{-4} 3×10^{-2}
Saturn	G-381	$\varnothing = 46$ $l = 90$	8			$60 \mu s$ $50 \mu s$
Félix	G-381	$\varnothing = 45$	continu	1 à 3	courant	∞
	G-379	$l = 270$	pulsé 5 24		continu ou 10^6 J	ou 4.7×10^{-3} 0.2

† G-381 signifie: Communication à la Conférence de Geneve No. P/381.

TABLEAU

Nom	Reference	Laboratoire	l longueur entre miroirs $\varnothing$ diamètre cm	B_0 au centre kG	Rapport de miroir (B_1/B_0)	Source de courant Puissance MW	Constante de temps
D C X	Geneve P/344	Oak Ridge U.S.A.	$l = 90$ $\varnothing = 45$	10	2	groupe tournant 3.5 installé	∞
OGRA	Geneve P/2298	Russie	$l = 1200$ $\varnothing = 140$	5	1.6	2.7 dans bobine groupe tournant 20 installé	∞
MAGNETRON À IONS		UCRL U.S.A.	$l = 100$ $\varnothing = 20$	14	1.4	4 dans bobine 1000 A 180 V	∞
MAGNETRON À IONS	Geneve P/2298	Russie	$l = 200$ $\varnothing = 70$	12.5	1.5		∞
HOMOPO- LAIRE II	Geneve 1064 et 373	UCRL U.S.A.	$l = 7$ $\varnothing_1 = 7.5$ $\varnothing_2 = 24.5$	10 à 20	2.5	Condensateurs 5×10^4 J	
IXION 3	Geneve 1860 et 2363	Los Alamos U.S.A.	$l = 86$ $\varnothing_1 = 3$ $\varnothing_2 = 24$	10 à 20	1.2 à 1.5	Condensateurs 3×10^3 J	

2

PYROTRONS

laboratoire de Livermore (U.S.A.)

Injection	Pression mm de mercure	Densité du plasma n_i	Remarques
capture des particules par un champ HF source à titane capture par collisions	Vide poussé		Abandonné Étude des pertes de particules et de la stabilité pour une valeur élevée de $\beta = \frac{8\pi nkT}{B^2}$ Rapport de compression élevé
3 sources à titane capture par collisions transfert			Compression très élevée température ionique estimée à 5×10^6 °K
sources à titane capture par collisions	2×10^{-7}	$n = 10^{14}$	
capture par collisions		$n > 10^{14}$	100 sources à titane $T_e = 1$ à 6 keV
source d'ion pulsée capture par montée du champ	10^{-9} – 10^{-10}		La source d'ions fournit jusqu'à 0.1 ampère sous une tension de 3 à 15 keV. La neutralisation de la charge d'espace n'est pas assurée

3

Champ électrique radial V/cm	Electrode centrale	Injection	Pression mm de mercure	Densité du plasma p/cm ³	Remarques
		Environ 1 mA–600 keV ions moléculaires dissocié sur arc au carbone 1A–200 keV ions moléculaires dissocié sur gaz résiduel	3×10^{-8} 10^{-8}	10^{12} à 10^{13} espéré 10^{12} à 10^{13} espéré	Courant critique non atteint jusqu'à présent Aucune information sur des résultats expérimentaux
1000	+	Colonne de plasma axiale de densité 10^{13} /cm ³ énergie des ions accélérés 10 keV	5×10^{-7} avant opération 2×10^{-4} pendant opération	10^{12}	
600 à 1200	+	Colonne de plasma axiale de densité 10^{14} /cm ³ énergie des ions accélérés 20 à 40 keV	10^{-7}	10^{13}	Emission de neutrons observée
500 à 1250	—	Ionisation du gaz dans la machine	10^{-4}	10^{13}	
50 à 2000	—	Canon de plasma à piston magnétique	2×10^{-5}	2×10^{13}	Emission de neutrons observée

particulières justifient l'intérêt. On réalise ainsi une injection à forte intensité (ordre de grandeur 1 ampère) de particules ayant une énergie de 10 à 40 keV, dont le temps de survie atteint quelques millisecondes.

Le plasma obtenu a une densité d'environ 10^{13} p/cm³, mais nous n'avons pas assez de renseignements sur sa composition exacte pour porter un jugement sur l'avenir de ces machines.

9. Les Machines à Plasma Tournant

Ce sont des machines à miroir dans lesquelles l'action d'un champ électrique perpendiculaire aux lignes de force magnétiques entraîne le plasma dans un mouvement de rotation.

La justification du procédé réside dans le fait que la force centrifuge peut réduire les pertes aux extrémités du miroir, en outre, le frottement visqueux entre les couches de plasma constitue un moyen de chauffage supplémentaire.

Malheureusement, dans toutes les machines réalisées jusqu'à présent, l'existence des électrodes nécessaires à la mise en rotation implique une pollution et un refroidissement du plasma. En outre, la force centrifuge diminue le confinement transversal et son influence sur la stabilité n'a pas été étudiée.

Les deux appareils réalisés à Berkeley n'ont aucune prétention thermonucléaire, mais ils ont bien mis en évidence un mouvement de rotation correspondant à une vitesse de 6×10^6 cm/s.

La machine IXION a permis de produire des neutrons par un mécanisme qui n'est pas expliqué.

Des progrès sont à espérer par accroissement de la vitesse de rotation et par suppression de la gaine électronique responsable d'un retard initial au démarrage.

PROBLEMES THEORIQUES RESTANT A RESOUDRE

Aucun des principaux problèmes théoriques posés par les machines à miroir n'a encore obtenu une solution pouvant être considérée comme définitive.

Parmi les effets ayant certainement une

influence sur les fuites ou sur la stabilité et dont on n'a pas encore tenu compte, mentionnons:

Effets non linéaires résultant de perturbations non petites.

Présence d'un plasma dont la pression ne soit pas négligeable vis-à-vis de la pression magnétique.

Dimensions finies des rayons de gyration. Présence de conducteurs métalliques au voisinage du plasma.

Répartition du champ électrique imposé par les phénomènes ambipolaires, tenant compte à la fois des pertes par les miroirs et de la diffusion transversale.

Mentionnons également l'intérêt qu'il pourrait y avoir à étudier les points suivants:

Théorie du miroir avec "Shear".

Possibilités du chauffage par pompage magnétique.

ORIENTATION DES EXPERIENCES

Le principal reproche que l'on puisse faire aux dispositifs à compression (Pyrotrons) réside dans le rapport des températures ioniques et électroniques qui est l'inverse de celui que l'on doit souhaiter. Inversement, les machines sans compression souffrent surtout d'un défaut de confinement et d'une densité faible.

Cette situation doit pouvoir s'améliorer par des combinaisons judicieuses des procédés énumérés.

Parmi les problèmes techniques à résoudre pour y arriver, citons:

1. Mise au point d'une injection d'ions rapides.

2. Mise au point d'une injection de plasma rapide.

3. Utilisation d'un chauffage par pompage magnétique.

4. Utilisation de la technique des champs magnétiques très intenses.

5. Mise au point de la technique des vides ultra-poussés.

6. Etude d'un mode de remplissage effectuant la capture de particules rapides par un plasma froid, peu dense, mais complètement ionisés.

7. Mise au point de procédés de neutralisation des charges d'espace.

Liste des symboles utilisés

μ :	moment magnétique	r_1 :	rayon de gyration des ions
E_1 :	énergie des ions exprimée en keV	v_0 :	vitesse moyenne des molécules du gaz résiduel
n :	nombre de particules par cm^3	N_0 :	densité du gaz résiduel
v_e :	vitesse des électrons	S :	aire de la surface limitant le plasma
v_i :	vitesse des ions	l :	longueur de trajectoire entre injection et retour à l'injecteur
v_r :	vitesse relative des électrons par rapport aux ions	V :	volume de l'espace contenant le plasma
m_e :	masse des électrons	R :	rayon du cylindre de plasma
m_i :	masse des ions	v :	vitesse des particules injectées
B_0 :	intensité du champ magnétique au centre du système	σ_{+0}, σ_{0+} et σ_1	sections efficaces de chocs respectivement de capture et perte d'électrons des particules injectées et d'ionisation du gaz résiduel.
$B_{\max}$:	intensité du champ magnétique au centre d'un miroir		
$R = \frac{B_{\max}}{B_0}$:	rapport du miroir		
T_e :	température des électrons		
T_i :	température des ions		
L :	longueur caractérisant les dimensions transversales de la configuration		Approximation CGL: voir Communication de Geneve No. 349.

6b. LES MACHINES À MIROIRS MAGNÉTIQUES

M. BINEAU, T. CONSOLI, CH. MAISONNIER, F. PRÉVOT et P. RICATEAU

Centre d'Etudes Nucléaires de Saclay, Gif-sur-Yvette, (S & O)

The aim of this report is to summarize our present knowledge of magnetic mirror machines without compression. Compiled to establish a working program, it gives detailed comparisons between different lines of approach. A critical survey of experimental and theoretical results is attempted.

The first part deals with confinement and stability problems in mirror machine geometries. The second part is devoted to injection of particles. In the third part different types of mirror machine devices are briefly described together with a discussion of experimental results.

Introduction

Ce rapport présente un résumé de nos connaissances sur les machines à miroirs magnétiques sans compression, telles qu'elles apparaissent d'après la littérature, à la suite de la Conférence de Genève de septembre 1958. Il a été rédigé dans le but de permettre la comparaison avec d'autres types de machines et d'établir un programme de travail. Dans cet esprit, les démonstrations ainsi que tout ce qui nous a semblé détail a été systématiquement omis. Par contre, les hypothèses ont été dégagées et les résultats théoriques ou expérimentaux sont donnés avec une tentative de critique.

Le lecteur trouvera dans la première partie ce qui a trait au confinement et à la stabilité dans ce type de configuration. La seconde partie est consacrée à l'injection des particules. Dans une troisième partie, les machines appartenant à cette catégorie sont sommairement décrites ainsi que les résultats expérimentaux.

Confinement et Stabilité

1. Confinement

L'étude de la trajectoire des ions dans une bouteille magnétique, ne représente que l'aspect individuel du confinement. Cet aspect très utile en ce qui concerne l'injection, n'est suffisant que dans la mesure où tous les gaz, gaz de neutre, gaz d'électrons et, bien entendu, le gaz injecté lui-même sont complètement négligés.

Dans le cas de l'injection d'un faisceau d'ion, cas le plus souvent envisagé dans cette étude, le gaz de neutre conduit à des pertes de particules par échanges de charge. Son action ne peut être négligée dans les machines à injection limitée dans le temps que si le nombre d'ions injecté est très supérieur au nombre de molécules de gaz présent au moment de l'injection, car les sections efficaces d'échange de charge ou d'ionisation sont très généralement supérieures à la section efficace de réaction DD ou TD. Dans le cas des machines à injection continue, le gaz de neutre ne sera négligé que si le flux injecté, dépasse une certaine fraction du flux de neutre rentrant dans la région du plasma. Cette action du gaz neutre est étudiée par ailleurs.

Si l'on excepte ces pertes sur les neutres, le confinement d'une particule déposée sur une trajectoire confinée pourra encore être détruit par deux grandes catégories de perturbations: des perturbations dues aux interactions coulombiennes soit avec le plasma complètement neutralisé soit avec les ions positifs injectés en excédent et des perturbations dues à des regroupements de particules de même espèce dans l'espace des phases faisant apparaître des champs électriques ou des courants macroscopiques.

Les premières seront abordées dans cette étude sous le nom de confinement, les deuxièmes sous le nom de stabilité.

¹⁾ R. F. Post, Summary of U.C.R.L. Pyrotron Program, Genève No. 377.

1.1. CLASSES DE TRAJECTOIRES - CÔNE DE PERTE

L'analyse de ces pertes nous amène à redéfinir ce qu'on entend par trajectoire confinée et non confinée. La topographie de la bouteille étant de révolution il est possible de montrer que toute particule se déplace dans un champ de potentiel qui lui est propre et qui est défini par ses deux invariants: vitesse et moment angulaire canonique. Les équipotentielles sont allongées parallèlement à l'axe. Partant du minimum de potentiel et allant vers les potentiels croissants on trouve d'abord des équipotentielles fermées puis une équipotentielle limite qui aboutit au plan des miroirs, enfin des équipotentielles ouvertes. Le miroir représente donc un col de potentiel. Les particules pourront être perdues, soit si leur énergie dépasse le potentiel du col, soit si leur équipotentielle limite traverse les parois latérales de l'enceinte.

Mais une particule bornée par une équipotentielle ouverte ou par une équipotentielle traversant les parois ne sera perdue que si sa trajectoire l'amène vers les hauts potentiels précisément dans la région des miroirs ou dans la région des parois. S'il n'en est pas ainsi, si sa trajectoire évite en fait ces régions dangereuses, on pourra dire que cette trajectoire est relativement confinée par opposition aux trajectoires absolument confinées bornées par des équipotentielles fermées et ne touchant pas les parois.

Cette méthode de classement des trajectoires quoique parfaitement correcte et utile pour exposer le problème des pertes n'a pu conduire pour le moment à aucun critère général de confinement. Un critère de ce genre ne peut être tiré que d'une approximation. L'approximation à laquelle il est fait appel est l'approximation adiabatique bien connue valable cette fois-ci pour un champ quelconque qu'il soit ou non de révolution. Cette approximation indique que sous réserve que le champ ne varie pas trop au cours d'une rotation cyclotron de l'ion, le potentiel analysé plus haut prend la forme

$$\frac{1}{2} mV^2 - \mu B$$

où V et μ représentent les conditions initiales (au lieu de: vitesse et moment angulaire canonique)

et B la valeur du champ. Dans ces conditions la forme des équipotentielles est devenue indépendante des données initiales. En contrepartie les équipotentielles orthogonales aux lignes de champ ne sont plus fermées et ne barrent que la région des miroirs. La région du plasma n'est plus fermée latéralement par des potentiels croissants mais par le fait que les trajectoires réelles s'inscrivent sur des tubes de flux, c'est un confinement relatif.

Le critère général de confinement dans l'hypothèse de l'invariance de μ est bien connu et se déduit immédiatement de la forme du potentiel:

$$\mu B_{\max} = \mu B \frac{B_{\max}}{B} \geq \frac{1}{2} mv^2$$

soit:

$$\sin \theta > \sqrt{\frac{B}{B_{\max}}}.$$

Dans l'espace des vitesses cette condition définit un cône dont l'ouverture est θ . Si la bouteille est clairement divisée en une grande étendue centrale à champ constant B_0 et en une région des miroirs où le champ monte à la valeur $B_{\max}$, on peut admettre que la quasi totalité des particules se trouve dans la région B_0 où le cône a une ouverture fixe, ceci constitue une deuxième hypothèse. Enfin, si le temps de transit le long de l'axe de la bouteille est inférieur au temps moyen de collision, on peut dire qu'une particule placée par choc à l'intérieur du cône est perdue. Ceci justifie le terme de cône de perte et permet de poser les bases du calcul des particules perdues aux miroirs.

Une étude particulière de A. Garren, R. J. Riddel²⁾ permet d'amorcer dans le cas des bouteilles une discussion sur la validité de la première des hypothèses, celle de l'invariant adiabatique. Sans entrer dans le détail de cette étude, en grande partie numérique, on peut dire qu'elle met en évidence le rôle important du paramètre

$$"V" = \frac{t_c}{t_1}$$

²⁾ A. Garren et R. J. Riddel, Individual particle motion and the effect of scattering in an axially symmetric magnetic field, Genève No. 383.

où t_c est la période cyclotron au centre et t_1 le temps de transit l/V pour une vitesse purement axiale.

Pour une série étendue de données initiales et pour une forme mathématiquement simple de champ l'hypothèse adiabatique reste légitime tant que :

$$|\nabla| \leq 0.2.$$

Au delà de cette limite le cône de perte tout en étant peu affecté par les valeurs du moment angulaire perd sa forme circulaire et son volume croît rapidement.

1.2. PERTES AUX MIROIRS

Ces pertes ont été étudiées par S. Persico et J. G. Linhart³⁾, dans l'hypothèse d'un grand rapport des miroirs (c'est-à-dire lorsque le cône de perte est petit) en assimilant les particules dans l'espace des vitesses à un gaz isotrope et en ne retenant que les collisions aux grands angles.

Le temps caractéristique de perte est alors donné par :

$$\frac{t_p}{t_D} = 2R$$

où t_D est le temps moyen de collision et

$$R = \frac{B_{\max}}{B_0}.$$

D. Judd, W. McDonald et M. Rosenbluth⁴⁾ ont analysé la même question à l'aide de l'équation de Fokker-Planck en supposant seulement, en dehors des hypothèses relatives au cône de perte, que la fonction de distribution se décompose en un facteur de la vitesse et du temps, et en facteur de l'angle.

Dans ces conditions le temps caractéristique s'écrit :

$$\frac{t_p}{t_D} = 3 K \log_{10} R$$

où K est un facteur de forme de la fonction de distribution :

$$K = \frac{1}{\bar{V}_0^3} \cdot \left(\frac{1}{\bar{V}} \right) \left(\frac{1}{\bar{V}^2} \right).$$

Des précisions intéressantes sur la forme de

cette distribution tirée de la même équation par la même méthode sont données dans l'article de A. Garren⁵⁾. Comme on peut s'y attendre cette distribution est disymétrique et peu fournie du côté des basses vitesses.

1.3. DIFFUSION AMBIPOLAIRE

Le taux de diffusion aux miroirs d'ions et d'électrons, de température donnée n'étant pas le même, un champ électrique se crée qui rétablit l'égalité de ces pertes. Kaufman⁶⁾ a montré que si la température des électrons est petite par rapport à celle des ions, cet effet était équivalent à une réduction du rapport des miroirs. La nouvelle valeur de ce rapport peut être calculée, elle s'écrit :

$$R_{\text{eff}} = \frac{R}{1 + \gamma \frac{T_e}{T_i}}$$

où γ est un coefficient voisin de 1. Dans le cas d'injection d'ions qui est le plus général dans les machines à miroir cette correction reste petite.

1.4. PERTES SUR LES PAROIS OU SUR L'INJECTEUR

Une série de collision sur les particules peuvent placer ces particules sur des trajectoires touchant les parois ou l'injecteur. Pour connaître ces pertes par diffusion en travers du champ magnétique il faut donc tenir compte de la distribution dans l'espace.

Le problème, à notre connaissance n'a pas été traité dans le cas des bouteilles. On peut se borner à remarquer que, le voisinage des parois refroidissant le gaz, les fuites aux miroirs seront proportionnellement plus importantes près des parois qu'au centre, la densité sera donc beaucoup plus faible.

Ces pertes sont généralement négligées dans

³⁾ E. Persico et J. G. Linhart, Plasma loss from magnetic bottles, Nuovo Cimento 8 (1958) 1668.

⁴⁾ D. Judd, W. McDonald et M. Rosenbluth, End leakage losses from the mirror machine, U.C.R.L. Report 2931.

⁵⁾ A. N. Kaufman, Plasma diffusion in a magnetic field, TID 7536.

⁶⁾ A. N. Kaufman, Ambipolar effects in mirror losses, TID 7520.

les machines à compression en raison de l'éloignement des parois.

Dans le cas des machines continues et tout particulièrement dans le cas du DCX les pertes d'énergie qu'elles provoquent ne sont certainement pas négligeables. Le calcul dans ce cas pratique n'a été fait qu'avec une approximation très grossière⁷⁾.

2. Stabilité

Après une revue des principales connaissances théoriques sur le problème de la stabilité nous essayerons d'indiquer l'application ou plus généralement la difficulté d'application de ces connaissances aux bouteilles magnétiques.

2.1. ÉTUDES THÉORIQUES DE LA STABILITÉ

Nous allons présenter succinctement les méthodes et résultats qui ont été développés jusqu'à présent et qui permettent d'aborder une étude de la stabilité. Les sujets choisis sont :

1. un principe d'énergie pour les problèmes de la stabilité hydromagnétique, proposé par Bernstein, Frieman, Kruskal et Kulsrud⁸⁾ ;
2. l'instabilité en cannelures (flute instability) d'après l'étude de Rosenbluth et Longmire⁹⁾ ;
3. des problèmes de stabilité magnétohydrodynamique traités par Berkowitz, Grad et Rubin¹⁰⁾.

A. Le principe d'énergie pour les problèmes de stabilité hydromagnétique

L'étude de Bernstein, Frieman, Kruskal et Kulsrud, s'applique aux équilibres hydromagnétiques dans lesquels la vitesse du fluide est nulle en chaque point. Le principe d'énergie procède d'une formulation variationnelle des équations du mouvement. Utilisant le fait qu'on cherche seulement un critère de stabilité

⁷⁾ C. F. Barnett et P. R. Bell, The Oak Ridge thermonuclear experiment, Genève P/344.

⁸⁾ Un principe d'énergie pour les problèmes de stabilité hydromagnétique par Bernstein, Frieman, Kruskal et Kulsrud, NYO 7315.

⁹⁾ Stabilité de plasmas confinés par des champs magnétiques par Rosenbluth et Longmire, Annals of Physics, 1, 2 (1957).

¹⁰⁾ Problèmes en stabilité magnétohydrodynamique par Berkowitz, Grad et Rubin, Conférence de Genève, P/376.

et non les taux de variation on obtient une formulation plus simple que celle du principe de Rayleigh.

Les hypothèses qui permettent cette étude sont essentiellement celles par lesquelles la force $\mathbf{F}(\xi)$ au premier ordre, associée au déplacement ξ est donnée par une expression qui contient ξ et non $\dot{\xi}$. Ces hypothèses sont les suivantes :

1. Les termes quadratiques en $\mathbf{v}$ et $\mathbf{j}$ sont négligeables. Ceci a lieu si la vitesse macroscopique est petite par rapport à la vitesse du son

$$\sqrt{\gamma p / \rho}$$

et à la vitesse de Alfvén $B/\sqrt{\rho}$.

2. Le courant de déplacement est négligeable, $v \ll c$ (vitesse de la lumière).

3. La loi d'Ohm peut s'écrire :

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = \frac{M}{e} \nabla \Phi$$

ou même :

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = \frac{M}{ep} \nabla \cdot P_1 + \frac{M}{e} \nabla \Phi$$

où P_1 est le tenseur de pression des ions. Ceci a lieu si :

le terme d'inertie des électrons est négligeable, t_e étant un temps caractéristique, pour

$$t_e^{-1} \ll \omega_p$$

le terme d'inertie des ions est négligeable

$$t_e^{-1} \ll eB/m$$

le terme de perte électrique η_j est négligeable ; temps de diffusion relative de la matière et du champ magnétique $\gg t_e$.

L'équation du mouvement s'écrit alors :

$$\rho \ddot{\xi} = \mathbf{F}(\xi)$$

avec

$$\mathbf{F}(\xi) = \nabla [\gamma p \nabla \cdot \xi + \xi \cdot \nabla p] + \mathbf{j} \times \mathbf{Q} - \mathbf{B} \times (\nabla \times \mathbf{Q}) + \nabla \cdot (\rho \xi) \nabla \Phi$$

où

$$\xi = \mathbf{r} - \mathbf{r}_0$$

(déplacement à partir d'une position d'équilibre).

$$\mathbf{Q} = \nabla \times (\xi \times \mathbf{B})$$

et la variation seconde de l'énergie potentielle est :

$$\delta W = -\frac{1}{2} \int \xi \cdot \mathbf{F}(\xi) dV.$$

Le théorème établi est le suivant :

La condition nécessaire et suffisante d'instabilité est qu'il existe un déplacement ξ qui rende δW négatif.

Applications du principe d'énergie

1. Si on considère un plasma sans champ magnétique et à pression constante entouré d'un champ magnétique dans le vide, si la concavité des lignes de force magnétique est tournée en un point vers le plasma, l'état est instable vis-à-vis des déplacements au voisinage de ce point.

2. On peut obtenir des critères de stabilité dans les deux cas suivants :

(a) cas du pinch où un courant longitudinal crée un champ magnétique radial qui supporte la pression de plasma,

(b) la pression de plasma est supportée par un champ magnétique à composantes longitudinale et radiale créées par un courant azimutal.

3. On peut appliquer le principe d'énergie à l'établissement d'un critère de stabilité pour un plasma avec des courants de volume. Calculons la variation seconde de l'énergie pour une perturbation limitée à un volume R et pour un fluide incompressible

$$\delta W = -\frac{1}{2} \int_R \xi \cdot F(\xi) dV.$$

Pour

$$\nabla \cdot \xi = 0$$

$$F(\xi) = \nabla[\xi \cdot \nabla p] + \mathbf{J} \times \mathbf{Q} - \mathbf{B} \times (\nabla \times \mathbf{Q})$$

$$\begin{aligned} \xi \cdot \nabla(\xi \cdot \nabla p) &= \nabla \cdot [(\xi \cdot \nabla p) \xi] - (\xi \cdot \nabla p) \nabla \cdot \xi = \\ &= \nabla \cdot [(\xi \cdot \nabla p) \xi] \end{aligned}$$

dont l'intégrale dans le volume R est nulle

$$\xi \cdot \mathbf{B} \times (\nabla \times \mathbf{Q}) =$$

$$= \nabla \cdot [\mathbf{Q} \times (\xi \times \mathbf{B})] + \mathbf{Q} \cdot \nabla \times (\xi \times \mathbf{B})$$

Le premier terme a une intégrale nulle et le deuxième est égal à Q^2 , or,

$$\delta \mathbf{B} = \nabla \times (\xi \times \mathbf{B}) = \mathbf{Q}$$

et

$$\delta W = \frac{1}{2} \int_R [(\delta \mathbf{B})^2 + \delta \mathbf{B} \cdot \mathbf{J} \cdot \xi] dV$$

une application de cette formule est donnée dans l'étude de Berkowitz, Grad et Rubin dont il est parlé plus loin.

B. Résultats de Rosenbluth et Longmire (Annals of Physics 1 (1957) 120, 140.

L'étude de Rosenbluth et Longmire est basée sur le principe d'énergie. L'énergie potentielle du système est la somme de l'énergie du champ magnétique :

$$B^2/2\mu$$

et de l'énergie interne du plasma :

$$pv/(\gamma - 1).$$

On considère un plasma infini, à symétrie axiale, périodique dans la direction de l'axe et confiné près de l'axe. On suppose la pression du plasma petite par rapport à la pression magnétique; une distorsion du champ magnétique accroît son énergie.

On pense donc que les perturbations dangereuses sont celles qui ne modifient pas le champ magnétique. Cette hypothèse conduit à la notion des cannelures. Ces cannelures sont construites de telle sorte que leur surface est bordée par des lignes de champ magnétique non perturbé. L'effet des cannelures est d'interchanger le champ et la matière entre deux tubes de flux voisins.

Pour que l'énergie magnétique ne soit pas changée les deux tubes voisins sont traversés par des flux égaux.

Le changement d'énergie du plasma peut se calculer de deux manières :

1. En calculant :

$$\delta \int \frac{pv}{\gamma - 1} dv.$$

En supposant le mouvement adiabatique on trouve que cette variation a le même signe que :

$$-\delta \int \frac{dl}{B}$$

et la condition de stabilité s'écrit :

$$\delta \int \frac{dl}{B} < 0 \quad \text{ou} \quad \int \frac{dl}{RrB^2} > 0$$

où R est le rayon de courbure de la ligne de force, positif si le centre de courbure est extérieur au plasma, et r la distance à l'axe.

2. En calculant le changement d'énergie d'une particule liée à une ligne de force lorsque cette ligne de force prend la place d'une ligne de force voisine.

Ce calcul donne :

$$\delta W \approx \frac{\int \frac{2W - \mu B}{RrB \sqrt{W - \mu B}} dl}{\int \frac{dl}{\sqrt{W - \mu B}}}$$

où W est l'énergie de la particule et μ son moment adiabatique. Ce calcul montre que certaines particules contribuent plus que d'autres à l'instabilité, en particulier celles pour lesquelles R est toujours négatif.

Si l'on tient compte de toutes les particules situées sur la ligne de force, le résultat donne une variation d'énergie proportionnelle à :

$$\int dl \frac{p_{\parallel} + p_{\perp}}{RrB^2}.$$

Le calcul de changement d'énergie d'une particule repose sur l'existence de deux invariants adiabatiques

$$\mu = \frac{W_{\perp}}{B} \quad \text{et} \quad J = \int v_{\parallel} dl.$$

On peut donc préciser les hypothèses de validité :

1. Le rayon cyclotron doit être beaucoup plus petit qu'une longueur caractéristique,
2. Les perturbations que l'on a considéré sont seulement celles dont la longueur d'onde est beaucoup plus grande que le rayon de Larmor, la période est beaucoup plus grande que la période de la particule sur son orbite.

C. Théorèmes de Berkowitz, Grad et Rubin (Genève P/376)

Les problèmes de stabilité magnétohydrodynamique étudiés par ces auteurs concernent principalement les configurations dans lesquelles un fluide sans champ et parfaitement conducteur est séparé du champ électromagnétique dans le vide par une surface avec charges et courants.

La méthode employée permet de séparer le problème de la stabilité en un problème purement électromagnétique et un problème de fluide. Le premier est alors statique si on néglige les courants de déplacement et le deuxième est thermodynamique.

La stabilité peut être considérée selon trois points de vue :

1. Recherche du minimum d'un certain potentiel vis-à-vis des perturbations finies,
2. Etude de la variation seconde de l'énergie potentielle,
3. Méthode des modes normaux.

Les équations utilisées pour le milieu électromagnétique, dans lesquelles on a négligé la force électrique et le courant de déplacement, et les conditions aux limites sont telles que la solution du système d'équations est déterminée par la position et non par la vitesse de la surface. On peut donc non seulement obtenir la variation de l'énergie électromagnétique mais aussi avoir une expression du potentiel pour des perturbations finies.

Les paramètres qui déterminent complètement la solution sont un certain nombre de flux constants et un certain nombre de courants constants.

Séparation des potentiels

L'équilibre d'un fluide non dissipatif :

$$\frac{1}{\rho} \nabla p + \nabla \Phi = 0$$

peut aussi s'écrire en utilisant l'énergie libre de Gibbs par unité de masse :

$$F = e - T\eta + p\rho$$

(e énergie interne par unité de masse et η entropie par unité de masse)

$$F + \Phi = \text{constante}.$$

Si on suppose Φ défini dans tout l'espace on peut étendre la définition de la pression à l'équilibre à tout l'espace en utilisant la relation :

$$F + \Phi = \text{constante}.$$

Cette fonction :

$$p_0$$

permet d'écrire l'intégrale d'énergie sous une forme qui sépare un potentiel du fluide et un potentiel magnétique.

$$K + P_I + P_M = \text{constante}$$

$$\text{où: } K = \int \frac{1}{2} \rho u^2 dV$$

$$P_I = \int_{V_f} (\rho e + p\Phi + p_0) dV$$

$$P_M = \int_{V_m} \left(\frac{1}{2\mu} B^2 + p_0 \right) dV - \sum_{\alpha=1}^m I_{\alpha} \Phi_{\alpha}$$

où I_α sont les courants constants et Φ_α les flux associés.

Les variations premières de P_I et de P_M sont nulles et leurs variations secondes sont :

$$\delta^2 P_I = \int (\operatorname{div} \delta \mathbf{x})^2 c^2 dV \quad \text{où} \quad c^2 = \frac{\partial p(\eta, \rho)}{\partial \rho}$$

(pour un milieu incompressible $P_I = \text{constante}$)

$$\begin{aligned} \delta^2 P_M = & \int_{V_m} \frac{1}{\mu} (\delta B)^2 dV - \\ & - \int_S \left[\frac{\partial}{\partial n} \left(\frac{1}{2\mu} B^2 \right) + \rho \frac{\partial \Phi}{\partial n} \right] (\delta \mathbf{x}_m)^2 dS. \end{aligned}$$

Remarque: Dans le cas simple où $\Phi = 0$ on voit immédiatement que :

$$\delta P_I = 0$$

en effet :

$$P_I = \frac{pV}{\gamma - 1} + p_0 V$$

$pV\gamma = \text{constante}$ donne :

$$\delta p = -\frac{\gamma p_0}{V_0} \delta V \quad \text{d'où} \quad \frac{\delta pV}{\gamma - 1} = -p_0 \delta V.$$

Quelques théorèmes relatifs à la stabilité

Théorème 1: Une condition suffisante de stabilité, par le critère de la variation seconde, est que $Q(S) > 0$ pour tout S . Une condition nécessaire est $Q(S) \geq 0$ pour tout S ; c'est-à-dire qu'une condition suffisante d'instabilité est $Q(S) < 0$ en n'importe quel point S .

$$Q(S) = -\frac{\partial}{\partial n} \left(\frac{1}{2\mu} B^2 \right) - \rho \frac{\partial \Phi}{\partial n}.$$

Ce critère d'instabilité est purement local, il a les applications suivantes.

Théorème 2: Quand $\Phi = 0$, une condition suffisante de stabilité (variation seconde positive) est que le centre de courbure des lignes magnétiques sur la surface de séparation se trouve toujours dans le vide; une courbure inverse en un point quelconque est suffisant pour l'instabilité.

Théorème 3: Quand $\Phi = 0$ il n'existe pas de configuration bornée stable pour un fluide séparé du champ magnétique par une surface partout lisse.

Un critère de stabilité avec des courants de volume

Le caractère local du critère du théorème 1 est dû à l'absence de champ dans le fluide. Pour un mélange de champ et de fluide on peut montrer qu'une perturbation suffisamment localisée est toujours stable.

Le critère utilise les quantités :

$$L = \int \frac{ds}{B} \quad \text{et} \quad \frac{dB}{dx} (\geq 0)$$

$$\text{où:} \quad \left(\frac{dB}{dx} \right)^2$$

est la plus grande des valeurs propres de la matrice :

$$\frac{\partial B}{\partial x_i} \frac{\partial B}{\partial x_j}$$

J est la densité de courant.

Le théorème est le suivant :

Pour un fluide incompressible, une configuration d'équilibre est stable contre toutes les perturbations confinées à une région R qui ne contient pas de ligne B fermée, si J ne s'annule pas identiquement dans une sous région de volume non nul et si le long de chaque ligne B dans R l'inégalité :

$$L \operatorname{Max} \left\{ \mu |J| + \frac{dB}{dx} \right\} < \pi$$

est satisfaite.

Un théorème semblable existe pour un fluide compressible.

Ce théorème s'applique aux bouteilles magnétiques, l'inégalité utilisée dans le théorème permet de démontrer que la variation seconde de l'énergie potentielle est positive. C'est-à-dire que :

$$\int_R (\delta B)^2 dV + \mu \int_R \delta \mathbf{B} \times \mathbf{J} \cdot \delta \mathbf{x} dV > 0.$$

Les hypothèses sur lesquelles repose le théorème sont donc celles du principe d'énergie exposées précédemment.

Berkowitz, Grad et Rubin ont appliqué ce critère à l'étude de la stabilité d'une bouteille magnétique de révolution limitée par deux plaques conductrices.

La configuration d'équilibre choisie dérive d'un potentiel vecteur de la forme :

$$A_0 = \frac{1}{2} r (1 + \frac{1}{2} r^2) B_0 - \frac{ak}{\pi} \cos\left(\frac{\pi z}{a}\right) I_1\left(\frac{\pi r}{a}\right).$$

Les paramètres sont :

β = énergie du plasma/énergie magnétique,

R = rapport de miroir moyen, et l'indice de forme = diamètre dans le plan de symétrie/longueur axiale.

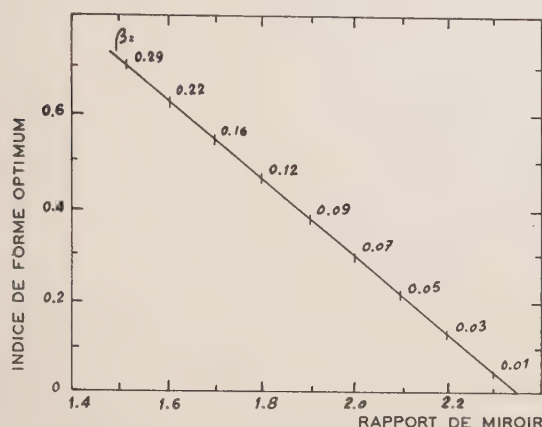


Fig. 1. Résultats graphiques de l'étude de la stabilité d'une bouteille limitée par deux plaques conductrices par Berkowitz, Grad et Rubin.

L'étude fait apparaître que "pour un indice de forme fixé, une augmentation du rapport de miroir moyen diminue β maximum pour lequel le critère de stabilité est satisfait. D'autre part, si le rapport de miroir est maintenu constant, il y a un indice de forme optimum pour obtenir le plus grand β compatible avec le critère de stabilité".

2.2. LA STABILITÉ DES BOUTEILLES MAGNÉTIQUES

Les théories précédentes ont certaines conclusions applicables aux bouteilles magnétiques. Cependant la confiance à accorder à ces résultats ne peut pas être grande à cause de l'absence de validité de certaines hypothèses. L'influence de l'anisotropie du plasma et de la présence des courants de Larmor a été en particulier envisagée par un groupe russe.

1. Conclusions des théories

L'intuition de Teller suivie d'une étude de Rosenbluth et Longmire a montré l'influence de la concavité du plasma sur la stabilité. Un

plasma à symétrie cylindrique et concavité tournée vers l'extérieur est instable. Ce théorème est démontré dans le cas d'un plasma sans champ magnétique, maintenu par la pression électromagnétique et dans le cas d'un plasma de faible rapport β entre l'énergie interne du plasma et l'énergie magnétique. Cependant on a besoin, dans ce dernier cas, d'un plasma infini pour construire une perturbation qui s'étende tout le long des lignes de forces.

Après que l'expérience eu montré la relative stabilité des bouteilles, l'étude de Berkowitz, Grad et Rubin a montré que l'application du même principe que celui de l'instabilité en cannelures conduit à l'existence de zones de stabilité dans le cas de bouteilles limitées par des plaques conductrices.

Ce résultat est d'un grand intérêt puisqu'il montre que le principe d'énergie n'exclue pas la stabilité d'une bouteille concave vers son centre. On ne peut pas néanmoins attribuer toute confiance à ce résultat à cause des hypothèses sur lesquelles il repose—Citons comme exemple, l'expérience "concombre" d'un plasma stable qui se situe loin de la zone de stabilité du diagramme de Berkowitz.

La machine Concombre, montée en 1954, avait pour but d'étudier le problème de la stabilité et de la diffusion au travers d'un champ magnétique. Pour avoir un β aussi grand que possible avec un plasma de température et de densité assez basses, que l'on sut faire, la pression magnétique fut aussi réduite que possible. Concombre avait un diamètre central de 45 cm, un champ de confinement de 20 gauss et des miroirs de 6000 gauss.

Un β de quelques pourcent et un rapport de miroirs de 300 donne un point du diagramme de Berkowitz, ... loin de la zone de stabilité. Or, le signal reçu sur une sonde montre un confinement de l'ordre de 1 à 2 ms. Et Post écrit¹¹⁾ "Un confinement stable et de longue durée du plasma fut observé dans des conditions où la pression du plasma fut estimée être plusieurs pourcent de la pression magnétique".

¹¹⁾ Aperçus sur le programme de pyrotrons à l'U.C.R.L. par Post, Conférence de Genève, P/377.

2. Raisons d'invalidité de la théorie

La raison la plus sérieuse semble être la dimension du rayon des orbites cyclotron par rapport aux dimensions du plasma. En effet, cette situation ne permet pas d'appliquer au plasma une loi thermodynamique comme par exemple, celle de l'adiabaticité,

$$\left[\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla \right] \left[\frac{p}{\rho^\gamma} \right] = 0$$

utilisée dans les différentes études; elle oblige à considérer le plasma dans son ensemble.

Les études dont nous avons parlé concernent uniquement les perturbations de fréquence basse, très inférieure à la fréquence cyclotron des ions. Un type d'instabilité à fréquence élevée est mis en évidence dans une étude dont nous parlerons plus loin.

Post¹¹⁾ résume de la manière suivante les difficultés que rencontre la théorie:

“(a) la nature finie d'un plasma réel dans une machine à miroirs (en opposition avec le plasma infiniment périodique de la théorie ancienne),

(b) l'existence de conducteurs ou presque-conducteurs aux extrémités de la machine,

(c) la nature non isotropique de la pression du plasma,

(d) la dimension non nulle des orbites des particules et peut être (e) l'existence d'effets ambipolaires contribuent tous à modifier le traitement de la stabilité”.

Nous avons vu que la théorie a pu considérer les points (a), (b), (c). Un critère simple d'instabilité par anisotropie sera en particulier indiqué plus loin.

Le physicien russe Kourchatov, souligne des points différents de ceux de Post de la manière suivante¹²⁾.

“Le plus grand danger est constitué par les instabilités liées aux interactions collectives des particules. Jusqu'à maintenant le problème de la micro-stabilité de plasma et du caractère de la diffusion dans un champ magnétique intense n'a pas trouvé de solution. Toute la théorie des pièges adiabatiques, ..., repose sur l'hypothèse des chocs binaires. Si des oscillations thermodynamiques non équilibrées prennent naissance

dans le plasma, la perte des particules à travers les miroirs magnétiques peut augmenter considérablement, comme l'a supposé Bohm, simultanément à un brusque accroissement de la diffusion transversale.

Le problème de la stabilité macroscopique a une grosse importance également pour les pièges adiabatiques. Bien qu'aucune tension électrique ne soit appliquée au plasma dans ce cas, il y a des courants qui le traversent. Ces courants qui ont reçu le nom de courants de Larmor, déterminent la stabilité du plasma. On ne peut affirmer, à priori que cet équilibre soit stable. Au contraire, dans nombre de travaux théoriques, on montre que dans certaines conditions l'équilibre du plasma devient instable. On a étudié attentivement, à l'Institut de l'Energie Atomique, l'instabilité du plasma de type Rosenbluth et Longmire. Un de nos théoriciens Sagdeev a découvert et a étudié un nouveau type d'instabilité, déterminée par l'anisotropie de la pression dans le plasma.”

3. Instabilité déterminée par l'anisotropie de la pression

L'influence de l'anisotropie de la pression peut être étudiée au moyen, soit, de considérations hydrodynamiques, soit de considérations cinétiques.

La première méthode a été employée en particulier par Bernstein *et al.*⁸⁾ qui ont calculé la variation d'énergie potentielle dans ce cas. Si l'on suppose qu'à l'équilibre qui précède la perturbation, la pression est isotrope mais que la perturbation conduit à une anisotropie, le calcul montre que la stabilité est plus grande que si l'isotropie demeure. Dans le cas d'équilibre isotrope il est donc suffisant de conserver l'isotropie.

Dans le cas d'équilibre anisotrope, Sagdeev¹³⁾ ont calculé par des considérations cinétiques les constantes de propagation d'une perturbation.

¹²⁾ Sur certains travaux de l'Institut de l'énergie atomique dans le domaine des réactions thermonucléaires contrôlées par Kourchatov, Atomnaya Energiya 5 (1958) 2.

¹³⁾ Dynamique d'un plasma raréfié dans un champ magnétique par Sagdeev, Kadomtsev, Rudakov et Vedyonov, Conférence de Genève, P/2214.

L'incrément p et les nombres d'onde $k_y k_z$, (l'axe z ayant la direction du champ magnétique non perturbé) sont liés par la relation

$$\frac{p}{k_z} = \sqrt{\frac{T_{\parallel}}{\mu}} \times \frac{\frac{8\pi n_0 T_{\perp}}{B_0^2} \left(\frac{T_{\perp}}{T_{\parallel}} - 1 \right) - \left(1 + \frac{8\pi n_0 T_{\perp}}{B_0^2} - \frac{8\pi n_0 T_{\parallel}}{B_0^2} \right) \frac{k_z^2}{k_y^2} - 1}{\sqrt{\frac{1}{2}} \pi \frac{8\pi n_0 T_{\perp}}{B_0^2} \frac{T_{\perp}}{T_{\parallel}}}$$

Posons

$$\frac{8\pi n_0 T_{\perp}}{B_0^2} = \beta.$$

Les perturbations croissent exponentiellement avec le temps si :

$$\beta \left(\frac{T_{\perp}}{T_{\parallel}} - 1 \right) > \left[1 + \beta \left(1 - \frac{T_{\parallel}}{T_{\perp}} \right) \right] \frac{k_z^2}{k_y^2} + 1.$$

Considérons le cas d'anisotropie dans les bouteilles magnétiques,

$$T_{\perp} > T_{\parallel}.$$

Ce type d'instabilité serait à craindre pour une valeur β telle que :

$$\beta > T_{\parallel}/T_{\perp}.$$

Cette instabilité est due au diamagnétisme du plasma.

4. Instabilité du plasma due à des courants ioniques

Ce type d'instabilité est étudié par Kadomtsev¹⁴⁾.

Cette instabilité est produite par l'existence de courants d'ions de sens contraires qui existent dans un magnétron à ions et dans une bouteille magnétique à injection d'ions rapides.

La période et la longueur d'onde des perturbations considérées sont petites par rapport à la période et au diamètre de mouvement cyclotron.

L'étude est très schématique. Deux courants contraires d'ions de direction Ox sont établis au milieu d'électrons en présence d'un champ magnétique de direction perpendiculaire Oz . En utilisant avec cette situation l'approximation

¹⁴⁾ Sur les instabilités de plasma dans un champ magnétique en présence de faisceaux d'ions par Kadomtsev, Physique des plasmas et le problème des réactions thermonucléaires contrôlées, 4 (Publication de l'Académie des Sciences de l'U.R.S.S. 1958).

hydrodynamique une relation de dispersion peut être établie pour une onde de la forme :

$$\exp \{ -i\omega t + iqz + ikx \} \quad \text{où } q/k \ll 1.$$

Cette relation montre qu'à condition que la vitesse du faisceau ionique soit supérieure à sa vitesse thermique (racine de l'écart quadratique moyen) il existe une série de valeurs de k et q qui conduisent à une fonction exponentielle du temps.

3. L'Injection dans les Machines à Miroirs

3.1. INTRODUCTION

Le remplissage d'un espace de confinement par le plasma peut être obtenu de deux manières :

- (a) l'ionisation d'un gaz dans cet espace,
- (b) l'injection de particules ionisées provenant de l'extérieur.

La première méthode nécessite un chauffage intense du plasma formé, pour le porter de la température ordinaire à celle exigée d'un plasma thermonucléaire. On sait que les moyens de chauffage proposés ou essayés jusqu'à présent souffrent d'un certain nombre de défauts : faible efficacité, domaine d'action limité, génération d'instabilités...

L'injection de particules, préalablement accélérées à une énergie suffisante, permet d'envisager de se passer plus ou moins complètement d'autres moyens de chauffage du plasma. Elle a, par contre, ses difficultés propres. La création de flux d'ions ou de plasma rapides d'intensité suffisamment élevée sera examinée dans d'autres rapports. Nous nous limiterons ici à l'exposé des problèmes relatifs aux méthodes d'injection et à la transformation du flux dirigé des particules injectées en un plasma.

Auparavant, nous croyons utile de souligner que le principe de l'injection est tout à fait distinct et indépendant du mode de confinement utilisé. Il se combine tout naturellement avec le principe d'un confinement par champ magnétique extérieur (miroirs magnétiques ou stellerator) qui est même nécessaire au départ pour créer un plasma par cette méthode. Il ne s'exclut pas forcément des confinements par champ propre (striction) avec lequel il ne semble

pas encore avoir été envisagé. Les méthodes d'injection décrites ici—bien que tout à fait générales en elles-mêmes—seront surtout envisagées sous l'aspect de leur application à une géométrie à miroirs magnétiques. Cette combinaison présente l'avantage supplémentaire d'éviter les pertes particulièrement importantes des miroirs magnétiques aux basses températures.

Certaines méthodes d'injection sont essentiellement limitées dans le temps et s'associent naturellement avec une compression adiabatique du plasma.

D'autres méthodes, au contraire, sont continues et permettent d'envisager la création puis l'entretien de plasma indéfiniment (au moins théoriquement).

D'autres, enfin, sont continues mais ne permettent pas la création d'un plasma à partir d'un espace vide et ne pourront être utilisées que pour l'entretien ou le chauffage d'un plasma préexistant.

Le choix d'une méthode particulière devra donc se faire en fonction de ses caractéristiques et de l'emploi visé.

Si l'on injecte un faisceau d'ions seulement, on devra se préoccuper des sources possibles d'électrons dans l'enceinte et, éventuellement fournir le complément nécessaire à la neutralisation du milieu. On peut également injecter simultanément ions et électrons sous forme d'un plasma.

3.2. MÉTHODES D'INJECTION

L'injection d'un ion rapide dans un champ magnétique n'est pas une chose aisée. En effet, la particule venant d'une zone de champ faible ou nul et pénétrant dans le champ magnétique va en ressortir après un parcours très réduit si l'on ne prend aucune précaution spéciale.

Une première amélioration consiste à augmenter le trajet de l'ion dans le champ magnétique. Pour cela, on peut conduire la particule jusqu'à une profondeur suffisante dans le champ soit par une source interne, soit en la soustrayant à l'action du champ à l'aide d'un dispositif convenable (blindage). On peut appeler

cela l'injection interne. Si, pendant le trajet de l'ion, aucune perturbation n'est apportée à son mouvement, il finira par retomber sur l'injecteur et sera alors perdu. Mais on peut profiter de l'allongement de sa trajectoire pour apporter une modification à son mouvement qui rendra sa capture définitive. L'injection interne n'est donc pas à proprement parler une méthode d'injection mais elle peut rendre possible ou plus facile une méthode, comme on le verra par la suite.

Comme il n'existe pas une théorie générale de l'injection interne, raisonnons sur un cas particulier.

En supposant, par exemple, l'orifice d'injection situé dans le plan central de symétrie d'une bouteille simple, on peut faire en sorte que l'ion soit animé d'une vitesse $v_{\parallel}$ juste suffisante pour éviter la source au premier tour de sa rotation cyclotron. La particule exécute un mouvement à trois périodicités: la période cyclotron, la période de va-et-vient entre les miroirs, la période de précession. Dans ces conditions, le centre de guidage balaye une surface cylindrique déformée à l'approche des miroirs et ne retouche l'injecteur, si la vitesse de précession est correcte, qu'après une précession complète. On peut donner une expression de la longueur parcourue l .

Soit
 ρ le rayon cyclotron,
 r le rayon d'encombrement de la source,
 R le rayon de l'enceinte,
 L la distance entre miroirs,
 $S = 2\pi RL$.

$$l \approx \pi \sqrt{2} S \frac{\rho^{\frac{1}{2}}}{r^{\frac{3}{2}}}. \quad (1)$$

A cette longueur l correspond un temps $t = l/v$. Supposons que les particules injectées pendant ce temps se répartissent dans la totalité du volume, alors la densité atteinte s'exprime par :

$$\frac{n}{n_{\text{source}}} = 2\pi^2 \sqrt{2} \frac{s_1}{s} \frac{\sqrt{\rho r}}{R} \quad (2)$$

où s_1 est la section du faisceau à la sortie de la source et

$$n_{\text{source}} = \frac{I}{Ze v s_1}.$$

Or, s_1/s est très inférieur à 1, de l'ordre de 10^{-1} et $\sqrt{pr}/R$ est lui aussi nécessairement inférieur à 1, quelque chose comme $\frac{1}{3}$, au maximum. On voit que la densité atteinte dans le volume de la bouteille est, au maximum, équivalente à la densité à la sortie de la source. Il n'y a aucune accumulation.

Pour dépasser ce niveau de densité dans une bouteille simple, il faut obligatoirement exercer sur la particule une action perturbatrice qui l'écarte suffisamment de sa trajectoire, pour qu'elle évite indéfiniment l'injecteur. Suivant une classification donnée dans le tableau 1,

précession si la contraction de la trajectoire est au moins aussi grande que le rayon injecteur. Tout calcul fait, on arrive à :

$$\frac{n}{n_{\text{source}}} \approx \left(\frac{2}{3}\right)^{\frac{3}{2}} \pi^2 \frac{s_1}{s} \sqrt{\frac{\rho}{\gamma}} \quad (3)$$

$$t = 2 \tau_m$$

avec

$$\tau_m = \frac{B}{\frac{\partial B}{\partial t}}$$

en supposant que l'angle d'injection n'est pas modifié au cours de la compression.

TABLEAU 1

Processus	Limité dans le temps	Périodique		Continu		
Mode d'injection	Champ magnétique variable	Miroirs mobiles	Interactions organisées	Champ électrique variable	Ionisation	Dissociation
Nature du gaz injecté						
Plasma	X ₂ ¹	X	X ¹			
Ions atomiques	X ²			X ₃ ²		
Ions moléculaires						X ₃ ²
Neutres rapides					X ₃ ²	

↓

Programme des Pyrotrons
U.C.R.L. Post

↓

Bostick

↓

Pyrotron

↓

U.C.R.L.
Gibson
Lamb
Lauer

↓

Oak Ridge
DCX and
U.R.S.S.
(OGRA)

1 = flux spécifique élevé.
 2 = température élevée.
 3 = accumulation.

nous classerons les méthodes d'injection en :

- injections limitées dans le temps,
- injections périodiques,
- injections continues.

3.2.1. Injections limitées dans le temps

On peut classer dans cette catégorie des méthodes employées par l'équipe qui a travaillé à Livermore sous la direction de R. F. Post et qui reposent sur l'augmentation du champ magnétique de la bouteille. Cette augmentation a pour effet de comprimer la trajectoire dans l'une ou l'autre des directions et de l'écarter du point d'injection. Reprenant le mouvement du paragraphe 3.2, l'injecteur sera évité après une

Si l'angle d'injection peut être modifié dans les meilleures conditions :

$$\frac{n}{n_{\text{source}}} \approx \sqrt{2} \pi^2 \frac{s_1}{s} \frac{\sqrt{\rho}}{\gamma} \quad (4)$$

$$t \gg \tau_m.$$

On voit que la densité par rapport au remplissage sans variation de champ a été multipliée par un facteur $0.136 R/\gamma$ pour (3) et $0.353 R/\gamma$ pour (4). Ce fait est suffisant pour que l'injection à champ variable soit plus intéressante car la dimension de la source est généralement faible devant le rayon de l'appareil.

Pourtant, la densité moyenne, ainsi que l'indique ces formules, reste, pour des machines

usuelles, de l'ordre de 1 à 5 fois la densité à la sortie de la source.

La densité locale, par contre, se répartit évidemment en couches concentriques de plus en plus denses vers l'intérieur.

Ainsi vue sous l'angle de l'efficacité du remplissage, cette méthode ne dépend que des qualités de la source et de l'injecteur proprement dit. Les efforts doivent viser à réduire l'encombrement injecteur et à accroître l'intensité de la source.

L'une des sources qui donnerait les meilleurs résultats, serait la source d'ions à titane dont l'encombrement est réduit et qui est capable de donner en impulsion des intensités de 400 mA.

Des essais ont été faits par C. C. Damm et F. S. Eby, de Livermore, avec une source de ce genre délivrant jusqu'à 100 mA, sous 3 à 15 kV. Leurs premiers résultats indiquent qu'à partir des intensités de dizaines de milliampères, la charge d'espace perturbe déjà gravement la trajectoire, au point de provoquer des fuites aux miroirs et de rendre certainement inadéquate la théorie de l'injection telle que nous l'avons esquissée. Aussi, un autre type de source, dans laquelle la neutralisation du faisceau serait réalisée, conviendrait mieux; il y a lieu de penser que les canons à plasma capables de très grandes intensités instantanées conviendraient bien, à condition que l'on sache en obtenir une bonne précision et une faible ouverture angulaire. Dans ce cas, il y aurait intérêt à augmenter la durée de cette émission jusqu'à la valeur t , sans pour cela réduire l'intensité de l'émission.

Quelle que soit la source choisie, la méthode d'injection par un champ variable est une méthode d'impulsion. Elle implique donc une température d'ions élevée, et d'autant plus grande que le temps d'injection est plus long, sous peine de voir une grande partie des ions injectés diffuser par les miroirs. Pour juger la source, il faudra donc comparer le temps d'injection au temps moyen de diffusion des particules, la meilleure source étant celle qui donne la plus haute densité pour un temps d'injection comparable au temps de diffusion.

L'utilisation de sources d'ions continues pour l'injection en champ variable n'est guère défendable car les densités moyennes que l'on obtiendrait seraient largement inférieures à celles que l'on obtiendrait dans l'injection continue par un procédé d'accumulation.

3.2.2. *Injection périodique*

Les deux types d'injections que nous signalons sous ce nom n'ont pas été réalisées pratiquement en tant qu'injections périodiques. Néanmoins, il nous semble qu'elles justifient ce nom car les phénomènes auxquels elles font appel peuvent être répétés à une certaine fréquence, avec accumulation des paquets à l'intérieur de l'enceinte.

Miroir mobile

Ce procédé est évoqué par R. F. Post (Genève No. 344). La source est placée à une extrémité de la bouteille; le miroir est alors déplacé de façon à traverser la zone de la source. Dans ces conditions, un certain nombre de particules ne peuvent retraverser le miroir et sont captées. Sous cette forme, le procédé s'apparente étroitement au transfert. Si maintenant, le miroir est remplacé par une onde progressive à au moins deux alternances, les particules sont prises par paquet et transportées irréversiblement dans la bouteille. Cette application, sans doute très coûteuse en énergie, doit être citée pour illustrer la possibilité d'injections périodiques, bien qu'elle n'ait fait l'objet d'aucune expérimentation.

Interactions de nuages de plasma

Ce procédé a été décrit par Bostick. Des plasmoids, projetés l'un contre l'autre dans une bouteille magnétique, fusionnent en transformant leur énergie cinétique cohérente en une énergie d'agitation thermique. En multipliant le nombre des injecteurs et, par conséquent, le nombre des plasmoids projetés, on a pu atteindre des densités de 10^{16} particules/cm², à des températures moyennes (de l'ordre de 5 keV). A de telles températures d'injection, le confinement du plasma peut être considéré comme assez long

pour que la densité s'accroisse par des projections successives; en effet, le temps de formation et d'expulsion du plasmoid est de l'ordre de 10^{-5} , alors que sa vie dans l'enceinte est de l'ordre de 10^{-4} s. Ici encore, l'injection pourrait être en principe répétée si l'on savait placer ces sources dans les conditions d'une décharge oscillante.

3.2.3. Injection continue

1. La première de ces méthodes consiste à provoquer la perturbation par l'action d'un champ électrique de fréquence, égal à la fréquence cyclotron des ions. Ce champ, appliqué dans la partie centrale de la bouteille à des particules injectées par le miroir, leur confère un accroissement de vitesse perpendiculaire.

Si l'apport d'énergie perpendiculaire au premier passage est suffisant pour orienter leur vitesse à l'extérieur du cône de perte, les ions sont confinés. Le rendement, en particulier, n'est appréciable que si la relation suivante est vérifiée:

$$\frac{1}{4} Z_e E \lambda \geq E_0 \operatorname{tg} \theta_0$$

où

Z_e est la charge des ions,

E le champ H.F.,

λ la longueur de la zone H.F.,

E_0 l'énergie d'injection,

θ_0 l'angle de cône de perte.

Malheureusement, le procédé ne peut être appliqué que dans la mesure où le milieu est suffisamment transparent à l'onde H.F.; son efficacité disparaît dès que la densité du gaz ionisé en empêche la pénétration. De ce point de vue, il peut être comparé aux méthodes d'ionisation classiques par haute fréquence mais avec les avantages considérables de laisser la liberté du choix de la température ionique et de pouvoir fonctionner aux pressions les plus basses.

Cette injection par champ H.F. a été utilisée par l'équipe de Livermore, à ses débuts; elle a été abandonnée peu après en raison de la densité très faible du plasma qu'elle permettait d'obtenir.

Une série d'autres méthodes d'injection

continue consiste à changer brusquement la nature des particules injectées, plus précisément leur rapport e/m , par un choc inélastique. Ces chocs se font entre le faisceau incident, d'une part et le milieu existant dans l'espace où se fait l'injection (gaz résiduel plus ou moins ionisé), d'autre part. Comme il s'agit de flux et de densités faibles, l'échelle de temps de constitution d'un plasma intéressant par cette voie est très grande, par exemple de plusieurs dizaines de secondes ou plus encore. Les deux types de chocs inélastiques qui ont été proposés sont la dissociation des ions moléculaires et l'ionisation des atomes neutres.

2. L'injection par dissociation des ions moléculaires a été réalisées sous deux variantes.

A Oak Ridge, dans la machine DCX, elle est faite avec un arc spécial entretenu à très basse pression dans un champ magnétique. Le rendement de dissociation est très élevé (20% ou plus sur un parcours de l'ordre de 1 cm) et indépendant de la pression résiduelle. Il en résulte que l'on n'a pas besoin de l'aide de l'injection interne et que les trajectoires des ions capturés sont toutes presque superposées (voir description de cette machine page 304).

Dans la machine russe OGRA, la dissociation commence sur le gaz résiduel. Comme on le verra au chapitre suivant, il est impossible d'augmenter la pression du gaz résiduel sans détruire le plasma au fur et à mesure de sa formation.

L'efficacité de la dissociation est donc très faible et il est nécessaire d'allonger la trajectoire des ions moléculaires dans le champ magnétique, par l'artifice de l'injection interne.

La longueur de cette trajectoire est:

$$l = \frac{2\pi Rl}{\sin \theta \sqrt{2rp} + r} = 0.84 \times 10^5 \text{ cm}$$

avec

R rayon de la bouteille = 140 cm,

l longueur entre miroirs = 1200 cm,

r rayon de l'injecteur = 5 cm,

ρ rayon de Larmor des ions = 18 cm

θ angle de l'injecteur avec le plan médian = 20° .

Les trajectoires des ions capturés sont, dans ce cas, réparties au hasard dans la bouteille

(voir description de cette machine page 305).

Remarquons que les deux méthodes précédentes ont en commun plusieurs inconvénients: le rayon de Larmor des ions injectés est voisin du rayon de la bouteille, ce qui conduit à des pertes importantes; et la neutralité du milieu n'est pas automatiquement assurée.

3. La méthode d'injection par ionisation d'atomes neutres consiste à faire d'abord un faisceau d'atomes rapides, soit par neutralisation d'ions atomiques, soit par dissociation d'ions moléculaires sur une cible gazeuse. Le faisceau neutre pénètre sans perturbation dans le champ magnétique et s'ionise à l'intérieur sur le gaz résiduel ou un plasma existant déjà. Les ions ainsi créés sont alors conservés dans le champ magnétique.

Cette méthode surmonte les difficultés précédentes en permettant une grande valeur rapport du rayon de la bouteille au rayon de Larmor d'une part, et en assurant automatiquement la neutralité du milieu, d'autre part. En revanche, on verra que le courant critique est tellement élevé qu'il semble très difficile de créer un plasma dans le vide par ce moyen. Par contre, cette méthode serait très convenable pour l'entretien ou le chauffage d'un plasma déjà existant et suffisamment dense.

4. Signalons enfin une méthode qui a été proposée par Post mais pas essayée: l'injection par choc élastique. La brusque déviation de la particule incidente, sur une particule du milieu, permet son confinement. Cette méthode, encore plus que la précédente, exige la constitution préalable d'un plasma de densité élevée.

3.3. ACTION DU GAZ RÉSIDUEL - COURANT CRITIQUE

Chaque fois que l'on veut remplir un espace vide avec des particules rapides, il s'établit une compétition entre elles et le gaz résiduel. En effet, les ions rapides éliminent le gaz neutre par: ionisation (section efficace σ_1),

échange de charge (section efficace σ_{+0}).

Mais ce dernier phénomène élimine également l'ion incident qui est neutralisé.

Si l'intensité du courant des particules

capturées dans le champ magnétique est trop faible, il ne parvient pas à éliminer le gaz neutre constamment renouvelé par le dégazage des parois matérielles du récipient. Dans ces conditions, la densité des ions rapides reste très faible en régime permanent. Il existe une valeur du courant injecté qui renverse brusquement cette situation. Alors, l'élimination du gaz neutre est totale et l'accumulation des ions peut se faire jusqu'à une limitation qui ne provient plus que de la pauvreté du confinement. Cette notion de courant critique est une conception valable pour toute injection dans un espace vidé. La valeur du courant critique dépend de la méthode d'injection utilisée. Nous considérons ici les cas de l'injection continue.

3.3.1. Injection indépendante du gaz résiduel

C'est le cas, par exemple, de la dissociation des ions moléculaires sur l'arc au carbone (DCX); (mais s'appliquerait aussi bien à l'injection par haute fréquence).

L'évolution de la densité d'ions n_+ et du gaz neutre résiduel n_0 est donnée par les deux équations différentielles:

$$\left[\begin{array}{ll} \frac{dn_+}{dt} = \frac{I_+}{eV} & \text{--- } n_+ n_0 \sigma_{+0} v \\ \text{apport d'ions} & \text{pertes par échange de charge} \\ \frac{dn_0}{dt} = \frac{1}{4} \frac{v_0 S}{V} (N_0 - n_0) & \text{--- } n_+ n_0 (\sigma_{+0} + \sigma_1) v \\ \text{apport de gaz neutre} & \text{élimination des neutres} \\ & \text{par les ions} \end{array} \right.$$

en négligeant les autres causes de perte que le processus précité (en particulier les pertes par les miroirs) où

I_+ est le courant effectivement injecté (en ampère),

e est la charge élémentaire (en coulomb),

v est la vitesse des ions,

N_0 est la densité du gaz neutre à l'extérieur du plasma, supposée constante,

S est l'aire de la surface limitant le plasma,

V est le volume de l'espace contenant le plasma,

v_0 est la vitesse des molécules du gaz neutre.

On en tire les solutions de régime permanent :

$$n_0 = N_0 - \frac{4}{v_0 S} \frac{I_+}{e} \frac{\sigma_{+0} + \sigma_1}{\sigma_{+0}}$$

$$n_+ = \frac{\frac{I_+}{eV} \frac{1}{\sigma_{+0} v}}{N_0 - \frac{4}{v_0 S} \frac{I_+}{e} \frac{\sigma_{+0} + \sigma_1}{\sigma_{+0}}}$$

et le courant critique :

$$I_{\text{crit}} = e \frac{v_0 N_0 S}{4} \frac{\sigma_{+0}}{\sigma_{+0} + \sigma_1}.$$

Le courant critique est proportionnel à la pression résiduelle (N_0).

Il augmente avec les dimensions de l'appareil (S).

Il dépend de l'énergie des ions $\sigma_{+0}/\sigma_{+0} + \sigma_1$; au-delà de 100 keV, σ_{+0} diminue très rapidement quand l'énergie augmente, tandis que σ_1 varie assez lentement. (On trouvera en annexe les valeurs de σ_{+0} et σ_1 en fonction de l'énergie.)

La valeur numérique du courant critique, dans le cas de l'appareil DCX, est 80 mA, pour une pression de 10^{-6} mm Hg—avec des ions D_+ dissociés de 300 keV.

3.3.2. Injection interne par dissociation sur le gaz résiduel

C'est le cas de la machine OGRA. L'injection interne est nécessaire pour allonger le trajet des ions moléculaires dans le champ magnétique et augmenter ainsi le rendement de dissociation. Il en résulte cependant que tout le courant moléculaire non dissocié retombera sur l'injecteur où il sera neutralisé, refroidi puis réémis sous forme de gaz neutre. Si le vide dans l'appareil est suffisamment bon, on peut admettre que le courant non dissocié fournit la principale source de gaz.

Dans ces conditions, les équations d'évolution n_+ et n_0 s'écrivent :

$$\frac{dn_+}{dt} = \underbrace{\frac{I}{eV} l \sigma_d (n_0 + n_+)}_{\text{ions dissociés}} - \underbrace{n_+ n_0 \sigma_{+0} v}_{\text{pertes par échange de charge}}$$

$$\frac{dn_0}{dt} = \underbrace{\frac{I}{eV} [1 - l \sigma_d (n_0 + n_+)]}_{\text{ions neutralisés sur l'injecteur}} - \underbrace{n_+ n_0 (\sigma_{+0} + \sigma_1) v}_{\text{ionisation du gaz résiduel}}$$

où σ_d est la section efficace de dissociation,

I le courant d'ions moléculaires,

l le parcours des ions moléculaires avant de retomber sur l'injecteur.

Les autres lettres ont la même signification qu'en 3.3.1.

En régime permanent, n_+ et n_0 sont solutions d'équations algébriques du second degré.

On peut montrer qu'il existe un courant critique :

$$I_{\text{crit}} = \frac{e}{4} \frac{V}{L^2} \frac{\sigma_{+0}^2 v}{\sigma_d^2 (2\sigma_{+0} + \sigma_1)}.$$

Cette expression est très voisine de celle donnée par Kourchatov (réf. 1.4.2., p. 307), si l'on néglige σ_1 .

On voit donc que, dans ce cas, le courant critique est indépendant de la pression initiale (sous la réserve que l'hypothèse faite au début est vérifiée). Il varie comme l'inverse de la longueur de l'appareil : v/l^2 . Il dépend de l'énergie des ions en diminuant rapidement si celle-ci augmente :

$$\frac{\sigma_{+0}^2 v}{\sigma_d^2 (2\sigma_{+0} + \sigma_1)}.$$

Les valeurs numériques pour le cas de la machine OGRA sont 2A pour des ions D_2^+ et 0.2 A pour H_2^+ de 200 keV.

On voit donc que la condition de courant critique pose des problèmes technologiques sérieux pour l'injecteur et pour le vide (il faut une pression initiale $\leq 10^{-8}$ mm Hg).

3.3.3. Injection par ionisation d'atomes rapides

C'est la méthode proposée par Gibson, Lamb et Lauer à Livermore. L'injection ne nécessitant pas d'injecteur dans le champ magnétique, le seul apport de gaz, en principe, est le dégazage des parois. Les équations d'évolution peuvent s'écrire sous la forme :

$$\frac{dn_+}{dt} = \frac{I_0}{eV} 2R \sigma_{0+} (n_0 + n_+) - n_+ n_0 \sigma_{+0} v$$

$$\frac{dn_0}{dt} = \frac{v_0 S}{4V} (N_0 - n_0) - n_+ n_0 (\sigma_{+0} + \sigma_1) v$$

où R est le rayon de la bouteille magnétique,

σ_{0+} la section efficace d'ionisation des particules injectées,

I_0 l'intensité du courant d'atomes injecté.

En régime permanent, n^+ et n_0 sont solutions d'équations algébriques du second degré. On peut montrer qu'il existe un courant critique:

$$I_0 \text{ critique} = \frac{eN_0V\sigma_{+0}v}{2R\sigma_{0+}}.$$

Le courant critique est proportionnel à la pression résiduelle. Il augmente avec les dimensions de l'appareil et diminue quand l'énergie du faisceau augmente à partir d'une certaine valeur. Cependant, on ne peut pas profiter pleinement de cette dernière circonstance parce que l'on a besoin d'une valeur assez élevée de σ_{+0} pour la création du faisceau de neutre, si l'on obtient ce dernier à partir d'un faisceau initial d'ions atomiques.

Application numérique: $R = 12$ cm, longueur 20 cm, pression 10^{-9} mm de Hg à 100 keV $\sigma_{0+} \neq \sigma_{+0}$

$$I_0 \text{ critique} = 0.5 \text{ A}.$$

Les difficultés technologiques de création d'un plasma par cette méthode sont encore plus grandes que dans les deux précédentes.

Par contre, cette méthode permettrait l'entretien et le chauffage d'un plasma déjà constitué et suffisamment dense, même s'il était initialement à basse température.

En conclusion à la méthode d'injection, sont associées deux techniques pour lesquelles des développements importants sont nécessaires. (Ces techniques font l'objet de rapports séparés présentés dans ce numéro):

Production de faisceaux d'ions et de plasma très intenses (voir page 376).

On a obtenu jusqu'à présent des faisceaux d'ions d'intensité jusqu'à 1 A environ.

Production de vides très poussés (voir page 367).

De grands progrès ont été réalisés, surtout par diminution du dégazage des parois, grâce à l'emploi de joints métalliques et au chauffage des parois à haute température sous vide (450°C). Des pressions de 10^{-8} mm Hg sont réalisables dans de très grands volumes. Des pressions de 10^{-9} et 10^{-10} mm Hg ne semblent pas impossibles à atteindre. Enfin, il serait très souhaitable de pouvoir ioniser aussi complète-

ment que possible les gaz à très basse pression, ce qui diminuerait dans certains cas la valeur du courant critique nécessaire.

3.4. THERMALISATION

Après l'injection, les ions vont tendre vers un équilibre, par suite des chocs élastiques.

Ces chocs se font:

avec les ions lents provenant du gaz résiduel,

avec les ions rapides injectés,

avec les électrons lents ou rapides, selon le cas, qui assurent la neutralité du milieu.

Peu de résultats ont été publiés sur ce point. Ces chocs conduisent à un refroidissement du milieu et à des pertes, si bien que si l'on maintient l'injection, on arrive à un état de régime permanent.

L'évolution du système peut être décrit par une équation de la forme:

$$\frac{dn_+}{dt} = \frac{I}{eV} - n_+^2 \sigma_{\text{eff}} v$$

où I est le courant effectivement capturé, et

σ_{eff} est une section effective de choc conduisant à une perte de particule.

On peut écrire:

$$\sigma_{\text{eff}} = \sigma_{\text{coulomb}} (P_1 + P_2)$$

P_1 est la probabilité qu'un choc envoie la particule dans le cône de perte. Il dépend essentiellement du rapport de miroir B_0/B_1 .

P_2 est la probabilité de perte par diffusion perpendiculaire au champ magnétique. Il dépend surtout de la géométrie et vaut environ $\rho S/V$.

La densité à l'équilibre sera:

$$n_{\text{eq}} = \sqrt{\frac{I}{eV} \frac{1}{\sigma_{\text{eff}} v}}$$

et le temps caractéristique pour atteindre cet équilibre:

$$\tau_c = \sqrt{\frac{eV}{I \sigma_{\text{eff}} v}}.$$

En application de ces formules, il semble que l'on puisse obtenir par injection continue des densités de 10^{12} à $10^{13}/\text{cm}^3$, avec des temps caractéristiques de l'ordre de quelques secondes.

BIBLIOGRAPHIE

1. Méthodes d'injection

- 1.1. Survie des particules injectées dans une machine à miroir
- 1.1.1. Constance de l'invariant adiabatique.
- Martin Kruskal, The spiraling of a charged particule, C.R. Congrès de Venise (1957) p. 562.
- J. Berkowitz et C. G. Gardner, Rigorous justification of Kruskal's asymptotic series, A.E.C. Report TID 7536-1 p. 10.
- Russel M. Kulsrud, The adiabatic invariant for the harmonic oscillator, A.E.C. Report TID 7536-1 p. 14; Phys. Rev. **106** (1957) 205.
- R. B. Theus et E. G. Harris, Numerical investigation of the adiabatic approximation, A.E.C. Report TID 7536-1 p. 39.
- 1.1.2. Pertes par non adiabaticité
- A. Garren, R. J. Riddell, L. Smith, G. Bing, L. R. Henrich, T. G. Northrop et J. E. Roberts, Individual particle motion and the effect of scattering in an axially symmetric magnetic field, Genève (1958) No. 383.
- O. B. Firsov, The repulsion of charged particles from regions of strong magnetic field, Plasma Phys. and probs of therm. reactions USSR vol. V p. 259.
- 1.1.3. Pertes aux miroirs d'un plasma dense
- R. F. Post, Summary of U.C.R.L. Pyrotron (Mirror Machines) Program, Genève (1958) No. 377.
- A. Garren *et al.* (cf. 1.1.2). Individual particle motion and the effect of scattering in an axially symmetric magnetic field, Genève (1958) No. 383.
- D. Judd, W. McDonald et M. Rosenbluth, End leakage losses from the mirror machine, A.E.C. Report 289, p. 158.
- Persico et Linhart, Plasma loss from magnetic bottles, Nuovo Cimento **8** (1958) 1668.
- 1.2. Captures par variation de champ
- 1.2.1. Capture par application d'un champ électrique
- F. C. Ford, An enhanced magnetic mirror machine, UCRL-4363 (1954).
- 1.2.2. Capture par variation du champ magnétique
- R. F. Post, Summary of UCRL Pyrotron program, Genève (1958) No. 377.
- 1.2.3. Injection externe
- F. H. Coengsen et F. C. Ford, Pyrotron plasma-heating experiments, Genève (1958) No. 378.
- 1.2.4. Injection interne
- C. C. Damm et F. S. Eby, Pyrotron high energy experiments, Genève (1958) No. 379.
- 1.3. Injection par miroirs mobiles
- F. H. Coengsen, Energetic plasma injection and transfer experiments, non publié.
- 1.3. Capture par chocs élastiques
- 1.3.1. Capture par collisions individuelles
- 1.3.2. Capture par collisions de nuages de plasma
- W. H. Bostick, Experimental study of ionized matter projected across a magnetic field, Phys. Rev. **104** (1956) 292.
- 1.4. Capture par chocs inélastiques
- 1.4.1. Capture par dissociation
- 1.4.1.1. Capture par dissociation sur arc
- C. F. Barnett, P. R. Bell, J. S. Luce, E. D. Shipley et A. Simon, The Oak Ridge thermonuclear experiment, Genève (1958) No. 344.
- A. Simon, Diffusion of arc plasmas across a magnetic field, Genève (1958) No. 366.
- J. S. Luce, Studies of intense gas discharges, Genève (1958) No. 1790.
- C. F. Barnett, The dissociation of diatomic hydrogen ions, Genève (1958) No. 1789.
- 1.4.2. Capture par dissociation sur neutre
- I. F. Kourchatov, Quelques travaux de l'Institut de l'Energie Atomique de l'Académie des Sciences de l'U.R.S.S. sur les réactions thermonucléaires contrôlées, Atomnaia Energia **5** (1958) 105.
- 1.4.2. Capture par ionisation
- G. Gibson, W. A. S. Lamb et E. J. Lauer, Injection into thermonuclear machines using beams of neutral deuterium atoms in the range from 100 keV to 1 MeV, Genève (1958) No. 380.

Annexe

CHOCs INÉLASTIQUES

Les collisions inélastiques jouent un rôle dans diverses questions de la physique des plasmas. On a vu que, dans la question de l'injection, les phénomènes de dissociation, d'échange de charge et d'ionisation des gaz par les ions rapides jouent un rôle important.

Nous avons rassemblé ici des résultats numériques, recueillis dans la littérature, des sections efficaces de ces phénomènes, en fonction de l'énergie de la particule incidente. La particule cible est à la température ambiante, c'est-à-dire au repos pour les énergies considérées. Ils sont présentés suivant les échelles d'unité: cm^2 pour

les sections efficaces—eV pour les énergies, en coordonnées logarithmiques afin de couvrir un vaste domaine d'énergie.

Le choix de l'énergie au lieu de la vitesse des particules incidentes nous a semblé d'un emploi plus pratique pour la physique du plasma, bien que pour la physique de ces chocs eux-mêmes, la vitesse est le paramètre intéressant. La plupart de ces résultats sont relatifs à l'hydrogène ordinaire; on peut en déduire les résultats relatifs au deutérium en admettant qu'à vitesse égale, les sections efficaces de chocs sont égales.

On pourra constater, par les désaccords entre les valeurs observées par différents auteurs ou entre les valeurs observées et les valeurs calculées, que ces grandeurs ne sont pas connues avec précision. Dans certains cas, ce désaccord atteint presque un facteur de 10.

Fig. 2. Dissociation des ions moléculaires H_2^+ . C. F. Barnett, Conférence de Genève (1958) P/1789. *Expérimental*:

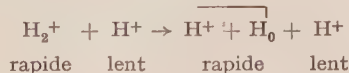


L'auteur donne une "working cross section", obtenue en considérant que toute la dissociation observée est due au premier processus. Courbe 1 gaz = H_2 . Courbe 2 gaz = He. Courbe 3 gaz = N_2 . Courbe 4 gaz = A.

On observe une décroissance de σ_d en $1/v$ à haute énergie.

E. E. Salpeter, Proc. of Phys. Soc. A **63** (1950) 337 et 1295. *Théorique*: Courbe 5 gaz = azote. $\sigma_d \propto (1/v^2)$ au-dessus de 2 MeV.

E. Gerjuoy, Research Report 60, 94439, 1, R2, Westinghouse Res. Lab., Pittsburgh, Penn. *Théorique*: Courbe 6: dissociation sur protons



décroissance en $1/v^2$ à haute énergie.

N. V. Fedorenko, Zhur. Tekh. Fiz. **24** (1954) 769.

Expérimental: Courbe 7 gaz = H_2 . Courbe 8 gaz = A.

K. K. Damodaran, Proc. Roy Soc. A **239** (1957) 382.

Expérimental: Courbe 9 gaz = H_2 . Courbe 10 gaz = A.

K. E. A. Effat, Proc. Phys. Soc. A **65** (1952) 433.

Il est à noter que les résultats expérimentaux de Barnett donnent des valeurs nettement inférieures (de près d'un facteur 10) à ceux de Fedorenko et Damodaran.

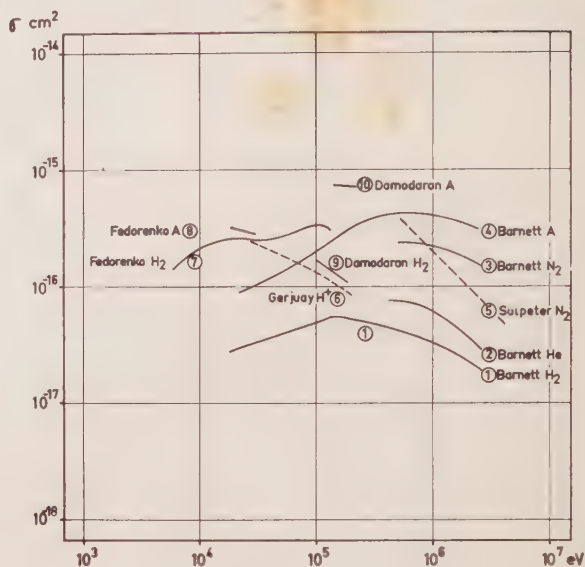


Fig. 2. Dissociation des ions H_2^+ .

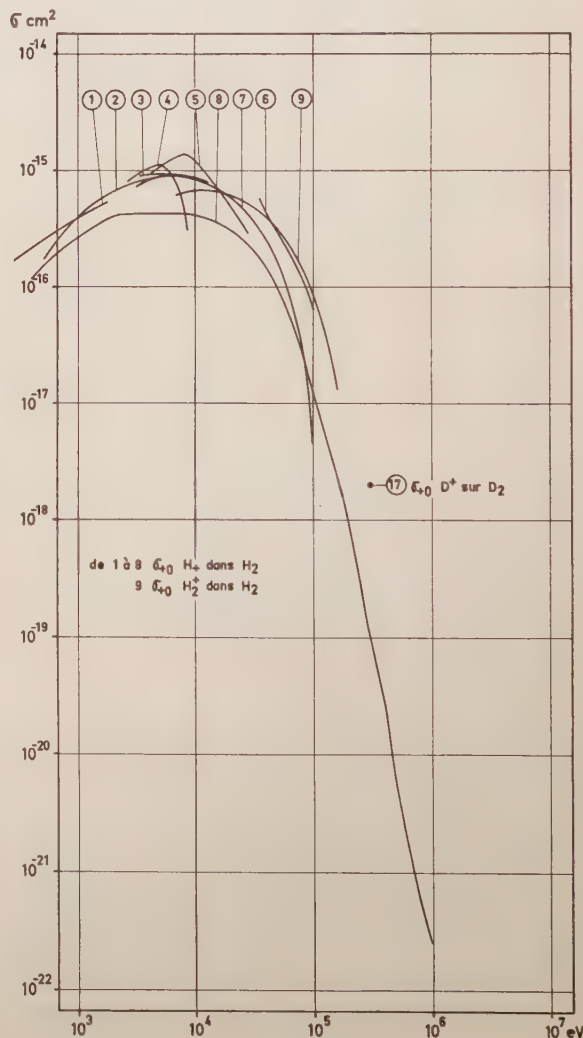


Fig. 3. Échange de charge.

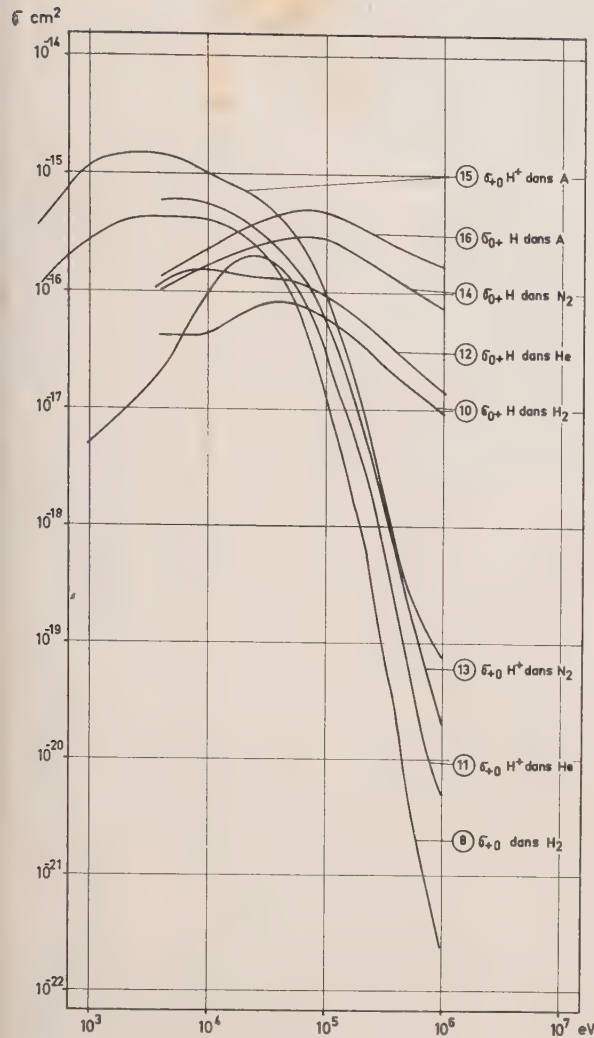


Fig. 4. Échange de charge.

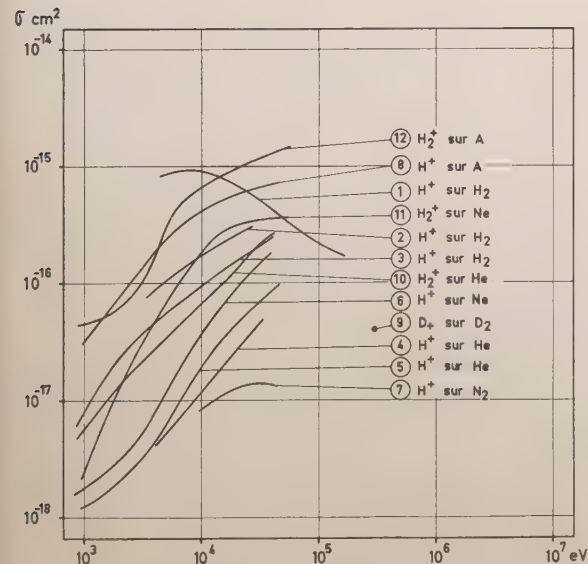
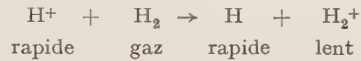


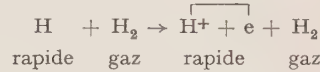
Fig. 5. Ionisation.

Figs. 3 et 4. 2. Échange de Charge. La réaction de capture d'électron (σ_{+0}) est symbolisée par l'équation suivante:



Tout autre molécule en atome que H_2 peut être utilisée.

Inversement, un atome neutre peut être ionisé (perte d'électron σ_{+0}) par un gaz.



Ces phénomènes font l'objet de nombreux travaux que nous en pourrions citer tous ici.

On trouvera dans les courbes 1 à 8 des résultats relatifs à σ_{+0} de H^+ dans l'hydrogène—par différents auteurs.

Certains de ces résultats sont assez anciens.

On notera une dispersion assez notable entre eux.

Les courbes 8 et 10 à 16 représentent les sections efficaces de pertes et de capture d'électron de l'hydrogène dans différents gaz. Elles sont prises dans l'excellente et très complète revue de cette question, récemment publiée par S. K. Allison dans *Revs. Mod. Phys.* **30** (1958) 1136, qui contient également une abondante bibliographie.

Courbes: 1. Wolf, *Ann. der Physik* **30** (1937) 313. 2. Goldman, *Ann. der Physik* **10** (1931) 460. 3. Keene, *Phil. Mag.* **40** (1949) 369. 4. Smith, *Proc. Cambridge Phil. Soc.* **30** (1934) 514. 5. Bartels, *Ann. der Physik* **6** (1930) 957. 6. Meyer, *Ann. der Physik* **30** (1937) 635. 7. Fedorenko, *Atomnaya Energija* **5** (1958) 105. 8. S. K. Allison, 1 à 8 σ_{+0} sur H_2 , résultats expérimentaux. 9. Fedorenko, $\sigma_{+0}\text{H}_2^+$ sur H_2 gaz, expérimental. 10. Allison, $\sigma_{+0}\text{H}$ sur H_2 gaz, expérimental. 11. Allison, $\sigma_{+0}\text{H}^+$ sur He gaz, expérimental. 12. Allison, $\sigma_{+0}\text{H}$ sur He gaz, expérimental. 13. Allison, $\sigma_{+0}\text{H}^+$ sur N_2 gaz, expérimental. 14. Allison, $\sigma_{+0}\text{H}$ sur N_2 gaz, expérimental. 15. Allison, $\sigma_{+0}\text{H}^+$ sur A gaz, expérimental. 16. Allison, $\sigma_{+0}\text{H}$ sur A gaz, expérimental. 17. Barnett, (Genève 1958 P/344) $\sigma_{+0}\text{D}^+$ sur D_2 gaz.

Fig. 5. 3. Ionisation par les ions rapides H^+ et H_2^+ dans différents gaz. 1. H^+ sur H_2 gaz, Fedorenko, *Atomnaya Energija* **5** (1958) 105. 2. H^+ sur H_2 gaz, Von Engel, *Ionized* (Oxford, 1955) p. 57. Ce résultat comprend l'ionisation par les électrons secondaires émis. 3. H^+ sur H_2 gaz, Gilbody et Hasted, *Proc. Roy. Soc. A* **240** (1957) 383. 4. H^+ sur He gaz, Keene, *Phil. Mag.* **40** (1949) 369. 5. H^+ sur He gaz, Gilbody et Hasted, *Proc. Roy. Soc. A* **240** (1957) 383. 6. H^+ sur Ne gaz, Gilbody et Hasted, *Proc. Roy. Soc. A* **240** (1957) 383. 7. H^+ sur N_2 gaz, Carleton et Lawrence, *Phys. Rev.* **109** (1958) 1159. 8. H^+ sur A gaz, Gilbody et Hasted, *Proc. Roy. Soc. A* **240** (1957) 383. 9. D^+ sur D_2 gaz, Barnett, Genève (1958) P/344. 10. H_2^+ sur He gaz, Gilbody et Hasted, *Proc. Roy. Soc. A* **240** (1957) 383. 11. H_2^+ sur Ne gaz, Gilbody et Hasted, *Proc. Roy. Soc. A* **240** (1957) 383. 12. H_2^+ sur A gaz, Gilbody et Hasted, *Proc. Roy. Soc. A* **240** (1957) 383.

4. Machines à Miroirs Magnétiques et à Injection (DCX-OGRA)

Les machines à miroirs magnétiques et à injection à haute énergie permettent l'espoir de créer un plasma à très haute température et à densité modérée—Comme de plus, elles sont des engins continus et où de nombreux paramètres sont directement accessibles elles se prêtent particulièrement bien aux conditions d'observation et d'expérimentation. Ceci explique que malgré la difficulté technologique et le coût élevé de ce genre de machines deux projets très importants aient été réalisés: le DCX aux Etats-Unis et la machine OGRA en U.R.S.S. Voisines par leur conception générale et leur but, ces machines diffèrent cependant entre elles par de nombreux côtés.

Il nous a paru intéressant de rapprocher leur description:

Les deux machines utilisent le procédé d'injection par dissociation d'ions moléculaires. Dans le DCX cette dissociation est obtenue sur un arc spécial créé et entretenu à basse pression à l'intérieur même de la bouteille. L'avantage de cet arc réside dans un rendement élevé de la dissociation des ions moléculaires qui le traversent. Cependant sa présence peut être un inconvénient ultérieurement.

Dans la machine OGRA au contraire la dissociation débute sur le gaz résiduel. Pour que le rendement ne soit pas trop faible on s'est efforcé d'allonger le plus possible le trajet des ions moléculaires dans le champ magnétique, ce qui conduit à une machine considérablement plus grande que le DCX—Malgré cela le courant critique est beaucoup plus élevé que dans DCX.

Ces deux machines sont terminées depuis assez peu de temps et ont encore peu fonctionné—De ce fait très peu de renseignements sur leurs performances sont encore parvenus jusqu'à nous.

Les problèmes relatifs à la thermalisation du faisceau, aux pertes de particules, à la stabilité hydromagnétique, nous semblent encore très loin d'être entièrement compris. Leur étude pourra beaucoup progresser grâce à ces deux machines†.

5. Les Magnétrons Ioniques

Les magnétrons occupent une place importante après le DCX et l'OGRA.

L'examen de ces machines présente d'abord un intérêt historique puisque le magnétron Russe a précédé et préparé la venue d'OGRA; elles gardent toujours cependant un intérêt d'actualité par leurs performances remarquables.

Les magnétrons ioniques sont effectivement des machines à miroirs magnétiques sans compression et à injection interne, avec cette distinction fondamentale que les particules confinées sont des ions positifs et non plus un plasma neutre. La compression n'est pas nécessaire car les particules que l'on se propose de confiner sont douées d'énergie relativement élevée (de 10 à 40 kV) dans les deux magnétrons réalisés respectivement à Berkeley, U.S.A., et en Russie.

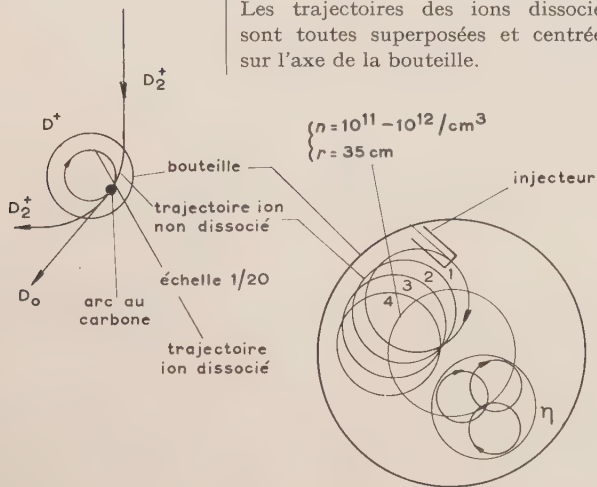
Les magnétrons ioniques utilisent comme les machines à plasma tournant l'action des champs croisés E et B , mais ces champs exercent leur action principale séparément sur deux zones différentes.

5.1. RAPPEL DU PRINCIPE DES MAGNÉTRONS SANS COMPENSATION DE CHARGE D'ESPACE

Le principe est connu depuis longtemps. On sait que l'action simultanée d'un champ magnétique radial, normal au premier, sur une particule chargée, par exemple positivement imprime à cette particule un mouvement que l'on peut analyser sommairement de la manière suivante:

En présence d'un champ électrique que nous supposons uniforme, la particule chargée, supposée sans vitesse est soumise à une force dans la direction de ce champ, force qui lui communique un mouvement uniformément accéléré, elle acquiert une vitesse, et par suite étant assimilable à un courant elle est soumise à la force $\mathbf{V} \times \mathbf{B}$, donc défléchie. On montre que son orbite est une cardioïde (voir fig. 2b).

† Voir tableau comparatif du DCX et OGRA à la page suivante.

	DCX	OGRA
<i>Caractéristiques de la bouteille</i>		
Rayon physique de la bouteille: R	21–22 cm	70 cm
Distance entre miroirs: L	40 cm (90 cm total)	12 m
Champ magnétique dans le plan médian H_0	10 kgauss	5 kgauss
Champ magnétique dans les miroirs: H_1	20 kgauss	8 kgauss
H_1/H_0	2	1.6
Puissance dans le confinement	2.7 Mwatt	4 Mwatt
Pression résiduelle minimum	3×10^{-8} mm Hg	3×10^{-8} mm Hg (pompes à mercure et titane)
Volume de la bouteille	$\begin{cases} 10^4 \text{ cm}^3 \text{ (cité P/334)} \\ 5 \times 10^4 \text{ (calculé)} \end{cases}$	$1.8 \times 10^7 \text{ cm}^3$
<i>Injection</i>		
Nature	D_2^+ dissocié $\rightarrow D^+$ (ou $H_2^+ \rightarrow H^+$)	D_2^+ dissocié $\rightarrow D^+$ (ou $H_2^+ \rightarrow H^+$)
Énergie des ions injectés	600 keV $\rightarrow$ 300 keV $v = 5.8 \times 10^8$ cm/s	200 keV $\rightarrow$ 100 keV $v = 3.1 \times 10^8$ cm/s
Intensité du faisceau D_2^+	max. 1 mA (source hf Moak...)	quelques cent mA (source à arc avec cathode chaude, champ magnétique transversal à l'extraction). Dispersion angulaire $2\alpha = 5^\circ$.
Processus d'injection	injection extérieure dans le plan médian—dissociation dans arc au carbone par un seul passage des ions—Les trajectoires des ions dissociés sont toutes superposées et centrées sur l'axe de la bouteille.	injection interne dans le plan médian: largeur du conduit magnétique 10 cm, angle avec plan médian 20° , precession des trajectoires, dissociation sur gaz résiduel au hasard dans le volume de la bouteille. Les trajectoires des ions dissociés sont réparties au hasard.
		
<i>Injection</i>		
Rendement de dissociation	40% mesuré 70% semblent possible	10% estimé
<i>Caractéristique de l'arc au carbone</i>		
	normal 200–300 A/150 V; longueur: 1 m environ; diamètre: 1 cm environ; maximum: 5000 A; densité dans arc: 10^{13} – 10^{14} /cm ³ ; énergie des électrons: 100 eV environ non Maxwellien	
Rayon de Larmor des ions D^+ dissociés	13 cm	14 cm
R	1.7	5

	DCX	OGRA
<i>Action du gaz résiduel</i> <i>p</i> en fonctionnement Courant critique	avec arc 10^{-7} mm Hg rapport critique théorique: $80 \text{ mA}/10^{-6} \text{ mm Hg}$ pour D^+ non observé jusqu'à présent d'un facteur $i/p = 14$ fois la valeur théorique ci-dessus pour: $i_{H^+} = 40 \text{ A}$ capturé étude de préionisation du gaz résiduel en cours	10^{-6} ($n = 10^{16}$) courant { $i_{D_2^+}$ crit. = 2 A critique { i_{H_2} crit. = 200 mA le courant critique est indépendant de la pression initiale à condition que celle-ci soit assez basse (10^{-8} mm Hg)
Thermalisation	σ_{coulomb} plus faible dans DCX que dans OGRA à cause de l'énergie d'injection: facteur 10 Trajectoires des ions dissociés $\parallel$ dans DCX et se coupant dans OGRA Volume plus petit dans DCX que dans OGRA. Pour le même courant injecté n est proportionnel à $1/\sqrt{V}$	
<i>Pertes des particules rapides</i> Observations	temps de survie moyen environ 10 msec mesuré avec: H^+ à $p = 5 \times 10^{-7}$ mm Hg (varie comme $1/p$) soit environ 10^6 tours $200 \mu\text{A } H_2^+ \rightarrow 40 \mu\text{A } H^+$ capturé, 4 A circulant	$n = 10^{13}$ correspond à un temps de confinement d'une particule plusieurs centaines de secondes, soit 100 000 km
Temps de constitution du plasma $\tau = \sqrt{\frac{eV}{I\sigma_{\text{eff}}v}}$	80 sec pour 1 mA D_+ , 25 sec pour 10 ml	60 sec pour 100 mA, D^+ 18 sec pour 1 A
Densité d'ions en régime permanent $n = \sqrt{\frac{I}{eV\sigma_{\text{eff}}v}}$	$4 \times 10^{12}/\text{cm}^3$ pour 1 mA injecté	pas d'information expérimentale estimé à $10^{13}/\text{cm}^3$ formule précédente $\rightarrow 6 \times 10^{11}/\text{cm}^3$ pour 10 mA D_+ capturé
Pertes dues à: échange de charge	plus faible dans DCX que dans OGRA, à cause de l'énergie d'injection mais de toute façon négligeable quand le plasma est établi	
diffusion (Coulomb scattering)	cette perte est probablement très importante surtout dans le DCX à cause de la faible valeur de R/q	
cône de perte (espace des vitesses)	situation analogue pour DCX et OGRA. La théorie exacte reste à faire. Les conditions initiales sont très favorables	
Instabilité	pas d'information	possibilité de créer des variations périodiques azimutales du champ magnétique
Confinement d'une particule isolée	dans DCX, R/q est plus petit que dans OGRA, mais les trajectoires sont centrées sur l'axe de la bouteille	

	DCX	OGRA
<i>Pureté de milieu</i>	les particules secondaires des ions non dissociés et des ions neutralisés peuvent être éliminés facilement (impact localisé) purification par élimination des particules lourdes dans le cône de perte commune aux deux appareils en principe, mais plus difficile à réaliser sur DCX que sur OGRA à cause des électrodes de l'arc de carbone l'arc au carbone lui-même est une source d'impuretés, donc une cause de refroidissement	les particules secondaires sont émises sur toute la surface et ne peuvent pas être éliminés facilement
<i>Divers</i>	Perte d'énergie possible par rayonnement des électrons dans le champ magnétique transfert à un autre espace de confinement, difficile à cause des électrodes de l'arc	transfert à un autre espace de confinement possible, en principe compression axiale possible, en principe
<i>Futur</i>	compression radiale prévue (ORION): (amélioration du confinement, chauffage et conservation des produits secondaires). Exige suppression de l'arc et injection injection de neutres d'énergie modérée (20 keV) ionisés sur le plasma	compression radiale possible, en principe possible, en principe

5.2. LES MAGNÉTRONS IONIQUES – CAS DE LA COMPENSATION PARTIELLE DE LA CHARGE D'ESPACE

Dans les deux magnétrons ioniques le mécanisme d'accélération identique, des particules est le suivant:

On sait que (voir figs. 7 et 6) autour de l'électrode centrale positive en cuivre du modèle de Berkeley, et de l'arc de plasma du magnétron Russe, il se forme une gaine d'électrons, où règne un champ électrique négatif intense. Ce champ accélère les ions positifs qu'il arrache au plasma dans le magnétron Russe, ou du deutérium ionisé dans le magnétron de Berkeley. Cette action accélératrice est continuée par l'action de l'électrode extérieure. Les ions atteignent ainsi des énergies élevées mais

inférieures à la tension appliquée sur la paroi et sous l'influence du champ magnétique ils décrivent par l'action superposée de E et B des trochoïdes, et reviennent dans la couche centrale où ils subissent le même cycle à moins que dans leur parcours ils n'aient subi un processus d'échange de charge avec le gaz résiduel, analogue à celui qui se produit dans OGRA. Dans ce cas les ions résultant de cet échange de charge forment avec les électrons qui ont pu diffuser à partir de la gaine centrale et avec les ions et les électrons résultant de l'ionisation du gaz résiduel une zone de plasma partiellement neutre. Quant aux ions qui n'ont pas subi de choc, ils conservent un mouvement rapide dans cette région (région 4, fig. 6).

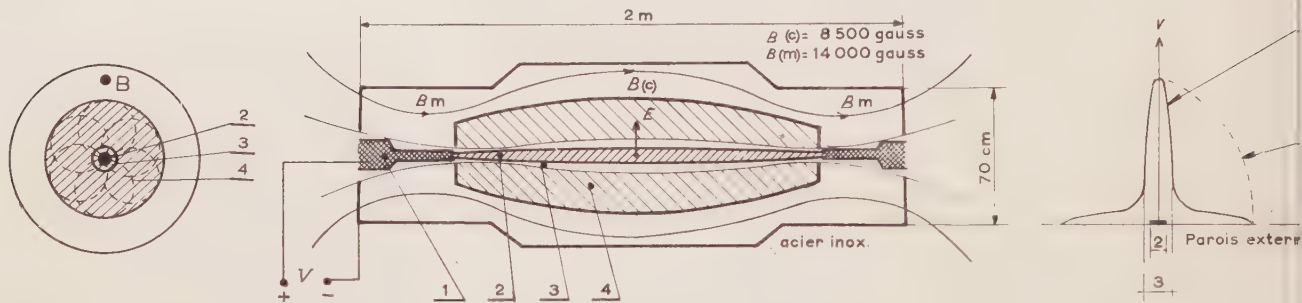


Fig. 6. Magnétron ionique Russe.

1. Source de plasma.
2. Faisceau de plasma.
3. Région de champ électrique intense.
4. Région du mouvement des ions rapides.

Répartition du potentiel dans une section transversale.

- a. Diagramme de V dans magnétron, à ions, avec charge d'espace partiellement compensée.
- b. Diagramme de V dans magnétron classique avec charge d'espace.

5.3. CARACTÉRISTIQUES GÉNÉRALES DU MAGNÉTRON DE BERKELEY ET DU MAGNÉTRON RUSSE - PARTICULARITES DES SOURCES DE PLASMA (Injection interne)

Les caractéristiques générales sont celles que nous avons pu recueillir à partir d'informations directes au cours de l'exposition atomique de la 2ème conférence internationale. Elles sont incomplètes et peut être partiellement inexactes. Nous les donnons dans le tableau suivant sous réserve de rectification. Le seul document extrêmement succinct sur ce sujet est la communication P/2298 d'Arzimovitch sur l'état de la recherche thermonucléaire en U.R.S.S.¹⁵⁾

¹⁵⁾ L. A. Artsimovich, Recherches sur les réactions thermonucléaires contrôlées en U.R.S.S., Conférence de Genève P/2298.

Les particularités des sources sont celles que l'on peut retrouver sur le tableau (5.4).

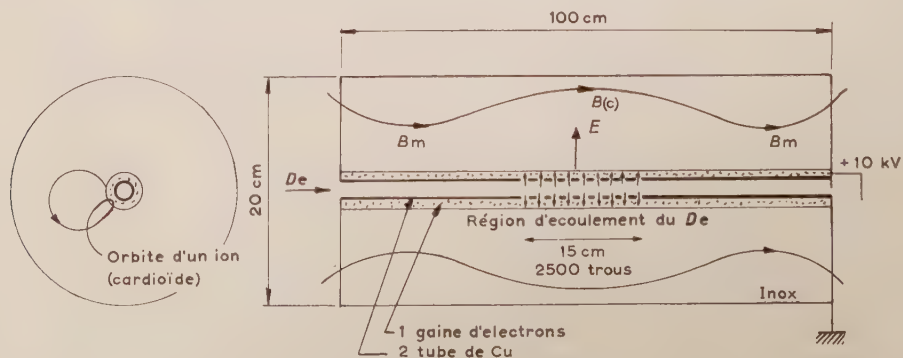
Ces sources sont à plasma.

Dans le magnétron de Berkeley, l'anode centrale en cuivre est percée sur une longueur de 7.5 cm de part et d'autre du plan de symétrie transversal de la machine, de 2500 trous d'où s'écoule le D_2 qui est ionisé.

La gaine d'électrons qui se forme autour du tube, accélère les ions et protège l'anode.

Dans le magnétron Russe, l'anode est l'arc central de plasma.

Les ions sont extraits dans les deux cas, d'un plasma créé dans la zone axiale par une source interne à la configuration. Ce type d'injection est qualifié d'injection interne.

Fig. 7. Magnétron ionique de Berkeley, U.C.R.L., $B_m = 14\,000$ gauss, $B(c) = 10\,000$ gauss, $V = 10$ kV.

5.4. TABLEAU COMPARATIF DES CARACTÉRISTIQUES DES DEUX MAGNÉTRONS IONIQUES

	Magnétron de Berkeley	Magnétron Russe
<i>Description</i>		
Forme de la chambre à vide	cylindre de révolution (voir fig. 7)	surface de révolution (voir fig. 6)
Diamètre	20 cm	70 cm
Longueur	100 cm	200 cm
Matériaux	acier inox	acier inox
<i>Les champs</i>		
(a) magnétique		
au centre de la bouteille	$B(c) = 10$ kgauss	8.5 kgauss
aux miroirs	$B(m) = 14$ kgauss	12 kgauss
loi du champ B	constant dans le temps	constant dans le temps
(b) électrique		
tension radiale appliquée	10 kV	20 à 40 kV
loi du champ E	?	pulsé, impulsion de 100 à 10 000 μ sec
<i>Alimentations:</i>		
(a) des bobines de champ B		
courant	1000 A	?
tension	180 volt	?
(b) de la tension radiale		
tension	10 kV	20 à 40 kV
courant	800 mA	1 A
<i>Le vide</i>		
Pression initiale	5×10^{-7} mm Hg (avant opération)	10^{-7} mm Hg
Pression en fonctionnement	2×10^{-4} mm Hg	?
Nature de la pompe	à diffusion d'huile avec piège à azote liquide	pompe à titane
<i>La source de plasma axiale</i>		
Nature de la source	écoulement continu de deutérium à travers 2500 orifices percés sur le milieu d'un tube de cuivre traversant axialement le magnétron. Le tube est porté à + 10 kV la décharge ionise le D_2	arc de D_2 suivant l'axe de la bouteille magnétique. Débit lent de 250 cm^3 de D_2 à l'heure
Densité du plasma de la source	?	10^{14} p/cm ³
<i>La source ionique</i>		
Densité ionique atteinte	10^{12} p/cm ³	10^{13} p/cm ³
Énergie des ions rapides	< 10 kV	< 20 à 40 kV
Temps de survie des ions dans la bouteille magnétique	?	1 msec environ

5.5. QUELQUES REMARQUES SUR LA STABILITÉ DANS LES MAGNÉTRONS IONIQUES

Nous les emprunterons à l'étude théorique de Kadomsev sur "les instabilités d'un plasma dans un champ magnétique en présence de faisceaux ioniques"¹⁸⁾, étude déjà citée dans

le chapitre "Stabilité" de ce même rapport.

Les conditions physiques envisagées par

¹⁸⁾ B. Kadomsev, Les instabilités d'un plasma dans un champ magnétique en présence de faisceaux ioniques.

Physique des plasmas et problèmes de réactions thermonucléaires contrôlées 4, p. 364 (Institut atomique de l'Académie des Sciences de l'U.R.S.S.).

l'auteur en question pour la naissance des instabilités se trouvent réalisées dans les magnétrons ioniques.

En effet, dans ce cas le plasma du magnétron à ions est créé par l'accélération des ions dans le champ électrique radial. La vitesse des ions au cours de leur trajectoire trochoïdale change en un certain point de direction et les ions reviennent vers le plasma central d'où ils ont été extrait.

Il apparaît ainsi deux courants d'ions inverses dans la direction radiale. Cette situation peut être une cause des instabilités. Elles sont de nature locales et dépendent peu des conditions aux limites.

Kadomsev, examine le cas des instabilités dont la longueur d'onde est beaucoup plus petite que le rayon de Larmor des ions et montre que son plasma est stable si la condition

$$0 < v_s^2 - C_i^2 < \frac{m}{M} C_e^2$$

est vérifiée.

En écrivant ce critère de stabilité sous la forme de :

$$\frac{1}{2} M v_0^2 < \frac{1}{2} m C_e^2 + \frac{1}{2} M C_i^2$$

on peut l'exprimer de la façon suivante :

Le plasma est stable vis-à-vis des perturbations de longueur d'onde plus petite que le rayon de Larmor des ions, lorsque l'énergie cinétique des ions correspondant à la vitesse d'extraction du plasma est inférieure à la somme des énergies thermiques des ions et des électrons.

5.6. CONCLUSIONS

Comme on l'a déjà fait remarquer dans l'introduction de ce chapitre, les magnétrons sont des machines qui ont précédé et préparé la venue du DCX ou de l'OGRA.

Ils gardent encore une importance relative qui les classe derrière celles-ci.

En effet, les avantages qu'on y voit sont :

- (a) absence de pertes de particules par injection,
- (b) densité ionique relativement élevée,
- (c) énergie des particules confinées acceptable, 10 à 40 kV (valeur maximum au départ sur leur trajectoire).

(d) durée de survie des particules de l'ordre de la milliseconde.

Les défauts sont ceux de toutes les machines à miroirs, à savoir :

- (a) pertes par les bouts,
- (b) pertes par diffusion,
- (c) pollution et refroidissement par rayonnement dans le cas d'un magnétron à électrode métallique centrale.

Enfin, il semble qu'il soit difficile d'augmenter la densité ionique et surtout l'énergie des particules d'où la nécessité de recourir à une injection externe, nécessité qui a conduit sans doute à la réalisation du DCX et de OGRA.

6. Confinement par Miroirs Magnétiques d'un Plasma Tournant

MACHINES HOMOPOLAIRES I ET II, IxIONS 1, 2 ET 3

6.1. RAPPEL DU PRINCIPE DE CES MACHINES

L'adjonction d'un champ électrique radial $\mathbf{E}$, normal au champ magnétique axial d'une bouteille magnétique, constitue un moyen supplémentaire de confinement et de chauffage du plasma¹⁷⁾.

Il en résulte en effet, une dérive dans la direction $\mathbf{E} \times \mathbf{B}$. Sous l'influence de cette rotation les particules sont éloignées de l'axe, ce qui entraîne une diminution des pertes par les bouts de la bouteille.

Cet effet gyroscopique a été utilisé par les équipes du laboratoire de l'U.C.R.L. (Berkeley) dans les deux machines Homopolaires I et II et par les chercheurs de Los Alamos dans les Ixions, 1, 2 et 3¹⁸⁾.

6.2. COMPARAISON DES ÉNERGIES AXIALES MAXIMUM CONFINÉES DANS LES BOUTEILLES MAGNÉTIQUES À PLASMA FIXE OU TOURNANT. RELATIONS FONDAMENTALES

Le calcul basé sur l'invariance adiabatique du moment magnétique donne :

- (a) en l'absence de rotation pour la compo-

¹⁷⁾ Study and use of a rotating plasma, P/373.

¹⁸⁾ Theoretical and experimental discussion of Ixion as a possible thermonuclear device, P/2363; A Summary of the Berkeley and Livermore Pinch Programs, P/1064; Review of Controlled Thermonuclear Research at Los Alamos, P/1860.

sante axiale $W_{\parallel}(c)$ de l'énergie des particules au centre du miroir† :

$$W_{\parallel}(c) = W - W_{\perp}(c) = \mu B(m) - B(c) = \\ W_{\parallel}(c) = \mu B(c) (R_m - 1) = W_{\perp}(c) (R_m - 1)$$

(b) dans le cas d'un plasma tournant

$$W_{\parallel}(c) = W_{\perp}(c) (R_m - 1) + \frac{1}{2} m_i V_D^2(c) \left(1 - \frac{1}{R_m} \right).$$

Discussion du terme d'accroissement

$$\frac{1}{2} m_i V_D^2(c) \left(1 - \frac{1}{R_m} \right).$$

Il représente l'accroissement dû à la rotation. Il augmente :

(a) avec R_m ; mais pour des raisons pratiques on se limite à des valeurs de R_m voisines de 2, ce qui conduit à :

$$\frac{1}{4} m_i V_D^2(c).$$

(b) avec

$$V_D \approx V_E = 10^8 \frac{E_{\perp} \text{ volt/cm}}{B \text{ gauss}} = \frac{1}{4} m_i \times 10^{16} \frac{E_{\perp}^2}{B^2}.$$

Il convient d'augmenter E et de diminuer B , en tenant compte de la relation ci-dessous dite "critère de confinement magnétique" qui lie le potentiel électrique sur l'électrode extérieure, au champ magnétique sur cette même électrode.

$$\left(1 + \frac{m_e}{m_i} \right) \left(\frac{\Phi}{\ln b/a} \right)^2 = \frac{B^4(b)}{8\pi \rho c^2} \frac{a^2 b^2}{b^2 - a^2}.$$

6.3. CONSÉQUENCES DE LA ROTATION DU PLASMA. CONCLUSIONS

La rotation du plasma entraîne outre l'amélioration de son confinement, raison première de cette rotation, l'apparition d'un ensemble d'effets caractéristiques.

(a) Effet diamagnétique

Le courant de dérive de densité j_D

$$j_D \approx -c^2 \frac{E^2}{B^3}$$

est dû à la dérive différente des électrons et des ions, il crée un gradient de pression magnétique, qui équilibre la force centrifuge et déforme les lignes de champ magnétique. Il apparaît une composante radiale.

Cette composante peut être mesurée au moyen de sondes magnétiques. Elle permet de déter-

† Voir pour les symboles et formules p. 321.

miner la vitesse de rotation du plasma. On vérifie qu'elle est indépendante du signe de E , mais qu'elle s'inverse avec B , comme l'indique l'expression de j_D .

(b) Effet capacitif

Ce même courant radial produit dans le plasma tournant un effet capacitif qui s'explique par l'analogie du courant de déplacement radial, avec le courant de déplacement dans le diélectrique d'un condensateur.

Le calcul donne pour la capacité du condensateur équivalent au plasma tournant

$$C = \frac{2\varepsilon}{\ln b/a} = 4\pi \left(1 + \frac{m_e}{m_i} \right) \frac{\rho c^2}{B^2} F(b/a)$$

C et ε étant dans les machines réalisées de l'ordre de :

$$\varepsilon = 10^6 \text{ à } 10^7$$

$$C = 10 \mu F.$$

Cet effet permettrait l'utilisation éventuelle des plasmas tournants comme condensateurs à décharge rapide, mais il constitue dans l'état actuel une difficulté à vaincre. Le courant de déplacement qu'il entraîne produit une charge d'espace locale qui fait effet d'écran sur le champ électrique appliquée et retarde l'établissement de la rotation du plasma.

(c) Effet électrostatique axial

Pour une même vitesse de rotation V_E , les électrons sont moins bien confinés que les ions, il apparaît un potentiel électrostatique axial perturbateur égligeable tant que la température des électrons reste inférieure à celle des ions.

(d) Effet Doppler

La rotation du plasma, déplace des raies spectrales. On peut en déduire une deuxième mesure de la vitesse de gyration.

(e) Action sur le gaz résiduel

L'interaction du plasma tournant sur les atomes neutres présents à l'origine, crée dans le plasma, un courant qui s'ajoute au courant de dérive dû à la rotation proprement dite. Il accroît la diffusion des particules sur les parois et réduit la durée du confinement.

(f) Chauffage du plasma

Il se produit par effet de diffusion des particules, (effet Joule) par frottement visqueux entre les couches de plasma par suite de rotation et par effet de striction.

La contribution la plus importante au chauffage est le frottement visqueux. Toutefois,

tous ces effets ne pourraient assurer une température thermonucléaire, on y supplée par le chauffage cyclotronique, obtenu en superposant au potentiel électrostatique constant une composante alternative de fréquence appropriée qui engendre des ondes hydromagnétiques.

TABLEAU COMPARATIF
des caractéristiques des deux principales bouteilles magnétiques à plasma tournant

Machines	Homopolaire II	Ixion 3
Forme géométrique	Cylindres coaxiaux	Cylindres coaxiaux
Hauteur du cylindre	7 cm	86 cm
Électrode intérieure: diamètre nature	7.5 cm métallique	3 cm plasma
Électrode extérieure: diamètre nature	24.5 cm métallique	24 cm métallique
Pression du gaz	10^{-4} mm Hg	2×10^{-5} mm Hg
Origine du plasma dans la bouteille	ionisation interne par décharge	bouffées de D_2 injectées au moyen d'une valve électromécanique et ionisées par ondes de chocs
Ordre de grandeur du champ électrique radial	500 à 1250 V/cm	50 à 2000 V/cm
Champ magnétique axial: $B(c)$, au centre $B(m)$, aux miroirs	4 à 10 kgauss 8 à 20 kgauss	8 à 10 kgauss 16 à 20 kgauss
Ordre de grandeur de la vitesse de dérive obtenue	3×10^6 à 6×10^6 cm/sec	3×10^6 à 6×10^6 cm/sec
Energie due à la rotation	20 à 60 eV pour D_2	30 à 50 eV pour D_2
Source extérieure: nature énergie	batterie de condensateur 48 μF 5 $\times 10^4$ joules	batterie de condensateurs 75 μF 3 $\times 10^3$ joules
Densité du plasma obtenu	10^{13} p/cm	2×10^{13} p/cm ³
Intensité de la décharge	50 kA	20 kA
Température du plasma	—	10^5 °K
Flux de neutrons obtenu	—	10^5 neutrons
Caractéristiques du condensateur équivalent au plasma tournant:		
Constante diélectrique	10^6 à 10^8	10^6 à 10^7
Capacité	6 μF environ	9 μF environ
Durée du confinement	80 μsec	probablement même durée

CONCLUSIONS

Les modèles étudiés par les laboratoires de Berkeley et U.C.R.L. ont prouvé la possibilité d'exploiter les effets dûs à la rotation du plasma pour le confinement, le chauffage du plasma tournant, comme aussi pour la réalisation d'un nouveau type de condensateur à décharge très rapide.

Ils ont permis plus généralement d'étudier les propriétés des plasmas tournants.

Par contre, leur développement en vue d'en faire des réacteurs thermonucléaires présente de nombreuses difficultés. Ce sont :

(a) l'apparition du potentiel électrostatique axial, dont les effets ne sont plus négligeables aux températures élevées.

(b) la nécessité d'utiliser des champs électriques de plus en plus intenses ce qui est très désavantageux au contact des électrodes au moment de la décharge.

(c) les pertes intenses par rayonnement, dues aux impuretés provenant des électrodes et des parois, limitent l'énergie qui peut être confinée,

(d) la stabilité d'un plasma tournant à température thermonucléaire est un problème encore non étudié,

(e) le retard à l'établissement de la rotation du plasma, dû à l'effet capacitif,

(f) la courte durée du temps de confinement à cause de la diffusion des particules.

Pourtant des perfectionnements prévus, permettront d'améliorer les résultats atteints dans les modèles expérimentés; dans ce but en vue de minimiser les effets nuisibles de la dérive due à la force centrifuge, réduire les effets de la diffusion des particules sur la paroi extérieure, et accroître le temps de confinement, les machines à plasma tournant devront être de diamètre plus grand. Elles pourront être munies d'une espèce de "divertor" ayant les mêmes fonctions que dans le "stellarator" celles d'entraîner les impuretés émanant des parois et réduire par suite les pertes par rayonnement.

Il faudra enfin, combattre l'effet d'écran qui retarde l'établissement de la rotation en fournissant aux électrodes des électrons et des ions par exemple par une préionisation.

7. Notes relatives à la Machine d'Osovetz

La classification ci-dessous permet de situer la Machine d'Osovetz parmi les machines à miroirs magnétiques.

Machines

1. à champ fixe (exemples: OGRA, DCX, magnétrons à ions, etc.).

2. à champ magnétique variable:

(a) où le moment magnétique $= mv_1^2/B$, reste constant (exemples: les machines de Post, etc.);

(b) où le moment magnétique n'est plus un invariant (exemples: le bétatron classique, la machine d'Osovetz, etc.).

Il est évident que la catégorie 2(a) n'est qu'un cas particulier de 2(b) et donc, a priori, certaines machines 2(b) devraient permettre d'obtenir des résultats, plus intéressants, par exemple un chauffage plus énergique. Mais évidemment, leur étude théorique est plus compliquée, et elle a à peine été effleurée.

A titre d'exemple, considérons ce qui se passe dans un bétatron classique (domaine non relativiste):

$$\frac{mv_1^2}{r} = ev_1B \quad (1)$$

donc:

$$\frac{mv_1^2}{B^2} = \frac{e^2 r^2}{m} = \text{constante}.$$

La température croît donc comme B^2 , alors qu'elle varie seulement avec B dans les machines où μ est constant.

Pourquoi alors ne pas utiliser un bétatron classique comme réacteur thermonucléaire? Tout simplement parce qu'une telle machine, qui peut donner une température très élevée fournit nécessairement une densité d'ions très faible. En effet, pour que la formule (1) soit valable, il faut que le self champ créé par les ions en mouvement soit négligeable par rapport au champ de guidage B , donc que le nombre d'ions soit très faible.

¹⁹⁾ Artsimovich, Conférence de Genève (1958) P/2298.

²⁰⁾ Osovetz *et al.*, Conférence de Genève (1958) P/2225.

²¹⁾ Physique des plasmas et problèmes de réactions thermonucléaires contrôlées. Institut atomique de l'Académie des Sciences de l'U.R.S.S.

Si les ions sont en quantité non négligeable, (1), s'écrit :

$$ev_1 B = \frac{mv_1^2}{r} + F_{\text{self, champ}}. \quad (2)$$

On calcule aisément que :

$$F_{\text{self, champ}} \approx \text{force centrifuge}$$

quand le nombre N de particules par cm de longueur du faisceau est de l'ordre de 10^{13} .

Pour faire de la fusion, on a avantage à avoir N le plus grand possible. Si en particulier $N \gg 10^{13}$, (2) se réduit à :

$$ev_1 B = F_{\text{self, champ}}. \quad (3)$$

La machine d'Osovetz est une machine satisfaisant à l'équation (3). La théorie fournit les conditions de fonctionnement suivantes :

pendant un temps limité t_0 . Passé ce temps, le champ de guidage devient trop fort et la spire de plasma se contracte.

La courbe $B(r)$ se compose d'une zone r_0 à r_1 où B reste presque constant, raccordée à une zone r_1 où B devient assez brusquement très grand. Après le temps t_0 , la spire de plasma se contracte donc lentement jusqu'au temps t , où son rayon est r_1 , puis s'effondre ensuite littéralement vers l'intérieur avec une très grande vitesse.

Pendant la période $0t_1$, le courant induit dans la spire de plasma croît continuellement, ce qui correspond à un emmagasinement d'énergie magnétique.

$$\frac{1}{2} L I^2$$

dans le système. Après le temps t , cette énergie

	Osovetz	Bétatron classique
Pour que: $r = c^{\text{te}}$	(4) $B_{\text{guidage}} = \frac{1}{4} B_{\text{moyen}} \left(1 + \frac{2}{l} \right)$	$B_{\text{guidage}} = \frac{1}{2} B_{\text{moyen}}$
Pour qu'il y ait stabilité	(5) $0 < n < \frac{3}{2} - \frac{l-2}{l(l+2)}$	$0 < n < 1$

$$\text{où } l = 2 \left[\log \frac{8r}{\pi} - 2 \right]$$

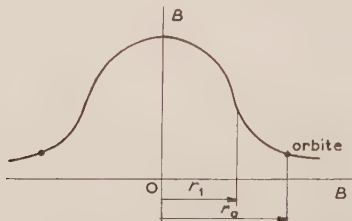


7.1. PRINCIPE DE LA MACHINE D'OSOVETZ

On décharge une batterie de condensateurs dans une spire unique de profil convenable créant un champ magnétique tel que (voir figure) :

le champ électrique induit soit suffisant pour ioniser le gaz présent,

en r_0 les conditions (4) et (5) soient sensiblement satisfaites au début du cycle d'accélération



Si l'on tient compte de la résistivité finie de la spire de plasma, la condition (4) reste satisfaite

est transformée en énergie cinétique radiale des particules (une partie de l'énergie disponible est en fait utilisée à comprimer le flux résiduel qui se trouve "gelé" à l'intérieur de la spire, ce que l'on peut éviter en créant au voisinage de l'axe un contre-flux constant dans le temps et égal justement à ce flux résiduel). Lorsque les particules arrivent au voisinage de l'axe, la densité est très grande et une interaction violente se produit, qui thermalise les vitesses. On aboutit donc à un état où une boule de plasma dense et chaud est localisée au centre de la machine.

7.2. RÉSULTATS EXPÉRIMENTAUX

batterie de condensateurs: $\begin{cases} 130 \mu\text{F} \\ 50 \text{ kV} \end{cases}$

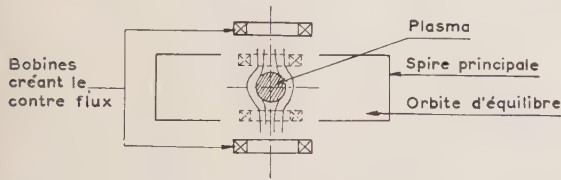
diamètre de l'orbite: 50 cm

pressions initiales: 10^{-1} à 10^{-2} mmHg

plasma final: $\begin{cases} T = 200 \text{ eV} \\ n = 10^{16} \text{ part./cm}^3 \end{cases}$

7.3. UTILISATIONS POSSIBLES

Dans son état actuel cette machine constitue un excellent injecteur de plasma dans un miroir



magnétique pulsé de type normal (schéma de principe ci-dessous). On sait en effet que la température en fin de compression est propor-

tionnelle à la température initiale, d'où l'avantage de la machine d'Osovetz qui permet de démarrer avec un plasma très chaud.

En traits pleins: machine d'Osovetz utilisée comme injecteur.

En pointilles: miroir magnétique pulsé.

Dans l'avenir: La machine d'Osovetz, dont les performances actuelles sont comparables avec celles des autres dispositifs de recherche thermonucléaire, semble pouvoir être améliorée sans qu'on n'aperçoive pour l'instant de limitation définie.

Symboles et Formules

$W_{\parallel}(c)$: composante de l'énergie cinétique parallèle au champ magnétique B , au centre du miroir.

$W_{\perp}(c)$: même chose... pour la composante normale.

R_m : $\frac{B(\text{miroir})}{B(\text{centre})}$ rapport du miroir.

V_D : vitesse globale de dérive.

V_E : vitesse de dérive due au champ électrique.

V_L : vitesse cyclotronique.

Φ : potentiel électrostatique appliqué.

l : longueur de la bouteille magnétique.

a : rayon de l'électrode interne (plasma ou métallique).

b : rayon de l'électrode externe.

C : capacité du condensateur équivalent.

ϵ : constante diélectrique.

N : densité ionique.

B : champ magnétique.

E : champ électrique.

j_D : densité du courant de dérive.

c : vitesse de la lumière.

μ : moment magnétique.

ρ : densité de masse.

$$V_E = \frac{cE}{B} = 10^8 \times \frac{E_{\perp} \text{ (volt/cm)}}{B \text{ (gauss)}}$$

$$V_L = \frac{eB\gamma}{mc} \quad \begin{matrix} m = m_e \text{ pour les électrons} \\ m = m_i \text{ pour les ions} \end{matrix}$$

$$E(v) = \frac{\Phi}{r} \frac{1}{\ln b/a}$$

$$j_D \approx -\frac{\rho}{B\gamma} V_E^2 \left[1 + \left(\frac{m_e}{m_i} \right)^3 \right]$$

$$V_D = \frac{1}{2} V_L [-1 + \sqrt{1 - 4V_E/V_L}]$$

Approximation

pour $V_E/V_L \ll 1$.

$$V_D \approx V_E.$$

7. NOTES ON FAST MAGNETIC COMPRESSION OF PLASMAS

H. L. JORDAN, H. KEVER and K. SCHINDLER

Technische Hochschule Aachen

In the first and main part of the report brief summaries are given of experimental work carried out at different labora-

tories on fast magnetic compression of plasmas. At the end, a short survey is presented of the theoretical work in this field.

1. Introduction

Earlier experiments at different laboratories have shown the possibility of obtaining plane shock waves of high Mach-numbers by magnetic acceleration¹). Further developments lead to experiments in which high temperature plasmas were obtained by fast magnetic compression of strongly preionized gases within magnetic coils. Temperatures achieved in some of these experiments appear to be in the keV range, temperatures of 10^8 °K are assumed to be obtainable. The high densities obtained in the compression phase are favorable for high thermonuclear reaction rates. Stability seems to be established to a rather high degree. These facts and the possibility of obtaining temperatures of some keV during times of the order of microseconds suggest the magnetic compression as an interesting heating mechanism for plasmas.

2. Methods

Magnetic compression experiments consist in general of condenser banks connected to coils of various geometries enclosing a discharge tube. During the discharge the time-dependent magnetic field induces an electric field and therefore currents in the plasma with current densities depending on the electrical conductivity of the plasma. If the conductivity is high enough, the magnetic field produced by these currents nearly cancels the external field. Currents are then limited to a surface layer (skin depth) of the plasma and the magnetic pressure drives the plasma towards the center. If the current rise time is much shorter than the time needed by small pressure disturbances to

travel from the plasma boundary to the axis, the process is essentially non-adiabatic and shock waves are expected. If the current rises slowly the plasma contracts adiabatically. At later stages of the discharge the first condition is not fulfilled and the further compression is adiabatic. Therefore both processes mix in the same experiment. Straight cylindrical coils are inefficient in confining the plasma in axial direction. In order to reduce the end losses magnetic fields with mirror geometry are used, produced by special coil shapes or additional coils.

3. Experiments

A great number of magnetic compression experiments exists which as far as known are briefly summarized:

AVCO, Research Laboratory, Everett, Massachusetts

G. S. Janes and R. M. Patrick²) used in their magnetic compression experiments a single turn flat copper coil with a preionization coil between the drive coil and the discharge chamber. The coil was connected to a 66 kV, 1.4 μ F condenser. By use of a special inductive energy storage system a current rise of 8×10^{11} A/sec was achieved. Smear camera pictures show that the velocity of the radial motion of the

¹) Magnetohydrodynamics, Ed. R. K. M. Landshoff (Stanford, 1957) Sect. 2, Magnetically driven shock waves: A. C. Kolb, Experiments at U.S. Naval Research Laboratory, pp. 76-91; S. W. Kash, Experiments at Lockheed Missile Systems Division pp. 92-98; H. E. Petschek, Experiments at AVCO, pp. 99-100.

²) G. S. Janes and R. M. Patrick, The production of high temperature gas by magnetic acceleration. Conference on extremely high temperatures, Boston 1958, Ed. H. Fischer and L. C. Mansur, p. 3.

plasma agrees with theoretical estimates of the plasma velocity.

A.W.R.E., Aldermaston

At A.W.R.E. both straight and toroidal magnetic compression devices ("Thetatron") have been examined but almost all practical work so far has been on a straight tube, 2 m long and 2-3" in diameter driven by a single turn straight coil of about 12" length. The driving unit is either a small fast bank or Maggi (25 kV, 40 kJ). The pressure range covered is from $\approx 20 \mu$ to 1000μ . The time to first compression is $\approx 1 \mu\text{sec}$ and about 10 pinches are observed in a half-cycle of the supply and load. Crowbar techniques have lengthened this time with promising results. Photographic techniques are employed and magnetic probes are being tried. No shaping of the coil for axial compression is normally introduced but neutrons ($\approx 5 \times 10^4$) are detected with the straight turn. (Personal communication by Dr. S. C. Curran.)

General Electric Company Research Laboratory, Schenectady, New York

Magnetic compression experiments are being made with a 160 000 joule, 70 kV condenser bank made up of 15 kV, $7.5 \mu\text{F}$ capacitors. Switching has been accomplished by means of a single pressurized spark gap. Single turn coils with mirror field geometry are used. With a 14 cm diameter coil the maximum field strength of 65 000 gauss is reached in under $4 \mu\text{sec}$. Over half the total voltage appears across the coil. The motion of the plasma is observed by smear camera; neutrons from nuclear reactions were detected. (Personal communication by Dr. Hurwitz.)

Institute of Physics, Techn. Hochschule Aachen

A single turn mirror coil is connected to a 25 kV, $4 \mu\text{F}$ condenser bank. The peak current is 300 kA, the current rise 10^{12} A/sec, peak field 30 kgauss in $0.4 \mu\text{sec}$, the ratio of coil to total inductance equals $\frac{1}{2}$. Spectroscopical measurements are carried out in the vacuum UV-region and UV.

Los Alamos Scientific Laboratory

J. W. Mather used an arrangement called "Orthogonal Pinch" with a 13 kJ condenser bank at 20 kV charging voltage. Fields of 17 kgauss were obtained in $5 \mu\text{sec}$. In these experiments breakdown occurs already at the second part of the first half cycle of the discharge. $B_z(r, t)$ is measured by means of magnetic probes and shows the existence of several current layers.

The "Scylla" experiment^{3,4}) consists of two condenser banks with $4.4 \mu\text{F}$ capacity and $0.13 \mu\text{H}$ inductance each, connected originally to two coils with $0.08 \mu\text{H}$ inductance. The magnetic field of the coils has a mirror ratio of 1.4. At 65 kV, the maximum charging voltage, the stored energy is 20 kJ. During the first half cycle of the discharge, field and plasma remain mixed and no ordered motion of the plasma is observed. At the second half cycle shock waves are moving in radial and axial directions. At the time of maximum current in the 2nd, to a lower extent in the 3rd and 4th half-cycles neutron pulses of $1 \mu\text{sec}$ duration and yields up to 10^6 - 10^7 neutrons per pulse are observed. X-rays are observed at times of maximum electric fields with maximum energies of at least 200 keV; they are probably produced at the wall by electrons escaping from the center of the discharge along magnetic lines of force. Neutron and X-ray emission are both pressure dependent. The neutron production depends on the mirror ratio and symmetry of the field; in case of a time delay between the pulsing of the two mirror coils the neutron production was lowered and vanished at delays of $0.5 \mu\text{sec}$.

Naval Research Laboratory Washington (A. C. Kolb)

We refer here only to the last experiments of the Naval Research Group. Previous experiments with magnetically accelerated shock waves in

³) W. C. Elmore, E. M. Little and W. E. Quinn, Neutrons from plasma compressed by an axial magnetic field (Scylla), Los Alamos Scientific Laboratory, University of California, A/Conf. 15/P/356.

⁴) W. C. Elmore, E. M. Little and W. E. Quinn, Phys. Rev. Letter 1 (1958) 32.

T- and H-shaped tubes are described in^{5,6,7)} and in the first part of the Geneva Report P/345⁸⁾.

In these experiments plasma is preheated by a high power rf-pulse and then heated by fast magnetic compression. The 2 μ sec rf-pulse is generated by discharging a 10 Mc/sec condenser (preheater) bank over the main coil. At 40 kV the preheater discharge gives an rf-amplitude of 20 kV with 10 Mc/sec frequency and 4×10^3 MW power superposed on 200 kA current oscillation with 1 Mc/sec frequency. The main discharge bank is triggered at the end of the rf-pulse with a time delay. The main discharge bank as 285 kJ at 20 kV, but only a part of this energy was used so far. By a multi-switch system a current rise of 8×10^{12} Amp/sec is reached. The coil consists of a long single turn coil with discs at the ends to produce mirror geometry, the coil inductance being in typical cases 0.007 μ H. Smear camera photographs of the interior of the discharge tube show without preheating during the first half cycle no changes in the structure while plasma and magnetic field are mixed. At the beginning of the second half cycle the photographs show a compression of the plasma; then bounces appear due to the expansion of the plasma boundary and the following compression of the plasma by the rising magnetic field. After the axial shocks collide at the center, the plasma expands again, is compressed once more and then returns to the wall as the discharge is damped out. With the preheater bank switched on previously to the main discharge the events observed without preheater bank in the second half cycle now appear already at the first half cycle of the main discharge. Without preheater bank there is only Balmer line emission in the first half cycle indicating neutral deuterium but already a high degree of ionization as the lines are strongly broadened by Stark effect. During the second half cycle the Balmer lines vanish and the bremsstrahlung continuum appears. During the compression after preheating no impurity lines appear in the spectrum, showing good isolation of the plasma from the wall.

Radial oscillations of the plasma beginning at the time of peak compression are observed, which extend over the whole plasma column and whose amplitudes are in general limited, so that the plasma does not reach the wall as long as the discharge is not damped out. With preheating the radial oscillations sometimes do not appear. The nature of these oscillations is not yet understood. One possibly meets here the same phenomena which in the case of slow pinch experiments lead to the enhanced loss of particles to the wall.

Measurements of the neutron yield with a plastic scintillation counter are considered to be consistent with a plasma energy of 1 keV, this energy estimate follows from considerations summarized in the next chapter.

Radiation Laboratory, University of California, Livermore

"Collapse"⁹⁾ was designed for studying strong hydromagnetic shock waves in the absence of particle collisions. For ionization and preheating adiabatic compression with a field of 2000 gauss was used. Bias coils served to establish mirror field geometry and to give the internal field necessary for the generation of shock waves in the collision free case. Magnetic probe measurements indicate that in the fourth half cycle of the slow collapse (adiabatic compression phase) the electrical conductivity becomes high enough to make the fast magnetic compression efficient (fast collapse). Measurements of plasma velocity by magnetic probes and by image converter photographs were in good agreement and gave 2×10^6 cm/sec when the fast collapse condenser (ringing frequency 2.5 Mc/sec) is charged up to 50 kV. Langmuir dual probe measurements gave radial voltage signals at the passage of radial shock waves in the order

⁵⁾ A. C. Kolb, Phys. Rev. **107**, II (1957) 345.

⁶⁾ A. C. Kolb, Phys. Rev. **107** (1957) 1197.

⁷⁾ A. C. Kolb, Phys. Rev. **112** (1958) 291.

⁸⁾ A. C. Kolb, Magnetic compression of shock preheated deuterium. Proc. 2nd U.N. International Conference on Peaceful Uses of Atomic Energy, Geneva 1958, Vol. **31**, p. 328.

⁹⁾ S. A. Colgate and R. E. Wright, Collapse—the shock—heating of a plasma. University of California Radiation Laboratory, A/Conf. 15/P/368.

of 20 volt corresponding to the ion energy behind the shock. This is in agreement with predictions discussed in an earlier paper of S. A. Colgate¹⁰).

4. Theory

The contributions to the theory of the magnetic compression are mostly concerned with the radial motion of the plasma. The problem has been treated approximately in several ways, which are shortly summarized as follows.

J. D. Jukes¹¹) considers the radial motion of a plasma cylinder as given by the similarity solution of G. Guderley for flows with strong cylindrical shocks. The penetration of the magnetic field into the plasma during the times considered are neglected. By equating the plasma pressure at the plasma boundary to the magnetic pressure he obtains an expression for the current needed at the plasma boundary to establish the assumed motion, a calculation of the change of magnetic flux outside the plasma gives an expression for the induced voltage. In the case of magnetic compression experiments the Guderley flow corresponds to an increasing current and an increasing voltage at the coil as the plasma moves inwards.

Calculations using the snow-plow model for the initial stages of the cylindrical plasma compression were performed by A. C. Kolb⁸). The compression times numerically computed from the equation of motion

$$\frac{d}{dt} \left\{ \rho_0 (a_0^2 - a^2) \frac{da}{dt} \right\} = -a \frac{H_z^2}{4\pi}$$

$a = a(t)$ plasma radius, $a_0 = a(0)$, and the circuit equation agree within 10% with the measured times, leading to the conclusion that one has a fairly good description of the dynamics of the system and the ion energy computed from this equation likewise agrees with the experiment. Under some conditions of Kolb's experiments the ion energy at the end of the first compression phase amounts to 80–90 eV. If the plasma undergoes an adiabatic compression with $\gamma = \frac{5}{3}$ in the second compression

phase a final temperature of approximately 7×10^6 °K at a density of 10^{17} ions/cm³ is calculated. Assuming an adiabatic exponent $\gamma = 2$, the circuit equation becomes integrable and current, detachment time, plasma-radius, pressure and temperature are easily computed. The temperature reached at maximum compression becomes

$$T_m = T_0 \left(\frac{W_c}{W_{pl}} \right)^{\frac{1}{2}} \frac{a_0}{R},$$

the maximum temperature T_m raising with the root of the initially stored condenser energy W_c ; T_0 , W_{pl} , a_0 are temperature, plasma energy and -radius at the beginning of the adiabatic compression, R coil radius. The external inductance is neglected as compared to the coil inductance.

The compression of a plane collision-free plasma initially in a homogeneous field B_0 parallel to the plasma boundary, by an increasing magnetic field has been treated by J. H. Adlam and J. E. Allan¹²).

Lagrange representation of the flow field equations for the electron and ion gas are used, friction terms and the thermal velocity as compared to the velocity of the ordered motions are neglected; quasi-neutrality is assumed throughout. The solutions are computed by numerical integration for a magnetic field increasing at the plasma boundary linearly with time

$$B = B_0 \left(1 + \frac{t}{\tau} \right)$$

with a specially chosen rise time

$$\tau = \frac{(m_i m_e)^{\frac{1}{2}}}{e B_0}$$

which is equal to the geometric mean of the ion and electron gyration period in the initial field B_0 .

¹⁰) S. A. Colgate, A description of a shock wave in free particle hydrodynamics with internal magnetic fields. University of California Radiation Laboratory Report, UCRL-4829 (Febr. 1957).

¹¹) J. D. Jukes, Atomic Energy Research Establishment Report AERE GP/R-2293 (Jan. 1958).

¹²) J. H. Adlam and J. E. Allen, Hydromagnetic disturbances of large amplitude in a plasma. Proc. 2nd. U.N. International Conference on Peaceful Uses of Atomic Energy, Geneva 1958, Vol. 31, p. 328.

The solution shows the increase of plasma density as the plasma boundary moves inwards; a space-charge field builds up and the magnetic field penetrates into the plasma. At $t \approx 1.6 \tau$ the particle density at the plasma boundary becomes infinite and the equations are no longer valid. The calculations show that during the rise time of the magnetic field the plasma reaches the velocity of the Alfvén-wave in the undisturbed plasma and field. For the diffusion or penetration depth of the magnetic field one obtains $d \approx c/\omega_p$, where ω_p is the plasma frequency. The same quantity appears as the thickness of plane steady hydromagnetic waves of large amplitude in the absence of collisions¹³).

Several papers deal with the question of the hydromagnetic shock transition zone thickness in the limit of great intercollision times¹⁴⁻¹⁷). If the plasma is collision-free, the thickness turns out to be under certain conditions of the order of the penetration depth mentioned above, but there exists no shock: the flow quantities ahead and behind the steady transition region are equal. If one inserts a frictional term, the flow quantities turn out to be different at the rear of

the wave; but the transition zone thickness is now of the order of the mean free path for ion-electron collisions rather than the penetration depth d or the geometric mean of the gyration radii of the ions and electrons. However, there are indications for the existence of damping mechanism for hydromagnetic waves without the intervention of particle collisions.

¹³) J. H. Adlam and J. E. Allen, *Phil. Mag.* **3** (1958) 448.

¹⁴) M. H. Rose, On the structure of steady hydromagnetic shock (New York University, 1956).

¹⁵) TID-7536 (pt. 1). *Proc. Conf. on Controlled thermonuclear Reactions*, 1957, pp. 32-38.

¹⁶) L. Davis, R. Lüst and A. Schlüter, *Z. Naturf.* **13a** (1958) 916.

¹⁷) L. Biermann, Recent work on controlled thermonuclear fusion in Germany: Magnetohydrodynamic shock waves, *Proc. 2nd, U.N. International Conference on Peaceful Uses of Atomic Energy*, Geneva 1958, Vol. **31**, p. 24.

¹⁸) C. S. Gardner, H. Goertzel, H. Grad, C. S. Morawetz, M. H. Rose and H. Rubin, Hydromagnetic shock waves in high-temperature plasmas, *Proc. 2nd. U.N. International Conference on Peaceful Uses of Atomic Energy*, Geneva 1958, Vol. **31**, p. 230.

¹⁹) R. S. Sagdeyew, B. B. Kadomtsev, L. I. Rudakov and A. A. Vedyonov, Dynamics of rarefied plasma in a magnetic field, Sect. 5. Non-linear uni-dimensional motion of rarefied plasma, *Proc. 2nd. U.S. International Conference on Peaceful Uses of Atomic Energy*, Geneva 1958, Vol. **31** p. 24.

8. CONFINEMENT OF PLASMA BY RADIO-FREQUENCY ELECTRO-MAGNETIC FIELDS

C. M. BRAAMS, W. J. SCHRADER and J. C. TERLOUW

FOM Institute for Plasma-Physics, Rijnhuizen, Jutphaas, Netherlands

A review is given of proposed methods for confinement of plasma by radio-frequency electro-magnetic fields.

Forces on charged particles in r.f. fields, the pressure balance, frequency limitations, the power balance of the

plasma, and external power loss are discussed. Briefly mentioned are confinement in combined d.c. and r.f. fields and other confinement schemes using alternating fields.

1. Confining Forces and Pressure Balance†

1.1. ARRANGEMENTS WITH LOW β -VALUES

The motion of a charged particle in a standing electro-magnetic field can often be described by splitting it in an oscillatory motion around a guiding centre and the motion of the guiding centre. Thus, the radius vector is given by

$$\mathbf{r} = \mathbf{r}_0 + \mathbf{r}_1 \quad (1)$$

where $\mathbf{r}_0$ locates the guiding centre and $\mathbf{r}_1$ describes the oscillation of the particle. If $\mathbf{r}_1$ and $\dot{\mathbf{r}}_0/\omega$ are small compared to the characteristic distance, L , over which the amplitudes of the field change appreciably, it can be shown^{1,2)} that, to a first approximation, the effect of the r.f. field can be described by

$$\mathbf{r}_1 = -\frac{e\mathbf{E}_0}{m\omega^2} \quad (2)$$

and

$$\ddot{\mathbf{r}}_0 = -\nabla \left(\frac{eE}{2m\omega} \right)^2 \quad (3)$$

where e and m are charge and mass of the particle, $\mathbf{E} \exp(i\omega t)$ is the electric field and E_0 is the amplitude of the electric field at the point $\mathbf{r}_0$. Eq. (3) is obtained by averaging the force on the particle over one period of the field variation, taking into account both the non-uniformity of the electric field, which results in a term $e(\mathbf{r}_1 \cdot \nabla)\mathbf{E}$, and the Lorentz force, $e\dot{\mathbf{r}}_1 \times \mathbf{B}$.

It follows from (3) that charged particles can, in principle, be confined in regions where E^2 has a minimum. The r.f. field acts mainly on the electrons, which, in turn, help to confine

positive ions by building up a negative space charge, giving rise to an additional term in eq. (3). Vacuum fields exist in which E^2 has minima; stationary confinement of plasma is possible if the presence of the plasma does not disturb the field too much and if the power dissipated in the plasma can be carried away by some mechanism at a temperature where kT is small compared to the height of the confining barrier.

The plasma will not disturb the vacuum field if the material currents are small compared to the displacement currents. This is the case, if

$$n_e e \dot{\mathbf{r}} \ll \omega \epsilon_0 \mathbf{E} \quad (4)$$

which, by virtue of (2), is equivalent to the requirement that the applied frequency is high compared to the plasma frequency³⁾:

$$\omega \gg \omega_p \equiv \left(\frac{n_e e^2}{m_e \epsilon_0} \right)^{\frac{1}{2}} \quad (5)$$

Confinement of particles of temperature T is only possible if the plasma is surrounded by a potential barrier which is appreciably higher than the mean thermal energy of the particles. Hence, for the electrons,

$$kT_e \ll \frac{e^2 E^2}{4m_e \omega^2} - e\varphi \quad (6)$$

where φ is the space-charge potential necessary for the confinement of the ions ($\varphi = 0$ at the point of minimum E^2). The values for E and φ

¹⁾ A. B. Gaponow and M. A. Miller, Soviet Physics JETP 7 (1958) 168 and 515, translated from JETP (USSR) 34 (1958) 242; and 34 (1958) 751.

²⁾ H. A. H. Boot *et al.*, J. of Electronics and Control 4 (1958) 434.

³⁾ L. Spitzer, Physics of Fully Ionized Gases.

† Rationalized MKS units will be used in this paper.

to be used in (6) are, of course, those of the lowest point of the confining barrier. For ion confinement:

$$kT_1 \ll Ze\varphi \approx \frac{n_e}{n_i} e\varphi \quad (7)$$

where Z is the charge number of the positive ions. Combination of (6) and (7) yields:

$$n_e kT_e + n_i kT_1 \ll \frac{n_e e^2 E^2}{4m_e \omega^2} = \left\{ \frac{\omega_p}{\omega} \right\}^2 \frac{\varepsilon_0 E^2}{4}. \quad (8)$$

From (8) and (5) follows, that the requirement that the plasma does not disturb the field involves that the plasma pressure must be low compared to the energy density of the confining field. The ratio between the energy density of the plasma and the time-averaged energy density of the confining field is sometimes called β ; thus,

$$\beta = (n_e kT_e + n_i kT_1) / \left(\frac{\varepsilon_0 \bar{E}^2}{2} + \frac{\bar{B}^2}{2\mu_0} \right). \quad (9)$$

The foregoing discussion, therefore, applies to the case that $\beta \ll 1$.

1.2. ARRANGEMENTS WITH HIGH β -VALUES

Configurations in which plasma is confined by an r.f. field may develop continuously from a vacuum-field by accumulation of charged particles in the regions where E^2 has minima. If the plasma frequency becomes comparable to the applied frequency, the field tends to be excluded from the plasma. This, on one hand, may extend the region of minimum E^2 , but, on the other hand, may disturb the intrinsic stability of the vacuum field. Modes in which the minimum of E^2 requires the presence of a conductor cannot develop continuously from a vacuum field. To excite these, pre-formation of the plasma is necessary. Also in this case, the stability of the configuration has to be considered.

If the density is so high that the field can only penetrate a thin boundary layer of the plasma, it can be said that the plasma is confined by radiation pressure. For a plane wave, reflected by a plane plasma boundary, the pressure-balance equation reads:

$$n_e kT_e + n_i kT_1 = \left(\frac{\varepsilon_0 E^2}{4} + \frac{B^2}{4\mu_0} \right) \quad (10)$$

where E and B are the field amplitudes outside the plasma. Eq. (10) also applies to confinement by standing waves of different geometries, as long as the field-penetration depth is small compared to other dimensions of the system; the case of finite depth of penetration in a cylindrical plasma has also been discussed^{2,4,5}. Since the plasma can usually be treated as a good conductor, E tends to be small at the surface, so that the confining pressure results mainly from the term $B^2/4\mu_0$. Assuming equal temperatures for electrons and ions, one thus obtains the pressure-balance equation

$$(n_e + n_i) kT = B^2/4\mu_0 = \bar{B}^2/2\mu_0 \quad (11)$$

which has been used by several authors^{6,7,8}.

2. Frequency Limitations

2.1. ARRANGEMENTS WITH LOW β -VALUES

It was noted already in sec. 1, that if it is required that the plasma does not disturb the field, the plasma frequency should be small compared to the frequency of the confining field. Hence,

$$\omega \gg \omega_p = \left\{ \frac{n_e e^2}{m_e \varepsilon_0} \right\}^{\frac{1}{2}}. \quad (5)$$

Furthermore, it was noted that the height of the confining barrier must be related to the temperature. Eq. (8) is equivalent to

$$\omega \ll \frac{eE}{\left\{ 4m_e k \left(T_e + \frac{n_i}{n_e} T_i \right) \right\}^{\frac{1}{2}}}. \quad (12)$$

Numerically, (5) and (12) yield

$$56n_e^{\frac{1}{2}} \ll \omega \ll 2.1 \times 10^5 E \left(T_e + \frac{n_i}{n_e} T_i \right)^{-\frac{1}{2}} \quad (13)$$

where temperatures are expressed in electronvolts.

A third restriction is that the amplitude of the electron oscillations in the r.f. field is small compared to the distance, L' , over which the

⁴) M. U. Clauser and E. S. Weibel, Geneva 1958, P/1774.

⁵) E. S. Weibel, J. of Electronics and Control 5 (1958) 435.

⁶) F. B. Knox, Australian J. Phys. 10 (1957) 566.

⁷) J. W. Butler *et al.*, Geneva 1958, P/350.

⁸) A. A. Vedenov *et al.*, Geneva 1958, P/2501.

field is applied. According to (2), this entails that

$$\omega \gg \left\{ \frac{eE}{m_e L'} \right\}^{\frac{1}{2}} = 4.2 \times 10^5 \left(\frac{E}{L'} \right)^{\frac{1}{2}} \approx \frac{4.2 \times 10^5 V^{\frac{1}{2}}}{L'} \quad (14)$$

where V has the dimension of a potential difference and is at least equal to the amplitude of the applied r.f. voltage. An equivalent formula follows from the requirement that the distance travelled by electrons of thermal speed in the time $1/\omega$ is small compared to L' .

The distance L' may be greater than the characteristic distance L , mentioned in sec. 1, if $\text{grad } E$ is perpendicular to $\mathbf{E}$. In a case where $\mathbf{E}$ is parallel to the axis of a long straight cylinder, eqs. (2) and (3) were shown³⁾ to be valid for $r_1 \ll L'$, rather than $r_1 \ll L$.

2.2. ARRANGEMENTS WITH HIGH β -VALUES

In most of the proposed arrangements for confinement by radiation pressure, the pressure exerted by the r.f. field on the plasma oscillates between zero and $2nkT$ with an angular frequency 2ω . In this case, one must require that expansion of the plasma with the thermal speed of the ions during one quarter cycle is small compared to a characteristic length R . Hence,

$$\omega \gg \frac{\pi}{2R} \left\{ \frac{kT_i}{m_i} \right\}^{\frac{1}{2}}. \quad (15)$$

This requirement is automatically fulfilled if the field is maintained inside a resonator, where ωR is of the order of magnitude of c . It is not applicable if confinement results from fields which are constant in magnitude but rotate in the plane of the plasma boundary.

For eq. (10) to be valid, it is necessary that the reverse of (5) is true, i.e., that $\omega \ll \omega_p$ and also that the skin depth, $d = c/\omega_p \ll R$. Further, equations (12) and (14) also apply to high- β arrangements.

3. Heating of the Plasma

3.1. ARRANGEMENTS WITH LOW β -VALUES

The oscillatory electron motions described by eq. (2) represent volume currents which give

rise to heating by Ohmic dissipation. The dissipated power per unit volume is

$$\overline{W_v} = j^2 \eta = \frac{1}{2} \left\{ \frac{n_e e^2 E}{m_e \omega} \right\}^2 \eta \text{ watt/m}^3 \quad (16)$$

where j is the current density and η is the volume resistivity of the plasma. Since the kinetic energy of the oscillatory motion of the electrons is of the order of kT , the applicability of the often used formula³⁾ for the resistivity of a hydrogen plasma:

$$\eta \approx 5 \times 10^{-5} T^{-\frac{3}{2}} \log A \text{ ohm meters} \quad (17)$$

is questionable. If, however, (17) is used for an order-of-magnitude calculation, (16) yields

$$\overline{W_v} \approx 2 \times 10^{-20} n_e^2 T^{-\frac{3}{2}} \ln A \frac{E^2}{\omega^2} \text{ watt/m}^3. \quad (18)$$

It is interesting to compare the power dissipation with the radiation by bremsstrahlung. Again for hydrogen, the radiated power is given by³⁾

$$\overline{W_r} \approx 1.5 \times 10^{-38} n_e^2 T^{\frac{1}{2}} \text{ watt/m}^3. \quad (19)$$

Hence the ratio of the radiated power to the dissipated power per volume element is roughly

$$\frac{\overline{W_r}}{\overline{W_v}} \approx 10^{-18} \frac{T^2 \omega^2}{\ln A E^2}. \quad (20)$$

To obtain this ratio for the whole plasma, $\overline{W_r}$ and $\overline{W_v}$ must be integrated over the whole volume. Comparison of (19) with (12), which gives ω/E for the lowest point of the confining barrier, shows that thermal equilibrium is only possible if the E^2 -barrier is restricted to a very small fraction of the total plasma volume.

3.2. ARRANGEMENTS WITH HIGH β -VALUES

The power balance of a plasma, confined in a resonator, heated by Ohmic dissipation of skin currents, and cooled by bremsstrahlung, has been discussed for spherical^{6,7)} and toroidal⁵⁾ cavities. The power dissipated per unit area of the plasma surface is

$$W_s = (\eta/\mu_0 d) (B^2/4\mu_0) \quad (21)$$

where η is the volume resistivity, d is the skin depth, and B is the amplitude of the magnetic field. In a quiescent plasma, the volume resistivity is approximately proportional to $T^{-\frac{3}{2}}$. If the applied frequency is large compared

to the collision frequency of the electrons, the skin depth is given³⁾ by

$$d = c/\omega_p \quad (22)$$

and is, therefore, proportional to $n^{-\frac{1}{2}}$. The pressure balance requires that $B^2/4\mu_0$ equals nkT . Consequently, for a plasma in a resonator of linear dimensions proportional to r , the power dissipated in the plasma is proportional to $r^2 n^{\frac{3}{2}} T^{-\frac{1}{2}}$. On the other hand, the power lost by bremsstrahlung is proportional to $r^3 n^2 T^{\frac{1}{2}}$. Hence, the ratio of the radiation loss to the dissipated power is proportional to $rn^{\frac{1}{2}} T$. Numerical calculations^{5,6,7)} show that very large and powerful equipment will be necessary to obtain stationary conditions at temperatures necessary for complete ionisation. However, it has been suggested that radiation from impurities⁷⁾ or heat conduction by neutral particles formed by recombination at a small metal surface in contact with the plasma⁶⁾ would make it easier to maintain a hydrogen plasma in power equilibrium in laboratory-scale apparatus.

4. Power Losses in the Walls of the Resonator

Since the temperature in an energy-producing thermonuclear reactor is mainly determined by the choice of the fuel mixture, it is fair to say that the gas pressure is proportional to n . This implies that $B^2 \propto n$ and, thus, that the surface currents in the resonator walls are proportional to $n^{\frac{1}{2}}$. The skin depth in the walls varies as $\omega^{-\frac{1}{2}}$, while $\omega \propto r^{-1}$. Hence, the power loss per unit area is proportional to $r^{-\frac{1}{2}} n$, and the total power loss in the walls, to $r^{\frac{3}{2}} n$. In comparison, the power from thermonuclear reactions is proportional to $r^3 n^2$, so that the ratio of the thermonuclear power output to the wall losses varies as $r^{\frac{3}{2}} n$. Most authors^{4,6,7,8,9)} agree that a power-producing reactor would have to be exceedingly large. This conclusion is generally based on the assumption that the linear dimensions of the plasma are a few times smaller than those of the resonator. It has been proposed²⁾ that a very thin plasma rod could be supported in a wide toroidal cavity with relatively low power losses, but this configuration was found⁵⁾ to be unstable.

5. Stability of the Confinement

Several authors have discussed stability for various special geometries, such as a plasma core in a spherical⁶⁾ or a toroidal^{4,5)} resonator, or a plane plasma boundary supported by a rotating¹⁰⁾ or a traveling⁸⁾ magnetic field. No experimental proof that a stable plasma, confined by rapidly alternating fields, can exist has yet been presented.

6. Combination of d.c. and r.f. Confinement

Most authors agree that skin losses in the walls make it impracticable to construct a power-producing thermonuclear reactor using pure r.f. confinement in a resonant cavity. It has been suggested^{4,8)}, however, that radiation pressure might be used as an additional means for stabilization or for plugging leaks in a stationary confining field. It should be noted that the discussions in the previous sections are all based on the assumption that the plasma is confined by r.f. fields only. The presence of a constant magnetic field has many interesting implications, which have not all been investigated.

The pressure exerted by a circularly polarized wave, with its wave vector along a stationary magnetic field, has been discussed⁸⁾ in detail. The pressure balance is not affected by the d.c. field if the r.f. frequency is intermediate between the electron and ion cyclotron frequencies. The depth of penetration of the r.f. field into the plasma, however, is increased by a factor $\sqrt{\omega_{ce}/\omega}$, where ω_{ce} is the electron cyclotron frequency. It has been shown experimentally⁸⁾ that an alternating magnetic field of 60 oersted can balance the pressure of a plasma with a density of 10^{19} particles/m³ and a temperature of 5×10^4 °K; the steady field in this case was up to 2000 oersted.

7. Proposed Experiments in Resonant Cavities

Several authors have been attracted by the possibility of creating a stationary high-temperature plasma inside a resonator, because of its value in basic plasma research. Proposals

⁹⁾ A. J. Hatch, Geneva 1958, P/351.

¹⁰⁾ J. L. Tuck, Geneva 1958, P/1860.

have been worked out on the basis of the following design parameters.

meters), which theory predicts for a quiescent thermonuclear plasma, could be obtained, the

TABLE
of resonator experiments

Reference	⁶⁾	⁷⁾	²⁾	⁸⁾
Density (at/m ³)	2.5×10^{17}	2×10^{18}	3×10^{24}	10^{19}
Temp. (°K)	4×10^5	10^7	10^6	5×10^4
Radius of cavity (m)	1.5	0.3	0.25	0.1
ω (radians/sec)	10^9	5×10^9	3×10^9	2×10^{10}
Power consumption (MW)	0.1	1	0.14	0.4†

† r.f. power only.

The experimental figures from reference⁸⁾ in the fourth column have been added by way of comparison; in this experiment, the r.f. field exerted a force on the plasma parallel to the lines of force of a d.c. magnetic field. The power figure quoted does not include the power needed to maintain the constant magnetic field.

8. Other modes of Confinement

8.1. MULTIPACTING PLASMA

Stationary confinement of positive ions in the potential well of a multipacting electron cloud has been discussed⁹⁾.

8.2. THE ALTERNATING I_z , B_θ AND I_θ , B_z PINCH

It was mentioned in sec. 2.2 that confinement in a magnetic field of constant magnitude whose direction rotates in the plane tangential to the plasma boundary is not subject to severe frequency limitations, apart from those which might follow from stability requirements. A possible way to obtain this field configuration is to induce alternating currents, one quartercycle out of phase, in the axial and the tangential directions in a cylindrical or toroidal plasma.

If the applied angular frequency is reduced to a value below the electron collision frequency, the confining field can penetrate the plasma by resistive dissipation, the depth of penetration being given by $d = (2\eta/\mu_0\omega)^{\frac{1}{2}}$. It has been argued¹⁰⁾ that this would lead to configurations which, in a particular phase of the cycle, are analogous to an I_z , B_θ pinch with entrapped B_z field and reversed external B_z , which is hoped to be more stable than an ordinary pinch.

If the low resistivity ($\eta = 10^{-9}$ to 10^{-10} ohm

applied frequency could be exceedingly low. However, small-scale experiments at intermediate temperatures would call for frequencies in the kc/s to Mc/s region. This alternating pinch would have the advantage that it would not involve the excessive skin losses which make resonant cavities seem unattractive for power producing reactors.

9. Conclusions

Confinement of plasma by radiation pressure with laboratory-scale apparatus is possible; it does not, for the moment, appear to be a useful proposition for the production of thermonuclear power, because of high skin losses in the resonator walls.

Heating of the plasma will occur by ohmic dissipation in the skin layer; excessive heating is a great difficulty in attaining stationary conditions in laboratory-scale experiments.

Stability of the confinement has been studied theoretically but has not been proved experimentally.

Alternating electro-magnetic fields might be used as auxiliary means for confinement and stabilization.

Acknowledgment

This work is part of the research programme of the "Stichting voor Fundamenteel Onderzoek der Materie" (Foundation for Fundamental Research on Matter—F.O.M.) and was made possible by financial support from the "Nederlandse Organisatie voor Zuiver-Wetenschappelijk Onderzoek" (Netherlands Organization for the advancement of Pure Research—Z.W.O.).

9. RELATIVISTIC ELECTRON BEAM DEVICES FOR FUSION

J. G. LINHART (section 1-3)

and

A. SCHOCH (section 4, 5)

CERN, Geneva

Equilibrium and stability of relativistic electron beams is studied with the view of evaluating the potentialities of such configurations for the confinement of hydrogen

plasma. Tentative criteria for the use of these in a fusion reactor are derived. Particular attention is given to the heating of plasma and the proposed Astron-scheme.

Introduction

Schemes for confinement of hot plasma by means of relativistic electron streams can be regarded as related either to the toroidal pinch confinement, in which the conduction current is replaced by a toroidal stream of relativistic electrons, or to a magnetic mirror confinement in which the diamagnetic currents in the plasma are augmented by a current of relativistic electrons.

In either of these cases, the relativistic electrons, owing to their small scattering cross-section, should possess a lifetime of the order of seconds and the relativistic beam could be, therefore, regarded as a type of super-conducting permeable fluid. It is this property and also an immunity to some types of instabilities that suggests their application to the fusion problem.

A serious consideration of the relativistic beam as a confining agent has been made, so far, only by very few groups. In this report we shall discuss first the equilibrium of a sheet and cylindrical beam for the case in which $T_e \geq T_p$ and also for the case in which $T_e \ll T_p$. In the second section we shall mention the main stability criteria for such streams.

In the third section we shall calculate the energy loss from relativistic beams and the generation of fusion energy followed by the discussion of the reactor equation.

In the fourth section we shall mention the heating of plasma by relativistic electrons and related problems.

The fifth section will be devoted to comments on the Astron scheme.

1. Equilibrium Configurations of Relativistic Electron Beams

The structure of partially neutralized relativistic streams has been analysed by several workers^{1,2}). Such an analysis represents only the second part of the history of a relativistic stream, the first part being its production, the third part being related to its stability. In some cases an equilibrium may not exist, as the stream may disintegrate due to instability even before an equilibrium is reached. Some of the problems related to the production of relativistic electron streams are analysed by Bennett³), de Packh⁴), Christofilos⁵) and Linhart⁶). The stability of such streams against some types of perturbations has been discussed by Budker⁷), Polovin^{2,8}), Sturrock and Finkelstein⁹). However,

¹) G. I. Budker and S. T. Belyev, Plasma problems etc., U.S.S.R., Vol. 2 (1954) p. 330.

²) R. V. Polovin and N. L. Tsintsadze, Jour. Tech. Fiziki 2 (1957) 1354; 2429.

³) W. H. Bennett, Phys. Rev. 45 (1934) 89; 98 (1955) 1584.

⁴) De Packh, Proc. of Conf. on el. discharges in Gases, Venice, 1957.

⁵) N. Christofilos, Astron Thermonuclear Reactor. Geneva Conference on Atomic Energy, Paper 15/P/2446 (1958).

⁶) J. G. Linhart, Proc. Roy. Soc. 249 (1959) 318.

⁷) G. I. Budker, Proc. Accelerator Conf., Geneva 1956.

⁸) R. V. Polovin and N. L. Tsintsadze, viz. ref. ²) and also the same authors: Jour. Tech. Fiziki 3 (1958) 637.

⁹) P. A. Sturrock and D. Finkelstein, Stability of Relativistic Self-focusing Streams, CERN PS Internal report (1959).

stability criteria often depend on the equilibrium configuration considered.

In the first section of this chapter we shall introduce the basic equations used in the study of equilibrium configurations.

In the second section we shall analyse an equilibrium of a relativistic current layer.

In the third part a similar analysis will be outlined for a cylindrical stream.

1.1. BASIC EQUATIONS

An approximate analysis of relativistic electron streams can be based on the two fluid model of plasma. Assuming that frames of reference exist in which the respective fluids, i.e. the electron gas and the positive ion gas, possess a Maxwellian distribution of random velocities (fig. 1) and that the velocity of the electrons is much larger than their mean random velocity,

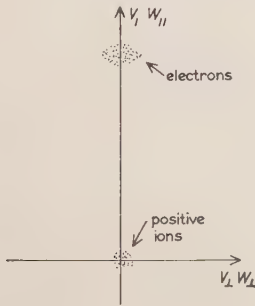


Fig. 1.

Electron and positive ion distributions in velocity space.

one obtains four equations of motion for the two fluids (written here for a hydrogen plasma, ref. 2,8)).

$$m\gamma^3 \frac{dv_{\parallel}}{dt} = -eE_{\parallel} - \frac{e}{c} (\mathbf{v} \times \mathbf{B})_{\parallel} - \frac{(\text{grad } p_e)_{\parallel}}{n_e} \quad (1a)$$

$$m\gamma \frac{dv_{\perp}}{dt} = -eE_{\perp} - \frac{e}{c} (\mathbf{v} \times \mathbf{B})_{\perp} - \frac{(\text{grad } p_e)_{\perp}}{n_e} \quad (1b)$$

$$M \frac{dw_{\parallel}}{dt} = eE_{\parallel} + \frac{e}{c} (\mathbf{w} \times \mathbf{B})_{\parallel} - \frac{(\text{grad } p_p)_{\parallel}}{n_p} \quad (2a)$$

$$M \frac{dw_{\perp}}{dt} = eE_{\perp} + \frac{e}{c} (\mathbf{w} \times \mathbf{B})_{\perp} - \frac{(\text{grad } p_p)_{\perp}}{n_p} \quad (2b)$$

where $\mathbf{v}$ and $\mathbf{w}$ respectively are the electron velocity and the positive ion velocity respectively. The p_e , p_p , and n_e and n_p are the respective pressures and particle densities.

Together with the equations of motion one has to consider the field equations, i.e.

$$\text{curl } \mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t}, \quad \text{div } \mathbf{E} = 4\pi e (n_p - n_e), \quad (3ab)$$

$$\text{curl } \mathbf{B} = \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t} - \frac{4\pi e}{c} (\mathbf{v} n_e - \mathbf{w} n_p), \quad \text{div } \mathbf{B} = 0. \quad (4ab)$$

Finally, as in all hydrodynamical problems, there are the equations of continuity

$$\frac{\partial n_e}{\partial t} = -\text{div} (n_e \mathbf{v}), \quad \frac{\partial n_p}{\partial t} = -\text{div} (n_p \mathbf{w}). \quad (5ab)$$

For some problems a formulation of these equations in terms of field potentials $\mathbf{A}$ and Φ is more convenient than that using field strengths $\mathbf{E}$ and $\mathbf{B}$.

1.2. SHEET STREAM

We shall treat briefly the equilibrium of a sheet electron-stream (fig. 2).

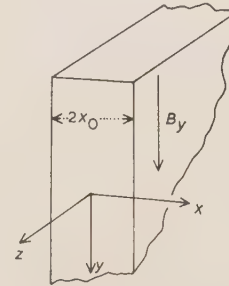


Fig. 2. Geometry of the plasma sheet equilibrium.

This model resembles a piece of the E layer as proposed in the Astron scheme (ref. 2)).

Neglecting the scattering and radiation loss and assuming that in a relativistic case $v_{\parallel} = \text{constant}$, an equilibrium is possible if

$$\begin{aligned} E_{\parallel} &= 0, & \frac{d}{dt} &= 0, \\ v_{\perp} &= w_{\perp} = w_{\parallel} = 0, & E_{\perp} &= E, \\ v_{\parallel} &= v = \text{const}, & B_{\perp} &= B_y = B. \end{aligned}$$

Equations (1)–(4) reduce to

$$eE = e \frac{v}{c} B - \frac{1}{n_e} \frac{\partial p_e}{\partial x} \quad (6)$$

$$eE = \frac{1}{n_p} \frac{\partial p_p}{\partial x} \quad (7)$$

$$\frac{\partial B}{\partial x} = -\frac{4\pi e}{c} n_e v \quad (8)$$

$$\frac{\partial E}{\partial x} = 4\pi e(n_p - n_e). \quad (9)$$

In an isothermal and Maxwellian ensemble of particles one can put

$$\text{grad } p_e = kT_e \text{ grad } n_e \quad (10a)$$

$$\text{grad } p_p = kT_p \text{ grad } n_p. \quad (10b)$$

From equations (6), (7), (8) and (10) one obtains

$$\frac{\partial}{\partial x} \left(\frac{T_e}{T_p} \frac{\partial n_e / \partial x}{n_e} + \frac{\partial n_p / \partial x}{n_p} \right) = -\frac{4\pi e^2}{kT_p c^2} v^2 n_e \quad (11)$$

whereas from equations (7) and (9) one has

$$\frac{\partial}{\partial x} \left(\frac{\partial n_p / \partial x}{n_p} \right) = \frac{4\pi e^2}{kT_p} (n_p - n_e). \quad (12)$$

These simultaneous non-linear differential equations are difficult to solve.

A particular solution is obtained if one assumes that (ref. 2))

$$n_p = \eta n_e$$

where η is a constant. In that case eqs. (11) and (12) are compatible if

$$\left(\frac{T_e}{T_p} + 1 \right) (\eta - 1) = \beta^2.$$

However, this set of particular solutions does not include some of the more realistic density distributions. We shall treat two cases which are of interest here

$$T_e \gg T_p \quad (T_p = 0) \quad (a)$$

$$T_p \gg T_e \quad (T_e = 0). \quad (b)$$

(a) In this case eq. (11) simplifies to

$$\frac{\partial}{\partial x} \left(\frac{\partial n_e / \partial x}{n_e} \right) = -\frac{4\pi e^2 \beta^2}{kT_e} n_e \quad (11a)$$

the solution of which is (fig. 3):

$$n_e = \frac{n_{0e}}{(\cosh x/x_0)^2} \quad (13)$$

where

$$n_{0e} x_0^2 = \frac{1}{2\pi} \frac{kT_e}{e^2 \beta^2} \quad (14)$$

which is the pinch relation for a sheet-beam. It then follows from eq. (12) that

$$n_p = \frac{n_{0p}}{(\cosh^2 x/x_0)^2} \quad (12a)$$

where

$$n_{0p} - n_{0e} \rightarrow 0 \quad \text{as} \quad T_p \rightarrow 0.$$

This leads us to a model of a quasi neutral relativistic plasma; we shall call this configuration a mixed beam model.

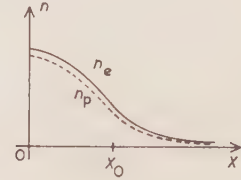


Fig. 3. Density distributions in the sheet for $T_e \gg T_p$.

(b) In this case the electromagnetic forces balance the transversal pressure of the positive ions only.

Eq. (11) gives

$$\frac{\partial}{\partial x} \left(\frac{\partial n_p / \partial x}{n_p} \right) = -\frac{4\pi e^2 \beta^2}{kT_p} n_e \quad (11b)$$

whereas eq. (12) is written as

$$\frac{\partial}{\partial x} \left(\frac{\partial n_p / \partial x}{n_p} \right) = \frac{4\pi e^2}{kT_p} (n_p - n_e). \quad (12b)$$

A solution satisfying both equations is possible only if

$$-n_e \beta^2 = n_p - n_e$$

i.e. if

$$n_e = n_p \gamma^2. \quad (15)$$

If such a solution were to hold for the whole of the space occupied by the beam, the linear densities of electrons and positives would have to be also in the ratio γ^2 , and the solution would be of the type $n_p = \eta n_e$ mentioned above. This corresponds to a non-physical situation, as the stored electric energy tends to infinity.

However, if one postulates an overall neutrality of the stream, the electron distribution becomes cut-off at a certain distance x_1 . For $x > x_1$ there is only a positive ion atmosphere

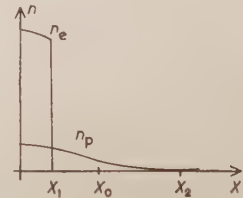


Fig. 4. Density distributions in the sheet for $T_e \ll T_p$.

confined by the electric field of the cold electron core (fig. 4). The equations (11b) and (12b) give for $x > x_1$

$$n_e = n_p \gamma^2 = \frac{n_{0e}}{(\cosh x/x_0)^2} \quad (16)$$

whereas outside $x = x_1$ one has to consider only eq. (12b) with $n_e = 0$. This gives

$$n_p = \frac{n_1}{\left(1 + \frac{x - x_1}{x_2}\right)^2} \quad (17)$$

where

$$x_2 = \frac{kT}{4\pi e^2 n_{0e} (1 - \gamma^{-2}) x_0 \operatorname{th} \frac{x_1}{x_0}}$$

$$n_1 = \gamma^{-2} \frac{n_{0e}}{(\cosh x_1/x_0)^2}.$$

Matching the n_p and $\partial n_p / \partial x$ distributions at $x = y_1$ one obtains for $\gamma \gg 1$

$$x_1 = x_0 \gamma^{-1}, \quad x_2 \cong x_0 \gamma \quad (18ab)$$

and a pinch condition

$$x_2 v = \frac{1}{\pi} \frac{kT_p}{mv^2} \quad (19)$$

where

$$v = 2 \frac{e^2}{mc^2} \int_0^{x_1} \frac{n_{0e} dx}{(\cosh x/x_0)^2}. \quad (20)$$

This equilibrium configuration will be called the cold electron core model.

1.3. CYLINDRICAL BEAM

Making the same assumptions as were made for the sheet beam one obtains a set of equations equivalent to eqs. (6), (7), (8) and (9). These are for $T_e = 0$

$$0 = -\frac{eE_r}{m\gamma} + \frac{e}{m\gamma c} v_z B_\phi \quad (21)$$

$$0 = \frac{eE_r}{M} - \frac{kT}{M} \frac{\partial n_p / \partial r}{n_p} \quad (22)$$

$$\frac{1}{r} \frac{\partial}{\partial r} (r B_\phi) = -\frac{4\pi}{c} e n_e v_z \quad (23)$$

$$\frac{1}{r} \frac{\partial}{\partial r} (r E_r) = -4\pi e (n_e - n_p) \quad (24)$$

From (21) and (22) one gets

$$B_\phi = \frac{kT}{\frac{e}{c} v_z n_p} \frac{\partial n_p}{\partial r}. \quad (25)$$

This equation is valid as long as eq. (21) is valid. The latter is limited to the region occupied

by the electron stream. For this region ($r < r_1$) one obtains using eq. (25) and (23) and assuming that $v_z = \text{const} = v$

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial n_p}{\partial r} \right) = -\frac{4\pi e^2}{kT c^2} n_e v^2. \quad (26)$$

From eq. (21) one has

$$E_r = \frac{v}{c} B_\phi.$$

Substitute this into eq. (24) and use also eq. (5). Thus one obtains

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial n_p}{\partial r} \right) = -\frac{4\pi e^2}{kT} (n_e - n_p). \quad (27)$$

Comparing eq. (6) and (7) one gets a condition for n_e and n_p

$$n_e = \frac{n_p}{1 - \frac{v^2}{c^2}} = \gamma^2 n_p. \quad (28)$$

Substituting eq. (28) into eq. (26) one has

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial n_e}{\partial r} \right) = -\frac{4\pi e^2}{kT c^2} n_e v^2$$

and the solution is

$$n_e = \frac{n_{e0}}{\left(1 + \frac{r^2}{r_0^2}\right)^2} \quad (29a)$$

where

$$\pi r_0^2 n_{0e} \frac{e^2}{mc^2} = \frac{kT}{\frac{1}{2} m v^2}. \quad (30)$$

All these relations apply to the region occupied by the electrons ($r < r_1$). Let us now solve eq. (22), (23), 24 for $r > r_1$.

In this region one encounters only an isothermal positive ion atmosphere for which

$$eE = kT \frac{\partial n_p}{\partial r} \quad (31)$$

$$\frac{\partial}{\partial r} (rE) = 4\pi e n_p r. \quad (32)$$

The solution of eqs. (31), (32) matching the solution given by eq. (29a) at the boundary of the electron core, is

$$n_p = \frac{kT}{2\pi e^2 \gamma^2} \left[\ln \frac{r}{r_1} - \frac{\sqrt{kT}}{\sqrt{2\pi} e r_1 \sqrt{n_1}} \right]^{-2} \quad (29b)$$

where

$$\frac{r_1}{r_0} = \frac{1}{\beta} (1 + \gamma^{-1}).$$

The ratio of linear charge densities of the electron and ion beams is

$$\eta = \frac{N_p}{N_e} \approx \frac{1}{\gamma}. \quad (33)$$

The cold-core model for cylindrical beams predicts, therefore, highly negative beams.

2. The Stability of Relativistic Streams

A partially neutralised electron stream can be suspected of many more types of instability than an ordinary pinch. Let us list these below.

Instability ←	hydromagnetic:	all the usual pinch instabilities ($m = 0, 1, \dots$)
	electromagnetic:	growing longitudinal electron plasma waves
	electric:	Webster's instability (ref. ¹⁰)
	hydrodynamic:	kink ($m = 1$) instability
	slow instabilities: ←	rise in electron temperature due to e-p scattering and inadequate radiation cooling runaway instability due to a decreasing scattering cross-section violation of the betatron condition.

It can be shown that the hydrodynamic forces F_h acting on a relativistic beam are in most cases stronger than the pinch forces F_m due to its own magnetic field. Thus for a cylindrical beam

$$\frac{F_h}{F_m} = \frac{\gamma}{v \ln \frac{R}{r_1}}$$

where R is the radial extent of the pinch field and r_1 is the radius of the electron beam. Thus the instability with respect to these should be investigated first.

An approximate but relatively simple analysis of this instability is based on the two-string model of a neutralised electron beam. The

electron beam is assumed to be bound to the positive beam by a restoring electrostatic force (fig. 5)

$$F_e = 2\pi e^2 n_p N_e (x_p - x_e) \quad (\text{dynes/cm}).$$

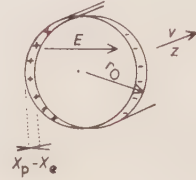


Fig. 5. The two-string model of a cylindrical neutralised electron beam.

The equation of motion for the two beams is

$$N_e m \gamma \left(\frac{\partial}{\partial t} + \frac{\partial}{\partial z} \right)^2 x_e = 2\pi e^2 n_p N_e (x_p - x_e) \quad (34a)$$

$$N_p M \frac{\partial^2}{\partial t^2} x_p = -2\pi e^2 n_p N_e (x_p - x_e). \quad (34b)$$

According to the usual small amplitude theory one may study an elementary harmonic disturbance in z -direction. The form of x_e and x_p is, therefore,

$$\exp j(\omega t + k z).$$

Eqs. (34ab) become

$$(\omega + vk)^2 x_e = -2 \frac{\pi e^2 n_p}{M} \frac{M}{m \gamma} (x_p - x_e) \quad (35)$$

$$\omega^2 x_p = 2 \frac{\pi e^2 n_p}{M} \frac{N_e}{N_p} (x_p - x_e). \quad (36)$$

From which one can derive a dispersion equation

$$(\omega + vk)^2 \left(\Omega^2 \frac{N_e}{N_p} - \omega^2 \right) + \frac{M}{m \gamma} \Omega^2 \omega^2 = 0 \quad (37)$$

where $\Omega^2 = (2\pi e^2 n_p / M)$.

In order that all the four roots for ω be real and therefore the perturbation a non-growing one, one has to satisfy

$$kv > \sqrt{\frac{2\pi e^2 n_p}{m \gamma}} \left\{ 1 + \left(\frac{N_e}{N_p} \frac{m \gamma}{M} \right)^{\frac{1}{3}} \right\}^{\frac{3}{2}} \quad (38)$$

or

$$\lambda > \sqrt{2} \pi r_0 \beta \sqrt{\frac{\gamma}{v_p}} \left\{ 1 + \left(\frac{v_e}{v_p} \frac{m \gamma}{M} \right)^{\frac{1}{3}} \right\}^{-\frac{3}{2}} \quad (38a)$$

where for cylindrical beams

$$v = 2\pi \frac{e^2}{mc^2} \int_0^\infty n r dr.$$

Let us apply this formula to the equilibrium configurations (a) and (b). For (a) we have $v_e = v_p = v$ and for $m\gamma < M$ one gets for stability

$$\lambda < 4.5 r_0 \beta \sqrt{\frac{\lambda}{v}} \quad (\text{cm}). \quad (39)$$

For (b) one should take as the positive beam only that portion of the positive cloud which is inside the electron stream. In that case it follows from eq. (15) that $v_p/v_e = \gamma^{-2}$ and therefore

$$\lambda < 4.5 r_0 \beta \gamma^{\frac{3}{2}} e^{-\frac{1}{2}} \left\{ 1 + \gamma \left(\frac{m}{M} \right)^{\frac{1}{3}} \right\}^{-\frac{3}{2}} \quad (\text{cm}). \quad (40)$$

However, the occurrence of strong electric stresses inside the cold-core configuration suggests that a type of Webster instability may develop.

Example: $v = 1$, $\gamma = 16$, $m/M = \frac{1}{3700}$, $r_0 = 1$ cm, then from eq. (39) follows $\lambda < 22.3$ (cm), whereas from eq. (40), $\lambda < 120$ (cm).

The presence of hot stationary plasma inside the relativistic stream may affect its stability. No analysis of this effect was available at the time of writing.

Most of the other instabilities do not appear as dangerous as the hydrodynamic one and here we shall only give references to generally or privately available treatments of these instabilities.

Electromagnetic (multiple stream mechanism) ref. ¹¹).

Electrostatic (a) in parallel magnetic field: ref. ¹⁰); (b) in a betatron field: ref. ¹²).

Slow instabilities (a) excessive scattering: ref. ⁶); (b) runaway instability: ref. ⁷).

3. Energy Loss from Plasma Confined by Means of Relativistic Electron Streams. Reactor Equation

There are probably many different energy losses from the relativistic stream. Three of these appear more important than others.

¹⁰) H. F. Webster *et al.*, I.R.E. Trans. ED-3, (1956) 172.

¹¹) J. R. Pierce, I.R.E. Trans. ED-3 (1956) 183;

A. W. Haeff, Proc. Inst. Radio Engrs. 37 (1949) 4;

J. R. Pierce, J. Appl. Phys. 19 (1949) 231.

¹²) C. E. Nielsen, Private communication (to appear shortly as an internal CERN Report).

(a) Cyclotron radiation from relativistic electrons.

(b) Scattering.

(c) Bremsstrahlung.

It is evident that if there is no scattering of relativistic electrons on particles stationary in the laboratory frame of reference the losses (a) and (c) do not occur. It is equally evident that in a e-p collision the bremsstrahlung emitted represent a smaller energy loss than the loss of longitudinal energy† suffered by the scattered particle. We shall therefore neglect (c) in comparison with (b).

The magnitude of the losses (a) and (b) depend on the type of equilibrium distribution. As expected the cold-core model, in which the electron beam and the positive cloud are almost separated, is much less lossy than the mixed beam model. In the following we shall consider both these models.

3.1. CYCLOTRON RADIATION

In a mixed cylindrical beam having the appropriate Bennett distribution we get in the system of reference of the relativistic electrons (primed symbols evaluated in this system) for the radiation output

$$W'_{\text{cyc}} = 2\pi \frac{e^2}{c^3} \int_0^\infty (\dot{v}_T)^2 \cdot n'_e r dr \quad (\text{erg/sec, cm}) \quad (41)$$

$$\text{where} \quad \dot{v}_T = \frac{e}{c} v'_T B' = \frac{e}{c} \sqrt{\frac{2kT'_e}{m}} B'. \quad (42)$$

Using a pinch relation for a mixed cylindrical beam (ref. ⁷) one finds for the cyclotron radiation output per electron:

$$W'_{\text{ec}} = \frac{2}{3} e^2 c (v\gamma)^3 \frac{\beta^2}{r_0^2} \left(\beta^2 - \frac{2kT_p}{vmc^2} \right) (\text{erg/sec, el}). \quad (43)$$

3.2. SCATTERING LOSS

This can be calculated using the coefficient of dynamical friction for the positives in the frame of the electrons

$$W'_s = \frac{4\pi e^4}{Mv} \ln A \int_0^\infty 2\pi r \frac{n_0^2}{\left(1 + \frac{r^2}{r_0^2}\right)^4} dr \quad (\text{erg/sec, cm}) \quad (44)$$

or per electron

$$W'_{\text{es}} = 6e^2 c \frac{r\gamma}{\beta r_0^2} \quad (\text{erg/sec, el}). \quad (45)$$

† Not toteneal rgy. See section 4.

3.3. TOTAL LOSS

The total loss of energy in the laboratory frame of reference can be evaluated as if it were due to a friction force.

$$F = \gamma^{-2} \frac{W'_{es}}{v} + \frac{v}{c^2} W'_{ec} \quad (46)$$

and the total loss per electron is therefore

$$W_e = vF = \frac{e^2 c}{r_0^2} \left[6 \frac{v}{\gamma \beta} + \frac{2}{9} (v\gamma)^3 \beta^6 \left(1 - \frac{2kT_p}{\beta^2 v m c^2} \right) \right]. \quad (47)$$

3.4. OPTIMUM THERMONUCLEAR OUTPUT

The thermonuclear output of a mixture of equal number of T and D nuclei at a mean energy of 100 keV is approximately

$$W_N = \frac{1}{2} \left(\frac{1}{2} n_p \right)^2 \sigma_{TD} \cdot v_{TD} \cdot Q \quad (\text{erg/sec, cm}^3) \quad (48)$$

where

$$\sigma_{TD} \approx 4 \times 10^{-24} \text{ cm}^2, \quad v_{TD} \approx 2 \times 10^8 \text{ cm/sec}, \\ Q \approx 2.8 \times 10^{-5} \text{ erg}.$$

Thus:

$$W_N \cong 0.28 \times 10^{-20} \cdot n_p^2 \quad (\text{erg/sec, cm}^3). \quad (49)$$

Integrating over the cross-section of the beam one gets

$$W_{N/cm} = 0.28 \times 10^{-20} \cdot n_0^2 \int_0^\infty \frac{2\pi r dr}{\left(1 + \frac{r^2}{r_0^2} \right)^4} \\ = 0.14 \times 10^{-22} N \cdot n_0 \int \frac{dx}{(1+x)^4} \\ = 4.7 \times 10^{-22} N \cdot n_0^\dagger. \quad (50)$$

The average output per particle is

$$W_{pN} = 4.7 \times 10^{-22} n_0$$

or

$$W_{pN} = 0.032 e^2 c \frac{v}{r_0^2} \quad (\text{erg/sec, part}). \quad (51)$$

3.5. REACTOR EQUATION

The reactor equation can be written

$$\alpha W_e \leq W_{pN} \quad (52)$$

where $\alpha \approx 3$.

Let us substitute into this equation an expression for W_e corresponding to the mixed beam model. There $T_e > T_p$ and, therefore, the bracket in the 2nd term on the r.h.s. of eq. (47) is very nearly unity. One obtains

$$\frac{18}{\gamma^3 \beta} + \frac{2}{9} v^2 \gamma^3 \beta^6 \leq 0.032 \quad (53)$$

† This can be written as $W_{N/cm} \approx 10^{-4} (v^2/r_0^2) (\text{watt/cm})$. Economic power production is possible only for $v/r_0 > 10^3$.

which can be satisfied only for large γ and $v \ll 1$ (e.g. $\gamma = 300$, $v = 10^{-4}$). However, this implies an insignificant nuclear output.

Let us consider the cold electron-core model. There $T_e \ll T_p$ and W_{ec} can be neglected, whereas

$$W'_{es} = 6e^2 c \frac{v}{\beta \gamma r_0^2} \quad (\text{erg/sec, el}) \quad (54)$$

as only γ^{-2} positives are found inside the electron beam. On the other hand the density of the positives had decreased γ^2 times. Eq. (52) becomes

$$\frac{18}{\gamma^3 \beta} \leq 0.032 \frac{4}{\gamma^2} \quad (55)$$

or

$$\beta \gamma > 140. \quad (55a)$$

Thus the cold-core model could work as a reactor. However, the considerations of economically interesting output require $v/(r_0 \sqrt{\gamma}) > 10^3$, which appears unrealizable.

One should finally consider a relativistic beam confining a stationary TD or DD plasma whose density is much larger than that of the relativistic beam. This seems to be a promising structure, however, it is doubtful that our formulae (13), (19), (39), (40) and (43) could be applied to it. Also other energy losses may appear, such as Čerenkov radiation, coherent radiation from electron bunches, etc. One can, therefore, make only an order of magnitude estimate, based on the following assumptions

$$\begin{aligned} & \frac{v_p}{kT} \\ & v \cong \frac{v_p}{\frac{1}{2} m v^2} \end{aligned} \quad (a) \quad (56)$$

i.e. the pinch forces are balanced by the pressure of the confined hot plasma.

(b) The electron temperature T_e of the relativistic electrons is so small, that the cyclotron radiation loss can be neglected.

(c) The relativistic electrons and the confined plasma have similar density distributions (three mixed fluids).

All these assumptions are to a certain extent interdependent.

The reactor equation can then be written as

$$18 \frac{v_p + v}{\gamma \beta} \leq 0.032 \left(\frac{v_p}{v} \right) v_p. \quad (57)$$

From eq. (56) follows that

$$\frac{v_p}{v} = v \frac{\frac{1}{2} mc^2 \beta^2}{kT_p} \quad (58)$$

and for $v > 1 \rightarrow v_p/v \gg 1$. Using this, eq. (57) may be written

$$\frac{v_p}{v} > 560 \gamma^{-1} \beta^{-1}. \quad (57a)$$

Applying eq. (58) to (57a) one has

$$v > 1120 \frac{kT_p}{mc^2} \gamma^{-1} \beta^{-3}. \quad (59)$$

As $T_p \approx 10^9$ °K we obtain finally

$$v > 220 \gamma^{-1} \beta^{-3}. \quad (59a)$$

Using eq. (58), the criterion for economic power output can be written

$$\frac{v^2 \frac{1}{2} mc^2 \beta^2}{r_0 kT_p} > 10^3$$

$$\text{or} \quad \frac{v}{\sqrt{r_0}} > 16. \quad (59b)$$

Thus, for example, for $\gamma = 28$ and $r_0 = \frac{1}{4}$ cm, eqs. (59a) and (59b) can be satisfied by $v = 8$.

This model of plasma confinement by a relativistic electron beam appears, therefore, promising, however, it is one which has been least analysed.

4. Heating of Plasma by a Relativistic Electron Stream

4.1. POSSIBLE SITUATIONS

The following limiting situations may be regarded: (a) the case of a plasma beam confined by self-forces where all electrons form a relativistic stream (purely relativistic plasma); and (b) the case of a neutral gas ionized and heated by a relativistic electron stream, providing at the same time a confining backbone inside an otherwise non-relativistic plasma.

(a) applies to configurations as described in sections 1 and 2; (b) applies e.g. to the "Astron" device, commented on in section 5 below. The cases (a) and (b) differ in the mechanism of heating: In case (a) the ions are heated directly by collisions with fast electrons, whereas in case (b) the fast electrons lose their energy mainly to the slow plasma electrons which in turn convey it to the ions.

4.2. ENERGY LOSS OF FAST ELECTRONS BY COLLISIONS WITH ELECTRONS AND IONS

A fast electron moving through neutral gas or plasma loses energy by collisions. Thus energy is transferred to the gas, which is first ionized, and thereupon heated. The energy transfer to electrons due to Coulomb interaction is calculated by the Bohr-Bethe formula

$$-\left(\frac{dE}{dt}\right)_{\text{coll}} = n_- \frac{4\pi r_e^2 \ln A}{\beta} cmc^2 \quad (60)$$

where

E the instantaneous electron energy

$r_e = e^2/mc^2$ the electronic radius

βc the velocity of the fast electrons

n_- the density of the scattering electrons and

$A = b_{\text{max}}/b_{\text{min}}$,

the ratio of maximum to minimum value of the collision parameters that cover the range of interaction.

Eq. (60) holds for plasma as well as for neutral matter (where the scattering electrons are bound), only the value of A is somewhat different in the two cases. Because $\ln A$ only enters, the energy transfer is not very sensitive to A . For electrons of energies $\gamma = 10, \dots, 100$, $\ln A = 25, \dots, 35$ in neutral gas. For plasma $\ln A$ decreases slightly with increasing plasma density, dropping to $\ln A \approx 20$ for $n = 10^{16}$ electrons/cm³ (by about 1 unit for each power of ten. These values are larger than those for non-relativistic electrons).

The energy loss due to collisions with heavy ions of mass M and charge Z is

$$-\left(\frac{dE}{dt}\right)_{\text{ion coll.}} = n_+ Z^2 \frac{m}{M} \frac{4\pi r_e^2 \ln A}{\beta} cmc^2 \quad (60a)$$

where n_+ the density of ions. Obviously the energy loss in collisions with ions plays only a part if practically no scattering electrons are present.

So relativistic electrons lose energy at a practically constant rate according to (60) or (60a). With $\beta = 1$, $\ln A = 25$, we can rewrite (60) in the form

$$-\left(\frac{d\gamma}{dt}\right)_{\text{coll}} \approx n_- \times 10^{-12} \quad (n = \text{electrons per cm}^3). \quad (61)$$

4.3. ENERGY LOSS OF ELECTRONS BY RADIATION

Apart from transfer of energy to the surrounding particles, an electron loses energy by bremsstrahlung at the rate

$$-\left(\frac{dy}{dt}\right)_{\text{brems}} = n_+ \beta \gamma \frac{Z^2 r_e^2 c}{137} f \quad (62)$$

where

n_+ the ion density,

Z the atomic number of the ions,

$$f = \begin{cases} \frac{1}{3} & \text{for } \gamma \lesssim 2 \text{ (non-relativistic electron)} \\ 4 \ln 2\gamma & \text{for } \gamma \gg 1. \end{cases}$$

The second value of f applies to free nuclei. If there is screening by bound electrons, f starts to grow as $4 \ln 2\gamma$ and approaches a limit of about 20 (at energies beyond $\gamma = 100$). Taking the limits $f = \frac{1}{3}$ and $f = 20$ we can rewrite (62)

$$-\left(\frac{dy}{dt}\right)_{\text{brems}} = n_+ \beta \gamma Z^2 \begin{cases} 0.9 \times 10^{-16} \gamma \approx 1 \\ 3.6 \times 10^{-16} \gamma \approx 100. \end{cases} \quad (63)$$

For $\gamma \approx 1$, $Z = 1$, and for equal electron and ion densities n_- and n_+ , the contribution by bremsstrahlung to the rate of energy loss is only about 10^{-4} of the collision loss to surrounding electrons, rising to $> 10^{-2}$ for $\gamma \gtrsim 30$.

If, however, collision loss is due to ions only, as in model (a), the loss by bremsstrahlung may be comparable or even greater than collision loss.

There is a further energy loss by radiation, if the electrons move in a magnetic or electric field bending their trajectories (continuously in distinction from the concentrated bends in Coulomb collisions). Such fields are the confining or focusing fields. The radiation loss caused by them amounts to

$$-\left(\frac{dy}{dt}\right)_{\text{e.m. fields}} = \frac{2}{3} r_e c \frac{(\beta \gamma)^4}{r^2} = 0.56 \times 10^{-2} \frac{\beta^4 \gamma^4}{r^2} \quad (64)$$

where r the radius of curvature of the trajectory. If in particular the curvature is caused by a magnetic field B , the radiation loss can also be written

$$-\left(\frac{dy}{dt}\right)_{\text{cyclotron radiation}} = \frac{2}{3} \frac{r_e^2 c}{mc^2} B^2 \beta^2 \gamma^2 = 2 \times 10^{-9} \beta^2 \gamma^2 B^2 \quad (64a)$$

For fast electrons ($\gamma \gg 1$), this radiation outweighs bremsstrahlung and even collision

loss in many of the devices proposed. It has been shown in section 2 to be prohibitive for the use of a strongly pinched relativistic beam in a fusion device.

Even for non-relativistic electrons, the cyclotron radiation is well known to compete seriously with bremsstrahlung, regarding the strong magnetic fields necessary in fusion devices.

4.4. HEATING EFFICIENCY

Case (a): All electrons form a relativistic beam

Whilst the electrons are transferring energy to the ions at a rate given by (60a), they at the same time produce bremsstrahlung at a rate given by (62). The ratio between the two energy rates is

$$\frac{\text{bremsstrahlung}}{\text{ion heating}} = \frac{1}{137} \frac{1}{4\pi \ln A} \frac{M}{m} \beta^2 \gamma f \approx \gamma \text{ (for } \gamma \gtrsim 10\text{)}.$$

Heating of an ion to a temperature T costs, therefore, $(\gamma + 1) \frac{3}{2} kT$ in electron energy. There is no competing mechanism of ion heat loss under ideal conditions.

If W_f is the fusion energy per ion, a self-sustaining reaction would require at least

$$(\gamma + 1) \frac{3}{2} kT < \frac{1}{\alpha} W_f / \alpha,$$

where $1/\alpha$ is the efficiency of the energy extraction. With $W_f / \frac{3}{2} kT = 50 \dots 100$, $\alpha > 3$, one finds

$$\gamma + 1 < 8 \dots 15.$$

Every additional waste of electron energy (e.g. cyclotron radiation and maintenance of beam for prolonged durations because of slow reaction rate) would deteriorate this figure still more. Thus fast electron heating cannot be considered interesting for a case (a) situation.

Case (b): Non-relativistic plasma heated by a relativistic electron stream

Here the ions are mainly heated by collisions with the non-relativistic plasma electrons. The latter are heated by the relativistic electrons, which lose energy to them at the rate given by (60). In comparison with this, direct transfer to ions as well as energy spent in the production of bremsstrahlung is negligible. The bremsstrahlung of the plasma electrons, however,

is important as it competes with the gain of heat, which is therefore governed by the following balance (per unit of plasma volume).

$$\frac{d}{dt} \left[(n_+ + n_-) \frac{\frac{3}{2} kT}{mc^2} \right] = 4\pi r_e^2 \ln A n n_- - \frac{16}{3} \frac{r_e^2 c}{137} \sqrt{\frac{2\pi kT}{3mc^2}} n_+ n_- \quad (65)$$

Here equilibrium between electron and ion temperature T has been assumed. n is the number of relativistic electrons per unit of plasma volume. The second term on the right-hand-side, the bremsstrahlung loss of the plasma electrons, corresponds to (62), a Maxwellian distribution of electron energies being taken account of (see ref. ¹³).

The asymptotic temperature attainable follows from (65)

$$\frac{kT_{\max}}{mc^2} = \frac{3}{2\pi} \left(\frac{4\pi \ln A \cdot 3 \cdot 137}{16} \frac{n}{n_+} \right)^2 \approx 0.5 \times 10^8 \left(\frac{n}{n_+} \right)^2 \quad (66)$$

In order to reach

$$\frac{\frac{3}{2} kT_{\max}}{mc^2} = \frac{1}{5} \left(\frac{3}{2} kT \approx 100 \text{ keV} \right)$$

a proportion

$$\frac{n}{n_+} \approx 0.5 \times 10^{-4}$$

of fast electrons would be sufficient.

The time required to reach a temperature $T = T_{\max}$ is found by integration of (65) (for $n_- = n_+$)

$$t = \frac{4}{\underbrace{\sqrt{\frac{4\pi}{9} \frac{16 r_e^2 c}{3 \cdot 137}}}_{4 \times 10^{-16}}} \frac{\sqrt{\frac{\frac{3}{2} kT}{mc^2}}}{n} \left\{ \ln \frac{1}{\left(1 - \frac{T}{T_{\max}}\right)^{\frac{1}{2}}} - \left(\frac{T}{T_{\max}}\right)^{\frac{1}{2}} \right\} \quad (67)$$

The value of the bracket $\{ \}$ is 0.2 for $T/T_{\max} = \frac{1}{4}$; 2 for $T/T_{\max} = \frac{9}{10}$, and ∞ for $T/T_{\max} = 1$.

Further it can be calculated from (65) that at reaching a temperature T , a proportion

$$\frac{T_{\max}}{T} \left\{ \ln \frac{1}{\left(1 - \frac{T}{T_{\max}}\right)^{\frac{1}{2}}} - \left(\frac{T}{T_{\max}}\right)^{\frac{1}{2}} - \frac{1}{2} \frac{T}{T_{\max}} \right\} \quad (68)$$

of the energy acquired by the ions has been spent on plasma bremsstrahlung. This propor-

tion is 0.27 for $T/T_{\max} = \frac{1}{4}$; 1.8 for $T/T_{\max} = \frac{9}{10}$, and ∞ for $T/T_{\max} = 1$.

The fast electron energy required in total to ionize and heat a plasma to a temperature $T = \frac{9}{10} T_{\max}$ is therefore per unit volume

$$n(\gamma - 1) \approx 2n_- \left(3 \frac{kT}{mc^2} + \frac{E_1}{mc^2} \right) \quad (69)$$

The ionization energy E_1 is practically negligible in this balance. Assuming again the figure of $\frac{3}{2} kT/mc^2 = \frac{1}{5}$ (i.e. $\frac{3}{2} kT = 100 \text{ keV}$), a ratio

$$\frac{n}{n_-} \approx \frac{0.8}{\gamma - 1}$$

of fast to slow electrons is found to be necessary from (69). For $\gamma = 10 \dots 100$ this is 1000 ... 100 times more than has been found necessary from (66) for reaching the proposed temperature. Thus either 100 to 1000 refillings of fast electrons are needed in order to complete the heating up, or the proportion of fast electrons must be 10^{-2} to 10^{-1} . Heating with replacement of fast electrons would of course require a continuously operating device.

A relativistic electron moving through (non-relativistic) plasma loses its energy in a time

$$t \approx \frac{\gamma - 1}{n \cdot 4\pi r_e^2 c \ln A} \approx \frac{\gamma - 1}{n_-} 10^{12} \quad (\text{seconds})$$

as follows from (60). For $\gamma = 10 \dots 100$, $n_- = 10^{12} \dots 10^{14}$ this is of the order of $10^{-1} \dots 10^2$ seconds. The actual lifetime of an electron may, however, be shorter, as fast electrons may be ejected from the beam focusing field by single large angle scattering events.

5. The Astron

5.1. BASIC IDEA

The most detailed proposition for the use of relativistic electrons in a fusion device has been made by Christofilos (ref. ²). In this device, a cylindrical current sheet formed by high energy electrons (called E-layer by Christofilos), circulating in the magnetic field inside a concentric

¹³ L. Spitzer, Physics of fully ionized gases (Interscience Publ., 1956).

coil, serves a two-fold purpose: (i) for modifying the field inside the coil so as to have lines of force closing on themselves. In this way, a "magnetic bottle" without holes would be obtained. (ii) for ionizing the gas and heating the produced plasma.

The field configuration is sketched in fig. 6.

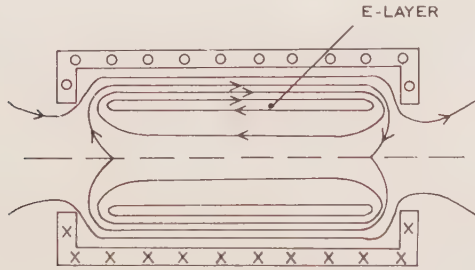


Fig. 6. "Astron" field configuration.

In order to obtain field lines closed inside the coil, the E-layer current has to be strong enough to reverse the field in the region enclosed by the layer. A steady state is maintained by continuous replacement of the electrons lost from the E-layer. This is effected by appropriate injection of energetic electrons from one end.

In the following notes, a few order-of-magnitude calculations are tried on the basis of simplified pictures, to obtain some idea on the feasibility of the proposal, and to show some of the problems arising. End effects and questions of stability will be ignored, as the primary problem of equilibrium of an infinitely long device seems not yet sufficiently settled.

5.2. E-LAYER EQUILIBRIUM

First we consider an E-layer of infinitesimal thickness, at radius r_1 (fig. 7). Without plasma present, the field is constant for $r < r_1$ and $r > r_1$, with values B_1 and B_2 resp. The following

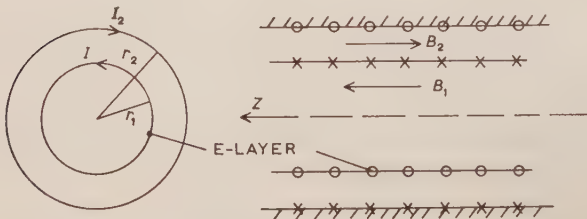


Fig. 7. E-Layer geometry.

conditions then hold for the E-layer current I (per cm) and the balance of the centrifugal force on the E-layer.

$$B_1 - B_2 = \frac{4\pi}{c} I = -\frac{4\pi}{c} \frac{N e v_e}{2\pi r_1} = -\frac{2N e \beta}{r_1} \quad (70)$$

$$\frac{B_2^2 - B_1^2}{8\pi} = \frac{N}{2\pi r_1} \frac{\gamma m v_e^2}{r_1} = \frac{N}{2\pi r_1} \frac{\gamma B^2}{r_1} m c^2 \quad (71)$$

where

N = number of electrons in the E-layer, per cm in axial direction,

$v_e = \beta c$ = electron velocity in azimuthal direction.

By virtue of (70), (71) is also equivalent to

$$\frac{e v_e}{c} \frac{B_2 + B_1}{2} = \frac{\gamma m v_e^2}{r_1}, \quad (72)$$

expressing that the mean field $\frac{1}{2}(B_2 + B_1)$ provides the right single particle guide field for the E-layer electrons.

Solving for B_1 and B_2 gives

$$B_1 = \frac{\beta \gamma m c^2}{r_1 e} \left(1 - \frac{N}{\gamma} \frac{e^2}{m c^2} \right)$$

$$B_2 = \frac{\beta \gamma m c^2}{r_1 e} \left(1 + \frac{N}{\gamma} \frac{e^2}{m c^2} \right). \quad (73)$$

The condition for field reversal is therefore

$$v \equiv N \frac{e^2}{m c^2} > \gamma. \quad (74)$$

In order to close as many field lines as possible on themselves, the magnetic flux through $r < r_1$ should cancel the flux between E-layer and wall (at $r = r_2$):

$$B_1 \pi r_1^2 + B_2 \pi (r_2^2 - r_1^2) = 0,$$

leading to a condition

$$\left(\frac{r_1}{r_2} \right)^2 = \frac{B_2}{B_2 - B_1} = \frac{1}{2} \left(1 + \frac{\gamma}{N \frac{e^2}{m c^2}} \right). \quad (75)$$

Thus, if zero total flux is required, always

$$\frac{1}{\sqrt{2}} < \frac{r_1}{r_2} < 1.$$

For example, $r_1 = 0.9 r_2$ would require

$$\frac{N}{\gamma} \frac{e^2}{m c^2} = \frac{5}{3}.$$

$r_1 = 0.75 r_2$ would require

$$\frac{N}{\gamma} \frac{e^2}{m c^2} = 8.$$

The external current necessary at $r = r_2$ is found to be

$$I_2 = \frac{cB_2}{4\pi} = -\frac{1}{2} \left(1 + \frac{\gamma}{N \frac{e^2}{mc^2}} \right) I.$$

I_2 is, therefore, between $\frac{1}{2}$ and the full E-layer current I .

In this analysis, it was tacitly assumed that the space charge of the E-layer was neutralized by positive ions. This will not be the case at the start of the machine, and then electrostatic forces add considerably to the centrifugal force. (71) has then to be replaced by

$$\frac{B_2^2 - B_1^2}{8\pi} = \frac{N\gamma\beta^2}{2\pi r_1^2} mc^2 + \frac{N^2 e^2}{2\pi r_1^2} \quad (76)$$

and (72) by

$$\frac{1}{2} e\beta (B_2 + B_1) = \left(\gamma\beta^2 + N \frac{e^2}{mc^2} \right) \frac{mc^2}{r_1} \quad (77)$$

giving

$$B_1 = \frac{\beta mc^2}{er_1} \left(\gamma + v \left(\frac{1}{\beta^2} - 1 \right) \right)$$

$$B_2 = \frac{mc^2\beta}{er_1} \left(\gamma + v \left(\frac{1}{\beta^2} + 1 \right) \right)$$

and no field reversal can occur. Furthermore the guide field required by the electrons at the start is considerably modified according to (77).

The equilibrium of the E-layer in the axial direction is provided by an axial component of momentum of the electrons (they have to have this for injection anyway). Thus an axial pressure in the E-layer balances the axial tension caused by the magnetic field. Stability requires that a compression (or increase of density) of the E-layer increases the net axial pressure. That this might be so, can be made plausible as follows:

The kinetic pressure in z -direction is

$$P_{\text{kin}} = N\gamma m v_z^2$$

(v_z = axial electron velocity). In adiabatic compression ($\gamma m v_z \cdot \text{length}$) would be invariant, resulting in a law

$$P_{\text{kin}} = P_0 \left(\frac{N}{N_0} \right)^2.$$

The magnetic pressure we calculate by assuming a linear dependence of B on r inside the E-layer, and its thickness $h \ll r_1$:

$$P_{\text{magn}} = -\frac{1}{8\pi} \int_{r_1 - \frac{1}{2}h}^{r_1 + \frac{1}{2}h} B^2 2\pi r dr = -\gamma mc^2 N \frac{h}{r_1}$$

using (70) and (72). In equilibrium

$$P = P_{\text{kin}} + P_{\text{magn}} = P_0 - \gamma mc^2 N_0 \frac{h}{r_1} = 0$$

and

$$\frac{\partial P}{\partial N} = \frac{2P_0}{N_0} - \gamma mc^2 \frac{h}{r_1} > 0.$$

In trying to understand whether an E-layer of the postulated type can exist, let us consider single electron orbits in the region of transition of the field from B_1 to B_2 , which of course must have a finite radial extension (fig. 8). The electrons circulating on orbits with radius r_1 after

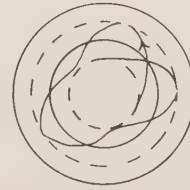
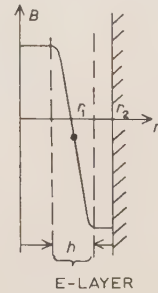


Fig. 8. Extended E-layer and electron orbits.

injection spiral inward due to the energy loss by collisions and radiation. They acquire an increasing amount of radial momentum owing to collisions, which makes them carry out radial "betatron" oscillations about their circular equilibrium orbits. In fact, the radial gradient of the field provides focusing, which can be quite strong if

$$k = \frac{r}{B} \frac{\partial B}{\partial r}$$

is large. One can imagine that these betatron oscillations enable the existence of a current density reaching into a region of reversed field (where no circular electron orbits in the same sense about the axis of symmetry would be

possible otherwise). In the lower part of fig. 8 a typical orbit corresponding to this situation is sketched.

The only more substantial argument for an equilibrium of the postulated E-layer is the existence of self-consistent distributions of mono-energetic electrons, showing field reversal in the centre, established by calculations by L. Tonks (ref. 14)).

It is, therefore, plausible that an E-layer for field reversal could be set up, although there is probably no answer to this question as yet.

5.3. PLASMA CONFINEMENT

For a plasma in equilibrium in the Astron field we have the pressure balance

$$-\frac{\partial}{\partial r} \left(\frac{B^2}{8\pi} + P \right) + \frac{n_e \gamma m v^2}{\rho} = 0 \quad (78)$$

where $P = n_+ k T_+ + n_- k T_-$ is the plasma pressure. The third term takes care of the centrifugal volume force in the E-layer ($1/\rho$ denoting the local curvature of electron orbits as to magnitude and sign). The gradient of the magnetic field has to balance the E-layer centrifugal force in addition to the plasma pressure gradient. In the presence of plasma the difference of B between the wall and the centre is therefore greater than for the E-layer alone. A distribution of plasma density as sketched in fig. 9 may be expected. A calculation of the $(8\pi n k T/B^2)$ attainable is difficult as high plasma densities upset the E-layer calculations in subsection 5.2.

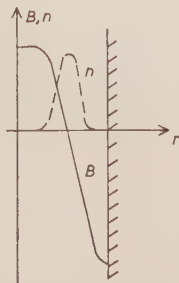


Fig. 9. Plasma density distribution.

¹⁴) L. Tonks, The self consistent field of single-type electrons in a uniform magnetic field, Univ. of Calif. Radiation Laboratory report UCRL 5335 (1958).

More rigorously there are also centrifugal forces on the plasma itself owing to angular momentum being transferred to it from the E-layer electrons.

Decisive for confinement is the diffusion of plasma to the wall. Christofilos attempts an estimate by constructing a plausibly looking field distribution (in analytical form) and deriving density distribution and diffusion flow from it, using the magnetohydrodynamic equations of motion. The assumed solution satisfies, however, the continuity equation only in the average, and ignores also the E-layer centrifugal forces. Thus a less elaborate estimate might be sufficient here. Magnetohydrodynamic theory gives for the flux density of ambipolar radial diffusion across the magnetic field:

$$n v_r = \frac{n_0^2}{r_2 - r_0} \frac{4\pi r_0 c m c^2}{\left(\frac{2kT}{mc^2}\right)^{\frac{1}{2}} B_2^2} = \frac{n_0^2}{r_2 - r_0} \frac{0.9 \times 10^{-7}}{\left(\frac{2kT}{mc^2}\right)^{\frac{1}{2}} B_2^2} \quad (79)$$

where v_r is the radial flow velocity, n_0 the density at some $r = r_0 \approx r_1$, and B_2 the magnetic field at the wall $r = r_2$ (boundary condition on the wall: $n = 0$).

5.4. HEATING BALANCE AND CONDITIONS FOR PARAMETERS

The input power supplied by collisions of the E-layer electrons is according to (61), section 3, (in units of mc^2 , and per cm)

$$-N\gamma = Nn \times 10^{-12}.$$

Competing with this is the energy loss by diffusion of particles to the wall (see (79)).

$$\begin{aligned} \text{Diffusion loss} &= 2\pi r n v_r \frac{\frac{3}{2} k T}{mc^2} \\ &= \frac{2\pi r_1 n_0^2}{r_2 - r_1} \frac{\frac{3}{4} \cdot 0.9 \times 10^{-7}}{\left(\frac{2kT}{mc^2}\right)^{\frac{1}{2}} B_2^2} \\ &= \frac{N_p n}{(r_2 - r_1) h} \frac{0.7 \times 10^{-7}}{\left(\frac{2kT}{mc^2}\right)^{\frac{1}{2}} B_2^2} \end{aligned}$$

and the energy loss by bremsstrahlung of the plasma electrons (see (63) and (65)):

$$\text{bremsstrahlung} = N_p n \times 10^{-16} \left(\frac{2kT}{mc^2} \right)^{\frac{1}{2}}.$$

Care should be taken that the diffusion loss does not exceed the bremsstrahlung loss. This gives a condition

$$B_2^2 (r_2 - r_1) h \gtrsim 10^9 \quad (80)$$

for magnetic field and dimensions. If it is fulfilled, temperature increases as long as

$$\frac{N}{N_p} > 10^{-4} \left(\frac{2kT}{mc^2} \right)^{\frac{1}{2}}. \quad (81)$$

We shall now examine whether interesting conditions could be realized with reasonable Astron parameters.

First we know from (75) and (70) that the radii of E-layer and external coil are linked by

$$\left(\frac{r_2}{r_1} \right)^2 = \frac{B_2 - B_1}{B_2} = \frac{2Ne}{B_2 r_1} = 2 \left(1 + \frac{\gamma}{N \frac{e^2}{mc^2}} \right)^{-1} \quad (82)$$

and that r_2/r_1 is between 1 and $\sqrt{2}$. Choosing

$$\frac{r_2}{r_1} = 1.2$$

as a reasonable ratio, we find from (82)

$$B_2 r_1 = 2eN \left(\frac{r_1}{r_2} \right)^2 = 0.67 \times 10^{-9} N$$

$$\frac{N}{\gamma} = \frac{1}{\frac{e^2}{mc^2} \left(2 \left(\frac{r_1}{r_2} \right)^2 - 1 \right)} = 0.9 \times 10^{13}.$$

Finally we assume that the radial width h of

the E-layer is approximately equal to $r_2 - r_1$ whereupon we can write (80)

$$B_2 r_1 \approx 1.6 \times 10^5.$$

The last 3 conditions fix

$$N = 2.4 \times 10^{14}$$

$$\gamma = 27$$

(to be considered as a mean value in the E-layer, due to the energy spread in it).

In order to fix B_2 and r_1 we require

$$\frac{2kTn}{B^2/8\pi} = \frac{2kT}{mc^2} \frac{n}{B_2^2} 2 \times 10^{-5} \ll 1, = 0.1, \text{ say}$$

in order that the plasma does not perturb the E-layer too much. Thus for

$$n = 10^{13} \text{ cm}^{-3} \text{ and } \frac{2kT}{mc^2} \approx \frac{4}{3} \cdot 0.2, \quad B_2 \approx 2 \times 10^4 \text{ gauss}$$

$$n = 10^{14} \quad \text{corresponds to} \quad B_2 \approx 7 \times 10^4 \text{ gauss.}$$

The E-layer radii become correspondingly

$$\text{for } n = 10^{13} \quad r_1 \approx 8 \text{ cm}$$

$$\text{and for } n = 10^{14} \quad r_1 \approx 2.5 \text{ cm.}$$

These orders of magnitude for the radii seem uncomfortably small. The other figures agree with those given by Christofilos. Condition (81) for heating would be amply satisfied.

One may conclude that the Astron might be a feasible device if the postulated E-layer can be set up in a stationary fashion.

10. REPORT ON HIGH-SPEED PHOTOGRAPHY

A. FOLKIERSKI

Imperial College of Science and Technology, London

The application of Kerr cells, image converters, rotating mirror cameras and drum cameras to the photography of fast electrical discharges is critically discussed. The per-

formance of representative examples of each of these techniques is given quantitatively.

1. Introduction

Thermonuclear discharges possess certain characteristics which are very relevant in selecting methods of photography. Some of these characteristics are:

- (1) they are self-luminous,
- (2) there are rapid variations both spatial and temporal of intensity of light emitted,
- (3) the discharges are triggered from some master timing unit, from which the camera may also be triggered.

Because of the rapid variations it is necessary to have short exposures. In order to follow the development of phenomena in time, one should have sequences of short exposure shots, or possibly smear pictures. It is necessary to have a fair amount of spatial resolution, and in general a reasonably wide angle of view.

2. Fast Shutters

2.1. INTRODUCTORY REMARKS

The Kerr effect, the Faraday effect, and gated image converters can in principle be used. The Faraday effect is too slow, because of the difficulty of establishing the magnetic field required sufficiently rapidly.

2.2. KERR CELLS (ref. ¹), Karolus, p. 78)

2.2.1. *Kerr Effect.* Nitrobenzene is commonly used. This is doubly refracting when placed in an electric field E , oriented at a right angle to the direction of propagation.

Light is first plane-polarized by a polaroid, the plane of polarization being inclined at an

angle of 45° to E . The nitrobenzene splits the incident light into two rays and there is a progressive phase difference between them, given by

$$\varphi = K l E^2 \quad \text{wavelengths}$$

where K = Kerr constant

l = length of path through nitrobenzene

E = field strength.

A second polaroid oriented at 90° to the first is placed at the far side of the nitrobenzene. In the absence of the electric field no light is transmitted. With the electric field on, the transmitted intensity is given by

$$J_2 = \frac{1}{2} J_1 \sin^2(\pi\varphi) = \frac{1}{2} J_1 \sin^2(\pi K l E^2)$$

where J_1 is the intensity of the incident light. It is seen that J_2 is a periodic function of E . Evidently it is easiest to work at the first maximum, but it is not necessary to have E precisely at the optimum value.

The above expression neglects losses due to absorption in polaroids and in the nitrobenzene. The optimum value of E is given by

$$E = \frac{1}{(2kl)^{\frac{1}{2}}}.$$

The Kerr constant of nitrobenzene is a function of wavelength which introduces a certain amount of colour-filtering. The values are

λ	4360	5460	5770	6200	$\text{\AA}$
K	52	36	34	31	$\times 10^{-6}$ c.g.s. e.s.u.

¹) Photographie et Cinématographie Ultra-Rapides. Actes du 2^{ème} Congrès International de Photographie et Cinématographie Ultra-Rapides, Paris 1954 (Dunod, 1956).

The nitrobenzene is strongly absorbing for $\lambda < 4800 \text{ \AA}$. The dielectric constant is approximately 36, the breakdown strength is $\approx 50 \text{ kV/cm}$, the specific resistance is $\approx 10^{10} \text{ ohm cm}$ and the refractive index is 1.8. These values depend strongly on the degree of purification, and are liable to alteration as techniques of purification improve.

2.2.2. Practical Kerr cell construction. From the point of view of ease of construction of electronic circuits for control of potentials applied to Kerr cells, it is inconvenient to deal with high voltages. The largest voltage which can be conveniently switched by means of readily available thyratrons is 15 kV. It is possible, however, to apply the voltage via a pulse transformer, in which case this limitation would not apply.

The light transmitting faces of the Kerr cell must be accurately parallel, and made of high grade glass, to avoid the introduction of spurious phase-shifts.

A successful design is described by E. W. Walker (ref. ¹), p. 91). Two optical plates are cemented by Araldite to the body of the cell in which are mounted the electrodes. There are four electrodes, in the form of plates, which are connected externally in pairs. This enables a high field to be obtained without the use of excessively high voltages. The length of the active path in nitrobenzene is approximately $\frac{1}{2} \text{ in.}$ and the spacing of plates approximately $\frac{1}{4} \text{ in.}$ Taking $K = 35 \times 10^{-6} \text{ c.g.s. e.s.u.}$, this gives for optimum field a voltage of $\approx 20 \text{ kV}$. Thus at 15 kV this cell should have a transmission of about 60% of the possible maximum. The crosssection of the available light path is approximately $\frac{3}{4} \times \frac{3}{4} \text{ in.}$

2.2.3. Polaroids. Polaroids vary a great deal in quality. With good ones the above Kerr cell gives a transmission ratio for open and shut conditions of $\approx 10^3$ (ref. ¹), Walker, p. 97). There is considerable difficulty in obtaining polaroids which are sufficiently homogeneous over an area as large as $\frac{3}{4}'' \times \frac{3}{4}''$.

2.2.4. Optical Arrangement. The simplest arrangement is to mount the Kerr cell in front of a camera objective. The disadvantage is that light paths are converging which introduces chromatic aberration, due to the rather high refractive index of nitrobenzene.

Thus—in so far as possible—the Kerr cell should be inserted at a point where light is parallel. A compromise solution is shown below:

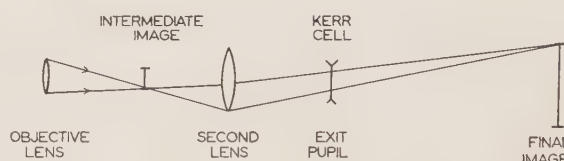


Fig. 1.

The objective lens forms a real image which is magnified by the second lens. The Kerr cell is placed at the exit pupil.

As an example, with an $f/1 \text{ 2''}$ lens giving 1:4 magnification as the second lens, and an $f/3 \text{ 3''}$ lens as the objective lens an overall $f/3$ can be achieved, with an object of $20''$ diameter at 8 ft distance.

2.2.5. Electronic circuits. The Kerr cell described above has a capacity of about 100 pF.

It is placed between the anodes of two thyratrons, both plates are initially at 15 kV. First one thyatron is discharged, opening the cell; then the other is discharged and the cell shuts.

Exposure times of $0.1 \mu\text{sec}$ can be readily achieved by this means, but the currents required are large. For the cell to be charged up to 15 kV in $0.01 \mu\text{sec}$ a current of

$$I = C \frac{dV}{dt} = 10^{-10} \frac{15 \times 10^3}{10^{-8}} \approx 150 \text{ A}$$

is required.

In practice it is usually necessary to use cables between the thyratrons and the cell which involves difficulties with matching.

Other methods of applying voltages to the Kerr cell include pulse transformers, and switching by gaps in air²).

²) E. Fünfer *et al.*, Zeits. f. Natur. 132 (1958) 524.

2.2.6. *Other doubly-refracting substances* (ref. 1), Karolus, p. 78). *Phenylisothiocyanate*, C_7H_5NS , has a lower Kerr constant $K = 30\%$ of $K_{\text{nitrobenzene}}$, but is suitable for use in the blue and possibly in the near U.V.

Uniaxial crystals, such as KH_2PO_4 , are birefringent in the presence of E parallel to the optical path and the optic axis of the crystal.

Two snags arise, firstly the light must be within 1° of arc of the axis, which limits attainable aperture. Secondly, the response time is poor. Uniaxial crystals are not suitable for exposure times shorter than about $10 \mu\text{sec}$.

2.3. IMAGE CONVERTERS

2.3.1. *Principle*. Image converters convert an optical image formed on the photocathode into an electron beam, which is then focussed electron-optically on a phosphor-coated screen. The picture is then photographed in the ordinary way. Shutter action is obtained by switching potentials so as to cut off the electron beam.

2.3.2. *Availability*. There are commercially available image converters made by Mullard. Type 1200 is a diode, type 1201 is a triode. Both are magnetically focussed³). It is believed that A.E.G. in Germany also manufacture them.

2.3.3. *Method of gating*. Normally the anode is maintained at a positive potential, say 6 kV. The cathode is also at + 6 kV, while the grid is at, say, + 5.9 kV. The electrons cannot proceed from the cathode to the anode. When the cathode is pulsed to, say, + 100 V the exposure begins; then the grid is pulsed to earth and the exposure ends.

2.3.4. *Comparison with Kerr cells*. Space resolution is generally not so good as with Kerr cells.

The size of image is dictated by the size of the photocathode, usually 1" diameter. The image converter itself provides some magnification (ca. 4).

On/off transmission ratio is essentially infinite, which is an advantage compared to Kerr cells. This is particularly so when a faint phenomenon

is to be photographed, occurring immediately before or after a bright one.

Photography is relatively easy as there is some persistence in the phosphor, so that the film need not have extremely fast response.

In principle image converters are capable of light gain. In practice commercial converters exhibit a loss of light, as measured by a comparison of film blackening directly and via converter (ref. 4), R. A. Chippendale, p. 116).

2.3.5. *Difficulties*. The image converters are sensitive to overexposure which causes distortion. Thus the aperture of the optical system in front of the converter must be adjusted according to the brightness of the object. This is a serious limitation in that it requires prior knowledge of what will be photographed. It also requires tests to be made of distortion from time to time.

2.3.6. *Electronic circuits*. These are similar to those required by Kerr cells, but are easier because:

- (a) the capacity of the converter is very much smaller than that of the Kerr cell. Consequently lesser currents are required to change voltages;
- (b) the operating voltages are lower.

2.3.7. *Special features*. It is possible to obtain a smear record on an image converter by the application of a continuously varying magnetic field (ref. 4)), Lunn, p. 102).

In principle the image converter can be used to convert U.V. to visible, but suitable tubes are not available commercially.

3. Rotating Mirror Cameras

3.1. INTRODUCTORY REMARKS

In a rotating mirror camera a beam of light from the object is rotated at a speed of 2ω , where ω is the angular velocity of the mirror.

The possible speed of rotation is limited in practice by the ultimate strength of the

³) Mullard Ltd.; Technical Service Dept., Valve Division: Image Converter Tubes, General Operational recommendations.

⁴) High Speed Photography. Proceedings of the 3rd International Congress, London 1956 (Butterworth, 1957).

material of the rotor. Speeds up to 400 000 r.p.m. are usable.

The drive is usually provided by either an air-driven turbine or a synchronous or induction electric motor. For the highest speeds it is easier to arrange a turbine drive.

It is not possible to synchronize the rotating mirror to an external master unit, but it is possible to use the mirror as master unit and thus ensure overall synchronization.

3.2. SMEAR CAMERAS

3.2.1. *Principle.* In a smear camera the image of a slit is formed by the objective lens on the film, the rotating mirror intercepting the beam between the lens and the film. The film has to be supported in a holder in the shape of an arc

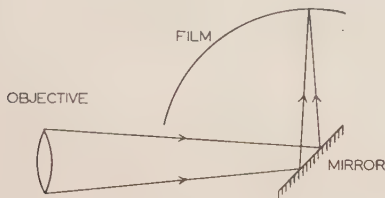


Fig. 2.

of a circle, centred on the mirror. In order to get a high writing speed a lens of long focal length F is required, the limiting writing speed being given by $2\omega F$. From the optical point of view the axis of rotation should lie in the plane of the mirror. This is not possible mechanically, at high speeds.

3.2.2. *Performance.* By way of example the performance of an A.W.R.E. camera is given:

Writing speed 40 mm/ μ sec

Relative aperture $f/22$

Angular extent of the film $\approx 70^\circ$

Length of rotating (optical) arm $\approx 19.5''$

Angular speed of mirror approx. 400×10^3 r.p.m.

Air-driven turbine.

3.2.3. *Construction* (ref. ¹), Walker, p. 92; Coates, p. 114). The mirror is made of high tensile strength steel, and is in the form of a lamella, about $\frac{1}{8}''$ thick and about 1–2 sq. in. area.

It is supported on two half-axles, "araldited"

to the mirror. The axles rotate in white metal bearings. One of the axles carries the turbine rotor. In order to reduce drag and avoid pitting of the mirror surface the mirror is put in a partly evacuated enclosure, formed in part by spherical glass walls. The bearings are lubricated by oil which is forced in by a head of oil and taken out by the vacuum pump.

3.2.4. *Synchronization.* An auxilliary lamp sends a ray of light which is reflected by the rotating mirror. Immediately before the beginning of the exposure the ray falls on a photomultiplier. Immediately after the exposure it falls on a second photomultiplier. Thus electrical pulses are available for recording the beginning and end of exposure. A third photomultiplier can be incorporated to give a master triggering signal. The auxilliary source is so positioned that the reflected ray falls on the side of the film, and not on the central part where the image sweeps.

3.3. ROTATING MIRROR CINE CAMERAS

3.3.1. *Principle.* The objective lens produces a real image of the object on the mirror. The mirror is looked at by an array of "taking" lenses disposed in an arc of a circle centred on the mirror. The lenses re-form the image on the film, which is mounted on a holder again in the form of an arc of a circle, but of greater radius.

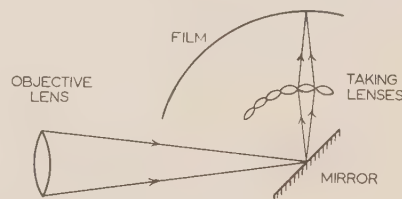


Fig. 3.

3.3.2. *Performance.* A camera of this type is available commercially from Beckman & Whitley Inc. (San Carlos 7, Calif.); for model 189 a maximum framing rate of 4.3×10^6 per sec is claimed. Its use for pinched discharge work is referred to by Aymar *et al.*⁵).

An A.W.R.E. camera using the same rotating mirror as their smear camera attains up to 80

⁵) Geneva Conference 1958; paper 15/P/1181.

exposures at 10^7 frames per sec with a 1:1 "mark-space" ratio. This mark-space ratio can be improved, i.e. the single exposure time cut down to 2×10^{-8} sec by the use of a repetitive Kerr cell shutter synchronized to the mirror (ref. 4), Walker, p. 133).

3.4. GENERAL LIMITATIONS OF ROTATING MIRROR CAMERAS

3.4.1. *Aperture and writing speed* (ref. 1), Walker, p. 91). The maximum possible aperture is limited by the relation

$$f = \frac{R}{X}$$

where R is the radius of the film track, X is the maximum usable transverse dimension of the mirror (diameter), f is the transverse relative f number. The longitudinal f number (parallel to axis) may be made larger. X is limited by the rotational speed and the strength of material. R has to be made large to get a high writing speed.

The writing speed $s = 2\omega R$, and the quotient s/f can be regarded as a figure of merit.

$$\frac{s}{f} = \frac{2\omega R}{R/X} = 2\omega X = 4 \times \text{peripheral speed of mirror.}$$

The maximum attainable peripheral speed is virtually a constant.

The steel used by A.W.R.E. is characterized by $s/f = 3 \text{ mm}/\mu\text{sec}$ and the maximum value of s/f realized in the A.W.R.E. camera is approx. $1.8 \text{ mm}/\mu\text{sec}$, the main limitation lying in the method of fixing of half-axes to the mirror.

3.4.2. *Framing frequency and the resolving power* (ref. 4), Schardin, p. 316). In a rotating mirror cinematographic camera the first image is formed on the mirror. The taking lenses re-form the image on the film.

By the usual telescope resolving power relation the maximum number n of resolved lines per cm on the film is given by

$$\frac{1}{n} \approx \lambda \frac{2}{D}.$$

The magnification from mirror to film is

$$\frac{X}{B} = \frac{l_1}{l_2}.$$

The duration of one frame is given by

$$\tau = \frac{D}{2\omega l_1}$$

where ω = angular frequency of mirror, and the peripheral speed of mirror is given by

$$v = \frac{1}{2} \omega X.$$

The product of the framing frequency and the total number of lines on film can be regarded as a figure of merit. It is given by

$$\frac{Bn}{\tau} = \frac{BD}{\lambda l_2} \frac{2\omega l_1}{D} = \frac{2B}{\lambda} \frac{2v}{X} \frac{X}{B} = \frac{4v}{\lambda}.$$

Thus this figure of merit is again a function of the ultimate tensile strength of the mirror material only, since λ is hardly a variable. Taking $v = 0.07 \times 10^6 \text{ cm/sec}$ again and $\lambda = 5000 \text{ \AA}$, we have

$$\frac{Bn}{\tau} = \frac{4 \times 0.07 \times 10^6}{5000 \times 10^{-8}} \approx 5 \times 10^9.$$

As an example, with a framing frequency of 10^7 frames per sec, and a picture size of 1" the maximum possible resolution is 20 lines/mm.

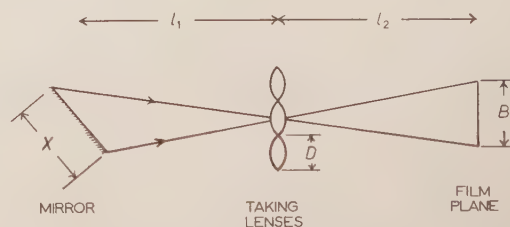


Fig. 4. X = transverse dimension of mirror; D = diameter of taking lens; B = size of image on film; l_1 = mirror to lens distance; l_2 = lens to film distance.

There exist two methods of partly defeating this relation. One is a Kerr cell shutter in front of the principal objective. This achieves an exposure time shorter than τ .

The other is to have several rows of taking lenses with the planes of their axes at an angle to the equatorial plane of the mirror. The result is that the product of framing frequency and exposure time becomes greater than 1.

4. Drum Cameras

Because of the large rotating mass involved the speed of these cameras is much smaller than is the case with rotating mirrors.

Writing speeds of the order of $0.1 \text{ mm}/\mu\text{sec}$ are readily attainable.

The Imperial College drum camera operates with that writing speed. The drum has 4" radius and rotates at 175 r.p.s. It is driven by an air turbine. The optics of a drum camera are very simple. At Imperial College a $14" f/5.6$ lens is used.

5. Film Selection and Processing

The requirements of speed in films vary with the photographic technique used. The image converter screen has a persistency, independently of the exposure time. All the other methods described result in a very short film exposure time.

At Imperial College the Ilford 35 mm recording film type 5G91 is used both with the drum camera and with the image converter camera. A 5-minute development in I.D.33 gives satisfactory results.

6. Omitted Topics

The following techniques are not treated in this report:

1. Image dissection by lenticular lenses.
2. Television-type cameras.
3. Mosaic cathode image converters.

All of these are referred to in ref. ⁴). It is considered that, in their present state of development, these techniques are not useful in the field of fusion.

A further topic of great importance, which is not treated in the present report, is time resolved spectroscopy.

11. PLASMA DIAGNOSTICS BY SPECTROSCOPICAL MEANS†

H. WULFF

Max-Planck-Institut für Physik und Astrophysik, München

This paper is principally devoted to a discussion of possible applications of spectroscopy in the study of high temperature plasmas, the two main sections treating, respectively,

measurements on spectral lines (intensity measurements line profile determination, temperature determination) and continuous radiation (e.g. bremsstrahlung).

1. Introductory Remarks on Plasma Radiation

Contents: Radiation in thermodynamical equilibrium
Thermal radiation. Non-thermal radiation.

We start with a plasma which we assume to be enclosed in a cavity the walls of which do not conduct heat at all. Under this condition the plasma is in thermodynamic equilibrium and from the point of view of thermodynamics it is completely specified by a set of fundamental variables, e.g. its temperature, pressure, volume and the relative number of atoms of the several kinds which may be present. The radiation in the cavity is in thermodynamic equilibrium too, i.e. the radiation within the interval $d\nu$ around the frequency ν emitted by a volume element of area df and height dh into the solid angle $d\omega$ $\perp$ to df is equal to the absorbed radiation within the same frequency interval and solid angle:

$$\varepsilon_\nu d\nu df dh d\omega = \kappa_\nu I_\nu d\nu df dh d\omega. \quad (1)$$

Here, ε_ν is the emission coefficient, I_ν is the intensity of the radiation in the cavity (energy per unit of time, area $\perp$ to the direction of the radiation, frequency and solid angle), κ_ν the absorption coefficient (dimension length⁻¹) which is the same quantity as it appears in the absorption law $I_\nu = I_{\nu_0} \exp \{ -\kappa_\nu h \}$ where h is the thickness of the absorption layer.

Eq. (1) can be written in the form

$$\frac{\varepsilon_\nu}{\kappa_\nu} = I_\nu = B(\nu, T). \quad (2)$$

† In refs. 1), 5) and 6) one finds systematic references to further literature, and also references to works of authors here mentioned only by their names.

For I_ν Kirchhoff's law holds: The intensity is a universal function of ν and T and does not depend on the particular properties of the cavity. In the following, therefore, this intensity shall be designated by $B(\nu, T)$. If the fundamental variables of the plasma in the cavity are known κ_ν and from this the emitted radiation of a volume element can be evaluated in principle according to eq. (2). The equilibrium in a cavity as specified above is an ideal case that in reality can only be approximated. As experimentally shown, however, there are various light sources, e.g. stabilized high-current arcs, containing at least in certain regions a plasma which is to a very good approximation in thermal equilibrium. In such a plasma, therefore, the velocities of the atoms, ions, and electrons obey the Maxwell-Boltzmann-distribution belonging to one single temperature; furthermore, the excitation follows the Boltzmann distribution and the ionization takes place according to Saha's equation again with the same temperature. Apparently, the radiation $\bar{\varepsilon}_\nu df dh$ emitted by a volume element $dV = df dh$ of such a plasma is not in equilibrium with any radiation coming from outside. $\bar{\varepsilon}_\nu$ is in spite of this completely specified under these conditions.

For example in the frequency range about a certain spectral line of a neutral atom, the radiation in a unit solid angle is given by

$$\int_{\text{line}} \bar{\varepsilon}_\nu d\nu df dh = \frac{1}{4\pi} h\nu A_{mn}^{\text{line}} n_0 \frac{g_n}{u} \exp \left(-\frac{\chi_n}{kT} \right) d/dh. \quad (3)$$

Here, A_{mn}^{line} means Einstein's transition prob-

ability for spontaneous emission, n the initial state, m the final state. Furthermore, n_0 is the number of all neutral particles per cc, g_n is the statistic weight, u the partition function, χ_n the energy of the upper level, $\chi_n - \chi_m = h\nu$.

This is the same radiation as emitted spontaneously by a volume element of the cavity mentioned above when it is filled with the same gas as the high-current arc chamber. This can be proved by calculating the spontaneous emission, this means the difference between the total and the induced emission. The spontaneous emission coefficient $\varepsilon_v^{\text{sp}}$ is given by

$$\varepsilon_v^{\text{sp}} = \kappa_v B(\nu, T) \left(1 - \exp\left(-\frac{h\nu}{kT}\right) \right). \quad (4)$$

$\varepsilon_v^{\text{sp}}$ differs from the total emission $\varepsilon_v = \kappa_v B(\nu, T)$ only if $h\nu/kT \ll 1$ (Rayleigh-Jeans region, viz. p. 354).

Integrating over the frequency interval around the line together with some rearrangements and multiplying by $dfdh$, eq. (4) yields the right-hand side of eq. (3). This means that with respect to the spontaneous emission the volume elements of a high-current arc radiate in the same way as the volume elements of the cavity if the particles of the plasma are in thermal equilibrium as far as their velocity distribution, their excitation and their ionization are concerned. It has become usual, therefore, to call this radiation thermal† in spite of the fact that for the radiation an equilibrium does not exist.

This fact seems to call for further explanation.

First, the statement should be mentioned which occasionally can be found in the literature on thermo-fusion, namely, Saha's equation does not hold because radiation can escape the plasma. This conclusion is definitely wrong as for example the high-current arc shows. Apparently, the non-thermal excitation and ionization is simply inferred from the fact that radiation is not in equilibrium.

The situation of thermal radiation not being

in radiative equilibrium is shown schematically as follows:

Excitation by photons ($\sim n$)	Emission of spectral lines ($\sim n_{\text{excited}}$)
Ionization by photons ($\sim n$)	Radiative recombination ($\sim n_{\text{ion}}n_e$)
Excitation by collision ($\sim n_e n$)	Super elastic collision ($\sim n_e n_{\text{excited}}$)
Ionization by collision ($\sim n_e n$)	Recombination by three-particle collisions ($\sim n_e^2 n_{\text{ion}}$)

Here, n means the number of particles at ground level per cc, n_e the number of electrons per cc, n_{excited} the number of particles in excited states per cc, n_{ion} the number of ions per cc.

In the cavity as specified above any process at the left-hand side of the table is in equilibrium with the corresponding process written at the right-hand side on the same line. In the case of a high-current arc, however, the cavity-radiation is not present. The rate at which ionization and excitation by photons occurs is, therefore, smaller by a factor equal to the ratio of the radiation density within the plasma to the radiation density within the cavity. Therefore, in the real plasma the particle density in excited states (ionized particles included) is smaller. On the other hand, because of the comparatively large particle density in high-current arcs the collisions are much more frequent than all processes in which photons are involved, i.e. the collision frequency is much larger than the transition probability (= reciprocal of the mean life time). Hence, an unmeasurably small decrease of the number of excited particles is already sufficient to reduce the number of the right-hand side collision processes (super elastic collision, the frequency of which is proportional to $n_e n_{\text{excited}}$ and recombination by three-particle collision which is proportional to $n_e^2 n_{\text{ion}}$) by an amount equal to the reduction of photo excitation and -ionization. Therefore, an equilibrium is now established between all processes at the right-hand side of the table and both collision processes on its left-hand side.

The situation is different if the densities of particles and photons are very small. Again we assume that excitation and ionization take place

† It should be noted that this does not mean the radiation is equal to the black body radiation of the same temperature. The radiation of a layer of finite thickness l obeys eq. (5), p. 355).

practically only by means of collisions. However, these two collision processes are now in equilibrium with both photo-processes on the right-hand side because due to the small density the number of super elastic collisions and three particle recombinations can be neglected as compared to their competitors. As assumed contrary to the state of pure thermal equilibrium, photo excitation and -ionization does not occur. Therefore, the radiation emitted by such a rarified plasma must be less intense; it is no longer thermal.

As is known, a situation like this exists in the corona of the sun. Ionization does not obey Saha's equation but follows the corona ionization formula; it is independent of the electron density. Electrons and ions have the same "kinetic temperature".

The intention of these remarks has been to demonstrate that every deviation from thermodynamic equilibrium needs to be investigated carefully and can be explained in terms of a more detailed treatment taking into account collision frequencies, transition probabilities, etc.

2. Possible applications of spectroscopy

Contents: Introduction. Measurement on spectral lines (Intensity measurements, determination of temperature, line profiles, dependence of the intensity on atomic number). Measurement on continuous radiation (Displacements of the series limits, free-bound and free-free radiation; plasma frequency).

2.1. INTRODUCTION

Analogous to the fundamental variables describing a real thermodynamic equilibrium we shall define quasi-fundamental quantities by means of spectroscopic measurements. The most important of these quasi-fundamental quantities will be a whole set of temperatures, e.g. Doppler-, ionization- and electron-temperatures. The difference between these temperatures, then, will give us clues on the properties of the plasma under investigation, especially on the deviation from thermodynamic equilibrium.

Since general statements can only be made for an equilibrium we will mostly start in the following with thermal radiation. As we have stated above such a radiation is by no means

necessarily identical with a black body radiation.

Let us first remember some fundamental relationships of thermodynamics: The intensity of the cavity radiation is given by Planck's equation

$$B(\nu, T) = \frac{2h\nu^3}{c^2} \frac{1}{\exp\left(\frac{h\nu}{kT}\right) - 1}$$

There are two limiting cases of this equation, viz., for $h\nu/kT \ll 1$ the Rayleigh-Jeans formula

$$B(\nu, T)_{R.J.} = \frac{2\nu^2}{c^2} kT$$

and for

$$\frac{h\nu}{kT} \gg 1 \quad \text{Wien's law} \quad B(\nu, T)_w = \frac{2h\nu^3}{c^2} \exp\left(-\frac{h\nu}{kT}\right).$$

In addition, Wien's displacement law holds giving the wave length $\lambda_{\max}$ at which $B(\nu, T)$ has its maximum as a function of temperature:

$$\lambda_{\max} = \frac{2.9 \times 10^7}{T} \text{ \AA}.$$

At $T = 10^7$ °K $B(\nu, T)$ reaches its maximum at $\lambda = 2.9$ Å. Therefore, in order to investigate the radiation of a hot plasma, which is only approximately thermal, it is necessary to extend the spectroscopic measurements to the region of extreme UV. They should be extended as far as it is at all possible experimentally.

The total radiation

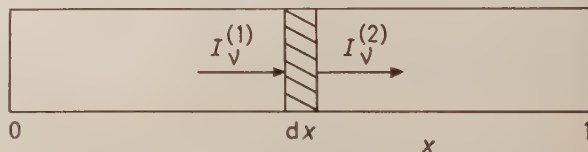
$$\int_0^\infty B(\nu, T) d\nu$$

is proportional to T^4 (Stefan-Boltzmann's law).

At temperature above 10^6 °K the visible radiation lies in the Rayleigh-Jeans region:

$$\frac{h\nu}{kT} = 2.8 \times 10^{-2} \text{ for } T = 10^6 \text{ °K and } \lambda = 5000 \text{ \AA}.$$

We will now turn to a thermally radiating plasma of *finite* thickness l .



The difference in intensities of the radiation entering the thin layer of thickness dx from the

left-hand side and that leaving the layer can be written as follows:

$$I_{\nu}^{(1)} - I_{\nu}^{(2)} = dI_{\nu}(x) = \kappa_{\nu} \left(1 - \exp \left(-\frac{h\nu}{kT} \right) \right) B_{\nu}(T) dx \\ + \kappa_{\nu} \exp \left(-\frac{h\nu}{kT} \right) I_{\nu}(x) dx - \kappa_{\nu} I_{\nu}(x) dx.$$

The first term represents the intensity of the radiation emitted by the layer by virtue of spontaneous emission; the second term gives the contribution of the induced emission. The third term represents the absorption within the layer. Together with the boundary condition $I_{\nu}(0) = 0$ integration of the latter equation yields

$$I_{\nu}(l) = B_{\nu}(T) \{ 1 - \exp(-\kappa'_{\nu} l) \} \quad (5)$$

where

$$\kappa'_{\nu} = \kappa_{\nu} \left(1 - \exp \left(-\frac{h\nu}{kT} \right) \right).$$

2.2. MEASUREMENTS ON SPECTRAL LINES

(a) Intensity measurements

The upper limit for the intensity of any thermal radiation of a homogeneous plasma is the intensity of the cavity radiation. In the case of very strong spectral lines this limit is frequently reached. This means a photon of the line frequency is contained in the plasma during a very long time or in other words the plasma behaves at this frequency like a black body: it is then optically thick. For example, by changing the thickness of the radiating plasma it can be decided whether a line is in this sense optically thick or not: If there is no increase in intensity on an increasing thickness, the question is positively answered. Correspondingly, eq. (5) yields for

$$\kappa'_{\nu} l \gg 1 : I_{\nu}(T) = B_{\nu}(T).$$

(In analogy to what is customary in astrophysics one might define from the preceding eq. a "radiation temperature".)

An optically thick line seems to be emitted in the laboratory ordinarily only by a body in thermal equilibrium. Naturally care has to be taken in order to avoid self-reversal.

The other limiting case is what is called an optically thin plasma. The term is used anal-

ogous to what has been explained above. Here, $\kappa'_{\nu} l \ll 1$ and eq. (5) becomes

$$I_{\nu} = \kappa_{\nu} l B_{\nu}(T) \left(1 - \exp \left(-\frac{h\nu}{kT} \right) \right).$$

The integration over the frequency around a spectral line leads to an expression which corresponds to the right-hand side of eq. (3):

$$\int_{\text{line}} I_{\nu} d\nu = I = \frac{1}{4\pi} h\nu A_{mn}^{\text{line}} n_0 \frac{g_n}{u_0} \exp \left(-\frac{x_n}{kT} \right) l \quad (6) \\ n_0 \frac{g_n}{u_0} \exp \left(-\frac{x_n}{kT} \right) = n_n$$

is the number of particles per cc in the upper level according to Boltzmann's formula†. Other symbols in this equation have been explained on page 353. Measurements of line intensities yield a temperature (excitation temperature) according to the latter equation. It doesn't seem to have much sense to define a temperature for a stationary state without thermal equilibrium in the restricted sense specified above (see p. 354) by means of the Boltzmann formula. Here, it is necessary to draw up a balance of all processes which create or destroy excited or ionized particles. The number of these particles, n_n and, hence, $\int I_{\nu} d\nu$ is then a function of cross-sections, transition probabilities, electron temperature (in most cases assumptions have to be made regarding the velocity distribution of the electrons) and the density of the electrons. The latter can be measured by other methods. These somewhat cumbersome calculations don't seem to be worthwhile in all cases because the cross-sections of meta-stable states which are often particularly important in a non-equilibrium state, frequently depend very sensitively on impurities.

Finally, a temperature may be determined by measuring the relative intensities of different lines emitted by the same element (or by different elements). According to eq. (6) one obtains for thermal radiation:

$$\frac{I \int I_{\nu} d\nu}{\Pi \int I_{\nu} d\nu} = V = \frac{v_I}{v_{II}} \frac{A_{mn} g_n}{A_{rs} g_s} \exp \left(-\frac{(x_n - x_s)}{kT} \right) \quad (7)$$

or

$$\frac{I \int I_{\nu} d\nu}{\Pi \int I_{\nu} d\nu} = V = \frac{v_I}{v_{II}} \frac{A_{mn}^I}{A_{rs}^{II}} \frac{n_0^I}{n_0^{II}} \frac{u_0^{II}}{u_0^I} \frac{g_n^I}{g_s^{II}} \exp \left(-\frac{(x_n^I - x_s^{II})}{kT} \right)$$

† More general instead of n_0, g_n, u_0, x_n one uses $n_r, g_{r,n}, u_r, x_{r,n}$ for the r th state of ionization.

for the elements I and II. In the more general case the lines are emitted by the same element but from different states, e.g. from the r th and the $(r+1)$ th states of ionization:

$$V = \frac{v_I A_{mn}^{r+1} u_r g_{r+1,n} n_{r+1}}{v_{II} A_{ls}^r u_{r+1} g_{r,s} n_r} \exp\left(-\frac{(\chi_{r+1,n} - \chi_{r,s})}{kT}\right) \quad (7a)$$

The relative error of a temperature measured in accordance with eq. (6), (7) or (7a) is given by

$$\frac{\Delta I}{I} = \frac{\chi_n}{kT} \frac{\Delta T}{T} \quad \text{and} \quad \frac{\Delta V}{V} = \frac{\Delta \chi}{kT} \frac{\Delta T}{T}$$

where

$$\chi_n - \chi_s = \Delta \chi \quad \text{or} \quad \chi_{r+1,n} - \chi_{r,s} = \Delta \chi.$$

The dependance on temperature of the partition function is in most cases weak and will be neglected here. One can see that a certain error in measurement implies an uncertainty of the measured temperature which decreases with increasing values of the quantity χ_n/kT or $\Delta \chi/kT$. Therefore this method can only be used if the energy of the upper level of the line χ_n or the difference $\Delta \chi$ is equal to or larger than kT . The lines suited for this method are the comparatively strong lines which correspond to transitions into the ground level or into levels near ground level. At high temperature most of these lines are in the far UV (for $T_{ev} = 11\,600^\circ\text{K}$ is $1\text{eV} = kT_{ev}$). Of course, there are also lines in the near UV or even in the visible spectral region which have sufficiently large values of χ_n ; these lines correspond to transitions into higher excited levels. The method becomes very convenient if two lines from different states of ionization can be used which lie near together. Under these circumstances $\Delta \chi$ is usually sufficiently large as compared to kT ; furthermore, no calibration with respect to the wave length dependence of the detector is necessary. E.g., such a pair of spectral lines for a temperature in the region of $\approx 5 \times 10^4^\circ\text{K}$ are the lines He II $\lambda = 4686 \text{ \AA}$ and He I $\lambda = 4713 \text{ \AA}$; here $\Delta \chi = 26 \text{ eV}$.

The description of the technique of measuring absolute and relative intensities of spectral lines is somewhat tedious and will be omitted. The difficulties should be pointed out, however,

which are connected with the problem of finding a radiation standard for extreme temperatures since such a light source has to radiate also in the far UV region with sufficient intensity. If this problem is solved the standard has to be calibrated in terms of thermodynamic temperature (cf. ref. ¹) p. 23 and 24).

Another problem is the identification of spectral lines. In order to identify the lines of highly ionized atoms a procedure given by B. Edlén (cf. ref. ²)) should be followed in which the levels are extrapolated along iso-electronic sequences. Then, all the quantities which influence the line intensities can be evaluated.

(b) Estimation of temperatures from ionization

Even before all the above mentioned conditions or magnitudes are known, which are necessary for the exact determination of relative and absolute intensities of spectral lines in the far UV region, it is possible to estimate the temperature of the electrons from the presence, in great intensity, of lines which belong to certain states of ionization, e.g., to the r th state, indicating the existence in large numbers of particles in this state.

Approximately this gives an upper limit: $\frac{3}{2} kT \leq \chi_{r-1}$ or

$$T \leq 7.7 \times 10^3 \chi_{r-1}$$

where χ_{r-1} in volts, T in $^\circ\text{K}$.

E.g., the appearance of the following lines gives an order of magnitude indication for the following temperatures:

O V	line	$T \leq 6 \times 10^5^\circ\text{K}$
Fe XIV λ 5303 $\text{\AA}$	(green corona line)	$T \leq 2.7 \times 10^6^\circ\text{K}$
Fe X λ 6374 $\text{\AA}$	(red corona line)	$T \leq 1.8 \times 10^6^\circ\text{K}$

The energy of ionization necessary for determining the "ionization temperature" can be found in two papers by W. Finkelburg (ref. ¹²)) and W. Finkelburg and W. Humbach (ref. ¹³)) or they can be calculated, according to the

¹) A. Unsöld, Physik der Sternatmosphären. 2. Aufl. (Berlin, Springer 1955).

²) B. Edlén, Z. Astrophysik 22 (1942) 30.

¹²) W. Finkelburg, Phys. Rev. 77 (1950) 303, 304.

¹³) W. Finkelburg and W. Humbach, Naturwiss. 42 (1955) 35.

authors, essentially by extrapolation along iso-electronic sequences (cf. ref. ¹), p. 90).

(c) Measurement of line profiles

Since the lifetime of an excited state of an atom is finite the energy level has a certain width according to the uncertainty principle. This "natural" line-broadening (in the order of 10^{-4} Å units) is masked by the (kinetic) Doppler broadening and the broadening due to the statistical interactions of the particles.

(c α) Doppler effect

Let us consider the velocity distribution of the particles in the direction of the axis of the spectrograph (x -axis). We assume the distribution to be Maxwellian. The fraction dN/N of atoms with their x -component of velocity lying between v_x and $v_x + dv_x$ is then given by

$$\frac{dN}{N} = \frac{1}{\sqrt{\pi}} \exp\left(-\frac{\frac{1}{2}mv_x^2}{kT}\right) \sqrt{\frac{m}{2kT}} dv_x.$$

The most probable velocity is $v_p = \sqrt{2kT/m}$ (m = mass of the atom). Substituting for $2kT/m$ the equation above becomes

$$\frac{dN}{N} = \frac{1}{\sqrt{\pi}} \exp\left(-\frac{v_x^2}{v_p^2}\right) \frac{dv_x}{v_p}. \quad (8)$$

By making use of the Doppler-relationship

$$\frac{\Delta\lambda}{\lambda} = \frac{\Delta\omega}{\omega} = \frac{v_x}{c}$$

the Doppler-width is defined according to

$$\frac{\Delta\lambda_D}{\lambda} = \frac{\Delta\omega_D}{\omega} = \frac{v_p}{c} = \frac{1}{c} \sqrt{\frac{2kT}{m}}.$$

Substituting for v_x and v_p in eq. (8) we see that the relative intensity should follow the law

$$\frac{dI}{I} = \frac{1}{\sqrt{\pi}} \exp\left(-\left(\frac{\Delta\lambda}{\Delta\lambda_D}\right)^2\right) \frac{d\lambda}{\Delta\lambda_D}$$

or

$$\frac{dI}{I} = \frac{1}{\sqrt{\pi}} \exp\left(-\left(\frac{\Delta\omega}{\Delta\omega_D}\right)^2\right) \frac{d\omega}{\Delta\omega_D}$$

where

$$I = \int_{\text{line}} I_\nu d\nu.$$

At half peak value of the intensity the equation

$$\exp\left(-\left(\frac{\Delta'\lambda}{\Delta\lambda_D}\right)^2\right) = \frac{1}{2}$$

holds. From this follows that the entire half width is given by

$$2\Delta'\lambda = 2\sqrt{\ln 2} \Delta\lambda_D = 1.67 \Delta\lambda_D$$

or

$$2\Delta'\omega = 1.67 \Delta\omega_D.$$

The table gives the values of $2\Delta'\lambda$ for a temperature of $T = 10^7$ °K at 1000 Å.

Atom	entire half width
Oxygen (atomic weight 16)	0.57 Å
Iron (atomic weight 56)	0.30 Å
Uranium (atomic weight 238)	0.15 Å

In addition, a line broadening is caused by statistically distributed macroscopic turbulence. In order to distinguish this influence from the genuine Doppler broadening one has to consider not only the half width but the entire line profile.

(c β) Line broadening by micro-fields of ions and electrons

In plasmas under the conditions we have to deal with in the field of thermonuclear fusion the energy levels will be predominately broadened by the influence of the electric fields of the ions and electrons at the point of the emitting atom or ion. The Stark effect disturbs the upper energy level much more than the lower one; therefore, the broadening of the spectral line is practically identical with the broadening of the upper level.

Historically the theory of this kind of broadening was developed along two different lines. The two resulting theories may be considered as limiting cases of one universal theory of line broadening. Unfortunately such a universal theory is still too complicated to be of practical use.

The Statistical Theory (Stark, Debey, Holtsmark, Verwey, Schmaljohann) is based on the assumption that the radiating particle is all the time influenced by the quasi static fields of the surrounding particles. The resulting displacement of the energy level yields the contour of the line if the statistical distribution of the micro-fields is taken into account. On the other hand the Collision Damping Theory (Lorentz, Lentz,

Weisskopf, Lindholm) considers the perturbation process to be an infinite short collision which interrupts the undisturbed emission. A Fourier analysis of such a statistically chopped but otherwise undisturbed radiation yields the profile of the line.

The borderline between the validity ranges of both theories has long been unknown. It was found independently then by G. Burkhardt (ref. ³) and L. Spitzer (ref. ⁴).

We shall describe the situation in the following way: In the Collision Damping Theory the so-called Weisskopf collision radius ρ_0 is of importance in so far as the theory shows that essentially only those perturbing particles which pass the emitting atom within the distance ρ_0 contribute to the broadening. The particle density is assumed to be small enough so that the time interval between two "collisions" is large as compared to the "perturbation time" (i.e. the time during which the perturbing particle remains within the circle of radius ρ_0). On the other hand, if the particle density is large there are always particles within the "Weisskopf circle"; there are no longer "collisions" but the emitting atom is all the time under the influence of disturbing particles. Under this condition the Statistical Theory has to be applied.

There is an intermediate density range, however, where on the average just one particle is found within a distance ρ_0 from the emitting atom, for which neither of both theories hold.

Quantitatively, this perturbation causes a frequency displacement $\Delta\omega$ (independent of the particle density!) given by

$$\Delta\omega = \frac{c_1}{\rho_0^2} \quad (\text{linear Stark effect})$$

$$\Delta\omega = \frac{c_q}{\rho_0^4} \quad (\text{quadratic Stark effect}).$$

At the distance $\Delta\omega$ from the center of the undisturbed line a frequency interval exists in which neither of both theories are applicable.

The Collision Damping Theory always leads to a dispersion profile:

$$I(\omega) = \frac{\alpha}{\pi} \frac{1}{(\omega - \omega_0 + \beta)^2 + \alpha^2}$$

($\omega = 2\pi\nu$ is the angular frequency, subscript 0 refers to the center of the undisplaced line).

The displacement β and the half width α are functions of the type of interaction during the collision (linear or quadratic Stark effect); furthermore, both quantities depend linearly on the electron density and weakly on temperature (linear Stark effect $\approx T^{-\frac{1}{2}}$, quadratic Stark effect $\approx T^{\frac{1}{2}}$).

In the Statistical Theory the probability distribution of the perturbing field and, hence, the statistics of the line, cannot be represented in closed forms. This probability profile contains implicitly the so-called "normal field strength" given by

$$F_0 = 2.6 e N_e^{\frac{2}{3}} \quad (N_e \text{ electron density}).$$

(F_0 may be interpreted as the field strength which corresponds to the mean distance between two disturbing particles: $F_0 = e/r_0^2$.)

In order to determine the electron density spectroscopically one has to vary the theoretical profile by variation of N_e until it is equal to the experimentally found profile. The measured profile has to be normalized like the calculated one ($\int I_\nu d\nu$ has to be equal in both cases). Any comparison between parts of a line-profile without normalization is worthless.

Almost always there are a number of Stark effect components. Our statements refer to every single component. The final profile of the line is obtained by summation of all components which are broadened according to the Collision Damping Theory or to the Statistical Theory.

(c γ) Splitting of lines by Stark effect. Faltung of line profiles. Splitting of lines in magnetic field

A general quantitative statement on the broadening by Stark effect cannot be made since for different lines the broadening may be different by orders of magnitude. This is due to the strong dependence of the Stark effect on the main quantum number of the line and on the effective atomic number Z (the broadening is

³) G. Burkhardt, Z. Physik 115 (1940) 592.

⁴) L. Spitzer, Phys. Rev. 58 (1940) 348.

$\approx 1/Z$ for linear Stark effect and hydrogen-like atoms or $\approx 1/Z^2$ for quadratic Stark effect). It can be said that due to this dependence on Z the broadening by Stark effect becomes smaller with temperature because $Z \approx T^{1/2}$ where T is the temperature at which ionization is essentially complete (see this page). A. Unsöld (ref. ¹) p. 320, 321) gives an estimation of the displacement for a hydrogen-like ion with atomic number Z :

$$\Delta\lambda = 0.019 \frac{\lambda^2 n^2}{Z^3} F$$

when

n = main quantum number of the upper term
 F = main statistic field strength = $1.6F_0$ (F_0 (see above) in e.s.u., $\Delta\lambda$ and λ in cm).

We have to expect widths of lines which may be larger or smaller than those due to Doppler effect. If neither the Doppler effect can be neglected as compared to the Stark effect nor vice versa the resulting line profile $P(\lambda)$ is obtained by faltung of the Doppler profile $D(\lambda)$ and the Stark profile $S(\lambda)$:

$$P(\lambda) = \int_{-\infty}^{+\infty} D(\lambda - \lambda') S(\lambda') d\lambda'.$$

If $S(\lambda)$ is a dispersion profile the result of the faltung is a so-called Voigt profile which has been calculated (e.g., c.f. ref. ¹) p. 259).

Finally, the splitting of spectral lines in magnetic fields shall be mentioned. As is known this splitting is given by

$$\Delta\lambda = g \frac{e\lambda^2}{4\pi mc^2} H.$$

In order to make an estimation we take Landé's g -factor equal to one and get

$$\Delta\lambda = 4.2 \times 10^{-13} \lambda^2 H \quad \Delta\lambda, \lambda \text{ in } \text{\AA} \quad H \text{ in gauss.}$$

E.g., for $\lambda = 1000 \text{ \AA}$ and $H = 10^4$ gauss we obtain

$$\Delta\lambda = 4.2 \times 10^{-3} \text{ \AA}.$$

(d) *Remark on the dependence of the line emission on the effective atomic number*

The energy loss due to the radiation depends strongly on the effective atomic number. We will demonstrate this, using as an example the

hydrogen-like ion of atomic number Z . The radiation per unit time and volume into the whole solid angle (isotropy is assumed) of an optically thin plasma with particle density N_n in the frequency interval of a line is given by

$$\int_{0, \text{line}}^{4\pi} \epsilon_r d\nu d\omega = h\nu_{mn} N_n A_{mn}^{\text{line}}.$$

Here, ν_{mn} is proportional to Z^2 because of Balmer's formula

$$\nu_{mn} = hRcZ^2 \left(\frac{1}{m^2} - \frac{1}{n^2} \right)$$

and A_{mn} is proportional to Z^4 (see e.g. ref. ¹), p. 348). Therefore, the total radiation in the frequency interval around a line is proportional to Z^6 .

Also let us estimate the atomic number of those atoms which at 10^7 °K still have just one electron (hydrogen-like atom). The ionization energy of such an ion is equal to $Z^2 \cdot 13.5$ eV. On the other hand at $T = 10^7$ °K $kT = 860$ eV. If we rely on thermal excitation we obtain practically complete ionization if $kT \approx \frac{1}{2} Z^2 \cdot 13.5$ eV. The factor $\frac{1}{2}$ arises from the fact that the fast electrons in the tail of the Maxwell distribution are responsible for ionization and is only an approximation.

Hence,

$$Z = \sqrt{\frac{860 \cdot 5}{13.5}} = 18.$$

If we assume the average transition probability for hydrogen for a fairly strong line to be 10^8 sec^{-1} we obtain for a hydrogen-like ion with atomic number $Z = 18$

$$A_{mn} = 18^4 \times 10^8 = 1.05 \times 10^{13} \text{ sec}^{-1}.$$

A plasma with a density such that the collision frequency is of the same order of magnitude that the above at $T = 10^7$ °K, is beyond technical realization. However, since the levels can be occupied only by means of collisions the number of particles in excited states must become small. The excitation is no longer thermal. These conditions are similar to those which are found in the corona.

As a result of continuity we might assume that

every atom should show a similar dependence on the effective atomic number.

2.3. MEASUREMENT OF CONTINUOUS RADIATION

(a) *Determination of electron density from the displacement of the series limit*

According to section 2 c β (p. 358) the electron density may be determined from line profiles. This method, is difficult in particular if applied to highly ionized atoms in the extreme UV region. The Stark effect constants have to be calculated (the experimental determination of these quantities does not seem to be possible). It is only then that the decision can be made whether one of both theories could be applied or not. For a number of reasons the procedure is also difficult from the standpoint of the experimentalist. In this section we shall describe a more simple method for the determination of electron density, if it is not possible to get this result out of quite different measurements (e.g. microwave measurements).

The micro-fields of ions and electrons cause a strong broadening of the higher energy levels so that they merge together before the theoretical limit of the series at $n = \infty$ is reached (n means the total quantum number of the upper term of the lines). Hence, the limit of the series now appears at a lower frequency. If we assume the terms of the series to be hydrogen-like the electron density of the plasma may be estimated from the last distinguishable line following a procedure by O. R. Inglis and E. Teller (cf. ref. ¹), p. 325). These authors assume that

$$\frac{\frac{3}{2} a_0}{Z} n_m^2 e F = \frac{\frac{1}{2} e^2 Z^2}{a_0 n_m^8}$$

where

a_0 is the Bohr radius of the atom

Z the effective atomic number

n_m the total quantum number of the upper term of the last distinguishable line

e charge of the electron

F mean field strength of the microfields at the point of the emitting atom.

The left-hand side of this equation represents the energy of an atomic dipole in the field F .

The right-hand side is equal to half the difference between the energy of the level with main quantum number n_m and the energy of the two adjacent levels. The electron density N_e is obtained by substituting for F in eq. (5) the quantity $3.7 e N_e^{\frac{2}{3}}$ according to the Statistical Theory. This method can only be applied if the "merging" of lines is really observed and is not masked by noise.

(b) *Free-bound and free-free radiation (bremsstrahlung)*

We distinguish the free-bound radiation (generated by capture of a free electron into a discrete bound state) from the bremsstrahlung which correspondingly may be called free-free radiation.

As in the case of discrete (line) radiation the intensity of free-bound radiation can only be evaluated if the eigenfunctions of the atoms and ions are known. The free-bound radiation, therefore, is characteristic for each atom. The dependence on frequency of this radiation may vary appreciably from one atom to another. In this respect the reader is referred to ref. ⁵), p. 105. Here we shall only give the emission coefficient for the free-bound radiation of hydrogen and hydrogen-like atoms. If we assume that only the electrons obey a Maxwellian velocity distribution we obtain an equation for the emission coefficient of capture of free electrons into the bound state with total quantum number n :

$$\epsilon_{\nu, n}^{\text{sp}} = c \frac{4\pi^2 e^4 m}{h^3 n^4} Z^4 \frac{N_e N_i}{(kT)^{\frac{3}{2}}} \exp\left(-\frac{h}{kT}(\nu_n - \nu)\right) \quad (9)$$

where ν_n = frequency of the series limits and

$$c = \frac{32 \pi^2 e^6}{3 \sqrt{3} c^3 (2\pi m)^{\frac{3}{2}}} = 6.36 \times 10^{-47} \quad \text{cgs units.}$$

At a certain frequency ν the contributions of all series limits with $\nu_n < \nu$ have to be added up.

In addition, the continuous bremsstrahlung

⁵) W. Finkelburg and Th. Peters, Hdb. d. Phys., Vol. 28 (Berlin 1957) p. 79.

⁶) W. Finkelburg and H. Maecker, Hdb. d. Phys., Vol. 22 (Berlin 1956) p. 254.

is superposed on all this free-bound radiation. Again under the assumption that only the electrons follow a Maxwellian velocity distribution the following equation holds for the bremsstrahlung†:

$$\varepsilon_v^{\text{sp}} = cZ^2 \frac{N_e N_i}{(kT)^{\frac{1}{2}}} \exp\left(-\frac{h\nu}{kT}\right). \quad (10)$$

By means of eqs. (9) and (10) which were derived at first by H. A. Kramers the electron temperature may be determined. As in the case of the line radiation any exact determination of temperature from continuous radiation is only possible if

$$\frac{h(\nu - \nu_g)}{kT} \geq 1 \quad \text{or} \quad \frac{h\nu}{kT} \geq 1.$$

If the ionization is thermal the product $N_e N_i$ may be expressed by means of Saha's formula.

The intensity of both types of continuous radiation decreases with increasing frequency according to eq. (9) and eq. (10) at any temperature. Furthermore

$$\int_0^\infty \varepsilon_{\nu,n}^{\text{sp}} d\nu \sim Z^4 T^{-\frac{1}{2}} \quad \text{and} \quad \int_0^\infty \varepsilon_v^{\text{sp}} d\nu \sim Z^2 T^{\frac{1}{2}}.$$

Referring to eq. (6) for line radiation, (9) for free-bound radiation and (10) for bremsstrahlung it should also be noted that these different expressions approach each other continuously in spite of their different appearance.

For example, the radiation around a line is proportional to Z^6 , whereas the free-bound continuous radiation is proportional Z^4 . However, if we distribute continuously the intensity of a line near the series limit (see p. 360 for the advancing of a series limit) over half of the frequency interval $\Delta\nu$ between the two adjacent lines (in order to do this we divide the intensity by $\Delta\nu$) we see that this distributed intensity is proportional to Z^4 since according to Balmer's formula $\Delta\nu \approx Z^2$ (viz. ref. ¹⁴), p. 167).

† From $\varepsilon_{\nu,n}^{\text{sp}}$ and $\varepsilon_v^{\text{sp}}$ the absorption coefficients $\kappa_{\nu,n}$ and κ_ν may be obtained according to

$$\varepsilon^{\text{sp}} = \kappa \left(1 - \exp\left(-\frac{h\nu}{kT}\right)\right) B(\nu, T).$$

(c) Plasma oscillation. Gaunt factor

As is known, a plasma may oscillate in the far infrared region with the electron eigenfrequency

$$\nu_{p1} = \sqrt{\frac{e^2 N_e}{\pi m_e}} \quad (m_e = \text{electron mass}).$$

This oscillation, however, does not appear at exactly the frequency but within a finite frequency interval about ν_{p1} as a "plasma oscillation line". This broadening of the plasma frequency is caused by acceleration of the electrons in the electric fields of the ions and the radiation damping connected with it.

According to G. Burkhardt¹⁴) the high frequency wing of the "plasma oscillation line" is represented by

$$\varepsilon_\nu = c \frac{\sqrt{3}}{\pi} Z^2 \frac{N_e N_i}{(kT)^{\frac{3}{2}}} \ln\left(\frac{kT}{1.44 e^2 N_i^{\frac{1}{3}}}\right).$$

This formula is also approached by eq. (10) according to G. Elwert¹⁵) if we take into account a hitherto neglected correction factor $g(\nu)$. On the other hand it does not seem to be possible to relate the intensity near ν_{p1} to any temperature since presumably this radiation is often emitted by a non-thermal, cooperative process.

According to a calculation by G. M. Berger¹⁶) the mentioned factor $g(\nu)$ (Gaunt factor) can no longer be neglected at high temperatures in a large spectral region. See fig. 1.

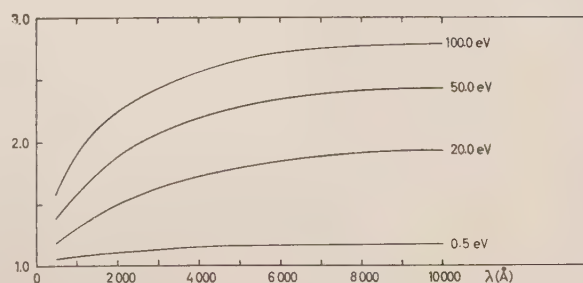


Fig. 1. The average Gaunt factor, $g(\nu)$, plotted as a function of wave length for various temperatures between 0.5 and 100.0 eV.

¹⁴) G. Burkhardt, Ann. Phys. 5 (1950) 373.

¹⁵) G. Elwert, Z. Naturf. 3a (1948) 477.

¹⁶) G. M. Berger, Astroph. Journ. 124 (1956) 550.

(d) Electron-Electron Radiation

In computing the radiation of a plasma, only electron-proton interaction is usually taken into account as is done also in the derivation of eq. (10). An electron colliding with a proton will produce dipole radiation but when colliding with another electron it will produce quadrupole radiation which can be neglected if the speed of the electron is not relativistic. For extremely fast electrons, however, the cross section for e-e collision is only slightly smaller than for p-e collision (ref. 7)). So it will play an important part in extremely hot plasmas. Also, relativistic corrections have to be applied to e-p collisions.

The cross sections have been computed by Redhead (ref. 8)) and Katzenstein (ref. 9)), the application for a plasma by Thompson (ref. 10)). The results have been critically discussed by

J. Stickforth (ref. 11)). If, according to the latter, the total emitted bremsstrahlung of a plasma is given by

$$W = \frac{32}{3} \sqrt{\frac{2}{\pi}} n_e^2 \frac{e^6}{\hbar mc^2} \sqrt{\frac{kT}{mc^2}}$$

the following corrections have to be applied to for e-e and relativistic e-p radiation:

kT (keV)	25	50	100	150
e-e radiation	6%	13%	34%	64%
rel. e-p radiation	6%	10%	22%	37%

7) W. Heitler, Quantum Theory of Radiation, Appendix 6.

8) M. L. Redhead, Proc. Phys. Soc. **66** (1953) 196.

9) I. Katzenstein, Phys. Rev. **78** (1950) 161.

10) W. B. Thompson, Harwell Report AERE T/M 73 (1954).

11) J. Stickforth, Die Electron-Electron-Bremsstrahlung in einem heissen Deuterium-Plasma (Fusionsgruppe Krupp, Essen, 1958).

12. DETECTION OF HEAVY PARTICLES IN THERMONUCLEAR REACTION EXPERIMENTS

L. H. TH. RIETJENS and C. M. BRAAMS

FOM Institute for Plasma-Physics, Rijnhuizen, Jutphaas, Netherlands

A survey is given of methods for detection of heavy particles resulting from thermonuclear reaction experi-

ments. Measurements of neutron emission as a function of time and energy are discussed in detail.

1. Introduction

The detection of heavy particles resulting from thermonuclear reaction experiments, and measurements of their energy, have been used in various laboratories to gain information about the behaviour of hot deuterium plasma.

Particularly, particles from the reactions:



will be considered in this survey, which intends to summarize the available data on this subject.

2. Neutron Spectroscopy

Neutron detection has been used in thermonuclear research to obtain information on the following points:

2.1. the total number of neutrons N emitted during one discharge,

2.2. the number of neutrons emitted as a function of time $\partial N/\partial t$, with a time-scale of the order of microseconds, and

2.3 the energy spectrum of the neutrons $\partial N/\partial E$, averaged over the total discharge time.

Other points of interest are the origin of the neutrons $\partial N/\partial x$, $\partial N/\partial y$, $\partial N/\partial z$, and the direction of emission $\partial N/\partial \theta$, $\partial N/\partial \varphi$.

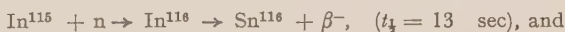
Ideally, one would like to be able to measure the total energy spectrum as a function of time, origin, and direction of emission. However, the low efficiencies of the detectors, the relative small neutron bursts, and the short discharge

times make it difficult to develop a neutron spectrometer that can perform all these measurements.

2.1. THE TOTAL NUMBER OF NEUTRONS N , EMITTED DURING ONE DISCHARGE

It has been found that the number of neutrons emitted in gas discharges depends on many factors, e.g. the applied voltage V , the discharge current I and dI/dt , the initial gas pressure, the ion density, the impurity percentages, and the strength of the externally applied magnetic field. To determine the dependence on these quantities, one measures the average value N over a number of discharges under the same conditions. For this purpose different detectors are used:

2.1.1. *Activation counter.* In these counters the neutrons, after being slowed down in a paraffin mantle, activate an indium or silver jacket surrounding the actual counter:



For the detection of the β -particles, scintillation counters and Geiger counters with a high detection efficiency for electrons can be used, refs. 1-4).

The advantage of these detectors is the

1) A. M. Andrianov *et al.*, USSR, P/2301 †.

2) G. N. Harding *et al.*, Harwell, UK, P/1520.

3) J. W. Mather *et al.*, LASL, USA, P/2394.

4) T. E. Allibone *et al.*, Aldermaston, UK, P/3.

† P refers to the papers of the Geneva conference 1958.

possibility of recording only during the time intervals between discharges. In this way there is no interference caused by electrical transients or by X- and γ -radiation, and there is no difficulty of screening the photo-multipliers. As with all other detection methods, the source of neutrons in the discharge chamber and the direction of emission can be determined by

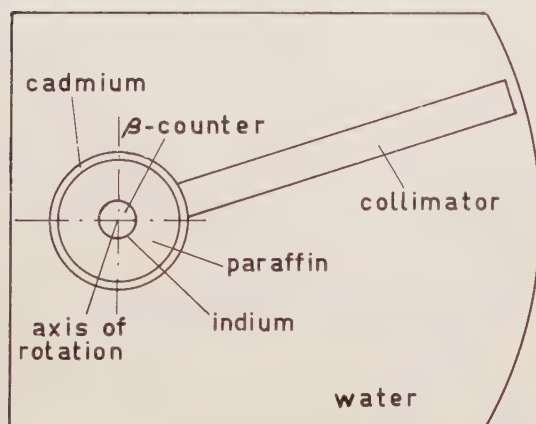


Fig. 1. Activation counter placed in a water tank to measure the number of neutrons per discharge as a function of place of origin and direction of emission.

placing the whole counter in a water tank with a collimator channel directed towards the place in the chamber to be investigated (see fig. 1).

Particularly In^{115} is suitable as material for the jacket because of its high activation cross section ($\sigma_{\text{act}} = 50$ barns), and because of the convenient 13-second half-life of In^{116} . It is possible to discriminate against the low-energy β -emission from the 54-minute $\text{In}^{116\text{m}}$ activity.

The overall detection efficiency depends on the size of the discharge tube, the distance between tube and detector, the detection efficiency of the counter, etc., and reaches in some experiments the value of 1.2×10^{-4} .

2.1.2. Gas proportional counter. These detectors are mostly filled with enriched boron-trifluoride (B^{10}F_3). As the detection efficiency for fast neutrons is extremely low, the large sensitive volume of the counter (about 1000 cc) is surrounded by a 5 cm layer of paraffin for slowing down the neutrons. Furthermore, shields of about 0.3 cm of lead to absorb the X- and

γ -radiation, and of 0.6 cm aluminium for electric and magnetic screening of the detector are present. Due to the slowing-down time, the neutrons are absorbed with a mean delay of about 200 μsec , so that the BF_3 counter is not suitable to measure the time dependence of the neutron emission^{2,5}). The 200 μsec delay is too short to permit extended recording. The overall detection efficiency is again about 10^{-4} .

2.1.3. Other methods. From data obtained by using the detectors which will be described in 2.2. and 2.3., N can be derived by integrating the results over time or energy respectively.

2.2. THE NUMBER OF NEUTRONS EMITTED AS A FUNCTION OF TIME

To determine the function $\partial N/\partial t$, almost all investigators are using scintillation counters of different type. In the major part of the counters, neutrons are detected through (n,p) collisions.

The resolving time of the scintillator and photo-multiplier is very short ($\tau \approx 10^{-8}$ sec). Generally, however, the RC times introduced by the electronics of the spectrometer (5×10^{-7} through 10^{-5} sec) are not short enough, compared with the discharge times (10^{-6} through 10^{-4} in fast discharges) to permit accumulation of more than a few pulses per discharge, and to make possible time-resolved spectroscopic analysis with fast scintillators.

According to their fast response, the scintillation counters are very suitable to determine $\partial N/\partial t$ on a microsecond scale.

2.2.1. Crystal scintillator. Several types of scintillators have been used, e.g. stilbene-, naphthalene-, anthracene-, terphenyl-loaded polystyrene crystals (with TPBD colour shifter), with dimensions up to $12\frac{1}{2}$ cm in diameter and a few centimeters thick. The crystals are connected to photo-multipliers with large photocathodes. The detection efficiency for fast neutrons is generally low, reason why large crystals have to be used to attain an acceptable overall detection efficiency of about 10^{-4} . One of the problems is that all scintillation detectors

⁵) J. Andreoletti *et al.*, France, P/1182.

for fast neutrons are X- and γ -sensitive. To prevent this interference, all crystals are shielded with lead of a thickness between 0.6 and 2 cm, while the photo-multipliers are surrounded by different layers of mu-metal and insulated layers of aluminium for magnetic and electric screening, refs. ^{1,2,5-8}). Another possibility to discriminate between neutrons and γ -quanta can be found by using scintillation crystals developed by Bonner and Hornyak¹⁶). In these large crystals, the size of the independent scintillation centra is correlated to the length of the proton recoil tracks from (n,p) collisions.

By using water or paraffin collimators, again the source of neutrons and the direction of emission can be localized.

2.2.2. Liquid scintillator. The liquid scintillator consists of a solution with a boron component and some kind of a scintillator. The total detection efficiency is of the same order of magnitude (5×10^{-5})¹¹), and again, the efficiency as well as the X- and γ -ray interference are the major problems. The advantage over the crystal scintillators is the large effective volume available for the detection, refs. ^{5,9,11}).

2.2.3. Gas scintillator. As the absorption of fast neutrons in gases is extremely low, no efficient use of gas scintillators can be made.

In all experiments using scintillation counters, the signals from the collector of the photo-multiplier have been recorded by multi-beam oscilloscopes, simultaneously with other quantities as $V(t)$, $I(t)$, etc., and photographed.

2.3. THE ENERGY SPECTRUM OF THE NEUTRONS AVERAGED OVER THE TOTAL DISCHARGE TIME

To measure $\partial N/\partial E$ in fast discharges, two methods are available, both without time resolution. In slower discharges, also scintillation counters can be used.

2.3.1. Nuclear emulsion. In spite of all the objections, already known for years, against nuclear emulsions, this method is used in nearly all experiments producing enough neutrons. The efficiency is low, and for reasonable

numbers of tracks, 10^6 neutrons per cm^2 are required³).

In some experiments 5000 discharges had to be executed to get a sufficient number of tracks on the photographic plates²). Then, the reading of the plates and the evaluation of the data is time consuming. Mostly Ilford C-2 emulsions have been used, refs. ^{1,2,4,11}). With the new Ilford fine grain L-2 emulsion, the background from X- and γ -radiation will cause less trouble.

Another new type of emulsion is Ilford L-4, detecting the protons from the $\text{N}^{14}(\text{n,p})\text{C}^{14}$ reaction, refs. ^{1,13,14}).

2.3.2. High pressure cloud chamber. This technique has as a major advantage the higher detection efficiency, while the energy resolution is still good. With these high efficiencies, $\partial N/\partial E$ can be measured already in experiments with neutron bursts of about 10^6 neutron/discharge, refs. ^{2,7,11}). In one experiment, 870 tracks were observed during 1600 expansions with neutron yields of 10^6 n/discharge. The cloud chamber in this experiment has a diameter of 30 cm and is shielded by 0.6 cm of lead to absorb X- and γ -radiation⁷). New methods developed for evaluating the photographs taken with cloud chambers will make this technique a very useful instrument in plasma diagnostics.

2.3.3. The scintillation counter. In slow discharges (10^{-4} sec up to a few milliseconds), the scintillation counter can be used for neutron spectroscopy. Until now, no efforts have been made to determine $\partial N/\partial E$ in this way. A high efficiency of the counter is not the main request. For example, an overall detection efficiency of 10^{-5} , and a neutron yield of 10^6 n/discharge will

⁶) A. C. Kolb, NRL, USA, P/345.

⁷) J. P. Conner *et al.*, LASL, USA, P/2488.

⁸) V. S. Komelkov, USSR, P/2302.

¹⁶) W. F. Hornyak, Rev. Sci. Instr. **23** (1952) 264; McCrazy, Taylor and Bonner, Phys. Rev. **94** (1954) 808.

⁹) W. C. Elmore *et al.*, LASL, USA, P/356.

¹⁰) S. C. Curran *et al.*, Aldermaston, UK, P/1460.

¹¹) O. A. Anderson *et al.*, UCRL, USA, P/2349.

¹²) C. Beets, Belgium, P/105.

¹³) W. C. Redmann *et al.*, Argonne Nat. Lab., USA, P/597.

¹⁴) S. Selim-Younis, United Arab Republic, P/1483.

give 10 counts/discharge in 10^{-4} sec, which gives the maximum permissible counting rate for a spectrometer with a dead-time of 10^{-6} sec. In discharges of 1 millisecond, 100 counts/discharge can be accepted.

3. Charged-Particle Spectroscopy

Analysers for charged particles emerging from the plasma region are constructed or are being constructed. Of particular interest is firstly the detection of protons originating from D on D reactions. A second point of interest is the measurement of the deuteron velocity distribution in a deuterium discharge.

For the analysis of charged particles, a narrow beam (≈ 0.1 cm diameter) of particles emerging from the plasma region is allowed to enter the analyser. With a high speed electromagnetic valve shutter, the time interval in which a sample is taken can be chosen, while a low operating pressure in the analyser can be maintained. In the analyser the particles are selected according to mv/e by using magnetic fields, and to v by using crossed electric and magnetic fields¹).

An experiment has been described in which, after about 25 discharges, parabolas corresponding to D^+ - and D^- -ions and to the ions from electrode and tube materials appear on the photographic plates, placed in the focal place of the analyser. After the polarity of the discharge had been changed, no D-ion tracks were found, indicating an acceleration process caused by a field with the same polarity as the applied field, refs. ^{1,15}).

A system like this should have a high efficiency and there is no need for a high resolution.

To determine the protons emerging from the plasma region, also nuclear emulsions have been

used. The detection efficiency of the plates is 100%; background from X- and γ -radiation, however, tends to fog the plates.

4. Conclusion

From this survey one may try to draw conclusions about the instrumentation which is suitable for the analysis of heavy particles occurring in thermonuclear reaction experiments.

Useful instruments are in the authors opinion:

(1) Neutron activation counters with indium jackets to measure N , or if it is placed in a water tank with a collimating channel, to measure $\partial N/\partial x$, $\partial N/\partial y$, $\partial N/\partial z$, and $\partial N/\partial \theta$, $\partial N/\partial \varphi$.

(2) Scintillation counters, e.g. a terphenyl loaded polystyrene crystal with colour shifter, or crystals as developed by Bonner and Hornyak, combined with photo-multipliers to measure $\partial N/\partial t$.

The crystal should extend over a large area and be shielded against X- and γ -radiation.

(3) High-pressure cloud chambers and nuclear emulsions to measure $\partial N/\partial E$.

In slow discharges (10^{-4} sec up to a few milliseconds), also fast scintillation spectrometers can be used.

Acknowledgment

This work is part of the research programme of the "Stichting voor Fundamenteel Onderzoek der Materie" (Foundation for Fundamental Research on Matter—F.O.M.) and was made possible by financial support from the "Nederlandse Organisatie voor Zuiver-Wetenschappelijk Onderzoek" (Netherlands Organization for the advancement of Pure Research Z.W.O.).

¹⁵) D. W. Kerst, John Hopkins Lab., USA, P/1062.

13. ULTRA HIGH VACUUM TECHNOLOGY

G. L. MUNDAY

CERN, Geneva

The production of ultra high vacuum ($\approx 10^{-9}$ torr) as a means of eliminating unwanted impurities is considered with reference to thermonuclear devices. The literature and some opinions are briefly reviewed on the methods of measuring and production of these low pressures. Material

characteristics are considered in relation to the special requirements of all metal systems which can be baked at $400-450^\circ\text{C}$. Methods of fabrication of chambers with demountable joints, windows and shut-off valves that can withstand repeated bake-out are reviewed.

1. Introduction

The importance of elimination of impurities, particularly atoms of high atomic number, from the containing chambers of thermonuclear devices has already been shown by several writers (see, for example, refs.^{1,2,3}). One of the techniques that may be used for reducing impurities is the initial production of ultra high vacuum within the confining chamber. Pressures† of 10^{-9} torr or below can be considered to define the vacuum in this context; even so many of the precautions and techniques to be described are necessary to achieve pressures an order of magnitude higher.

Alpert and his co-workers^{4,5} were the first to put this new technology on a sound basis. They were then concerned with small (glass) systems in which pressures of about 10^{-10} torr were achieved but the procedures apply equally well to larger systems. More recently Redhead^{6,7} has produced and measured pressures in the region of 10^{-13} torr in small laboratory installations. Reviews of the general aspects of ultra high vacuum techniques have been given by Alpert⁸⁻¹¹, Baker and Yarwood¹², Yarwood¹³ and more recently by Venema and Bandringa¹⁴ and Grove¹⁵. The latter is written with special reference to the now considerable experience of moderately large U.H.V. systems embodied in

the Stellarator project at Princeton. In addition the Westinghouse reports¹⁶⁻¹⁸ give excellent reviews of the progress and difficulties of the work at its various stages and in particular the more fundamental aspects.

At present the pumping equipment available

¹) For example, R. F. Post, Summary of UCRL Pyrotron Program. Proceedings of the Second International Conference on the Peaceful Uses of Atomic Energy, Vol. 31 and 32 (United Nations, Geneva).

²) P. R. Bell, J. S. Luce, E. D. Shipley and A. Simon, Description of the Oak Ridge Thermonuclear Experiment. Ibid.

³) L. Spitzer, *Astrophys. J.* **95** (1942) 329.

⁴) D. Alpert, *J. Appl. Phys.* **24** (1953) 860.

⁵) D. Alpert and Buritz, *J. Appl. Phys.* **25** (1954) 202.

⁶) P. A. Redhead, First International Vacuum Symposium, Namur, 1958, to be published shortly (The Pergamon Press, London); afterwards referred to as Namur, 1958.

⁷) P. A. Redhead, Paper given at the Fifth National Vacuum Symposium, San Francisco, U.S.A., 1958. To be published about mid 1959 (Pergamon Press, London) afterwards referred to as San Francisco, 1958.

⁸) D. Alpert, Vacuum Symposium Transactions (Pergamon Press, London, 1954).

⁹) D. Alpert, *Science* **122** (1955) 729.

¹⁰) D. Alpert, *Handbuch der Physik* (Springer Verlag, Berlin + Heidelberg) **12** (1958).

¹¹) D. Alpert, Namur, 1958.

¹²) F. Baker and J. Yarwood, *Vakuum Technik* **7** (1957) 1947; **7** (1957) 186.

¹³) J. Yarwood, *J. Sci. Instrum.* **34** (1957) 297.

¹⁴) A. Venema and M. Bandringa, *Philips Tech. Rev.* **20** (1958) 145.

¹⁵) D. Grove, San Francisco 1958. (This also appears under the title, Project Matterhorn, Scientific Paper No. 15 - NYO 8750.)

¹⁶) Westinghouse Research Report, No. 100 FF 1054-R1.

¹⁷) Westinghouse Research Report, No. 100 FF 1054-R2.

¹⁸) Westinghouse Research Report, No. 403 FF 220-R1.

† All pressures, speeds and outgassing rates referred to in this note are equivalent nitrogen pressures, speeds and outgassing rates unless otherwise noted.

for reaching 10^{-9} torr or below is of low speed, consequently it is necessary to reduce the system outgassing to a minimum. This can only be achieved by careful selection of materials and components all of which must be subject to a bake-out under vacuum at $400-450^{\circ}\text{C}$. It should be noted that the bake-out at $400-450^{\circ}\text{C}$ for 10–20 hours gives a factor of between 10^5 and 10^6 lower outgassing rate per unit area as compared with pumping without bake-out for the same length of time. The outgassing rate per unit area is defined as the quantity of gas evolved per unit time at a given pressure per unit area of surface and is generally quoted in units of torr l/sec $\cdot$ cm ^{2} .

Even if pumps of exceptionally high speed at low pressures were now available (e.g. $\approx 10^5$ l/sec at 10^{-9} torr) the system ultimate pressure would nevertheless be limited to some relatively high value if the chamber were unbaked as the impedances associated with apertures, manifold and valves would appreciably lower the pumping speed at the mouth of the chamber. This effect might be overcome if the pump were made integral with the plasma confining chamber; an example of this arrangement has been described by Milleron and Popp¹⁹) as part of the Pyrotron programme. In this work bake-out was used and it must be considered as a general rule for all ultra high vacuum purposes.

The basic components and factors influencing ultra high vacuum system design will be reviewed briefly in the subsequent paragraphs. A bibliography has been included which is fairly complete in so far as the review articles and example references cited give further references to the earlier work. Little is known of the Russian work on these problems at the moment but it is reported that ion-getter pumps of very high speed are being used in addition to diffusion pumps in "Ogra".

2. Measurement of Pressures below 10^{-8} Torr

The conventional ionisation gauge (the hot cathode triode valve) has a lower limit of

¹⁹) N. Milleron and E. Popp, Report UCRL 5258 and San Francisco 1958.

pressure measurement around 10^{-8} torr; the limit is set by unwanted currents due to photoelectric emission from the collector (anode) which arise from soft X-rays generated by the electrons which strike the grid. The photoemission from the collector gives a current of the same sign as the positive ions arriving at the collector.

Leck²⁰) and Alpert^{4,5}) give reviews (with references) of the methods that were tried before the advent of the Bayard-Alpert gauge²¹). The latter is in general use in U.H.V. systems and is only limited by the X-ray effect at pressures between 10^{-10} and 10^{-11} torr. This gauge is now available commercially along with suitable control units but it is nevertheless necessary to check the operation and calibration in the way laid down by Alpert. Schulz²²) discusses the properties of the gauge at higher pressures.

Two new cold cathode gauges have been reported which have extended pressure measurements into the 10^{-13} torr range. These are called the "inverted magnetron gauge"^{23,6,7}) and the "magnetron gauge"⁷) by Redhead who has invented them. These gauges are not stable in operation.

3. Constructional Materials

Glass parts of the system generally limit the bake-out temperature to 450°C , thus all parts should be able to withstand this without loss of mechanical strength, melting, decomposition or the production of high vapour pressures. Materials such as brass, zinc, cadmium, lead and aluminium and all organic gasket materials should be excluded.

Vapour pressure data are summarized by Smithells²⁴) and Dushman²⁵); the latter collates

²⁰) J. H. Leck, *Pressure Measurement in Vacuum Systems*. London, 1957 (The Institute of Physics) p. 77 and p. 125 *et seq.*

²¹) R. T. Bayard and D. Alpert, *Rev. Sci. Instr.* **21** (1950) 571.

²²) G. J. Schulz, *J. Appl. Phys.* **28** (1957) 1149.

²³) J. P. Hobson and P. A. Redhead, *Canad. J. Phys.* **36** (1958) 271.

²⁴) C. J. Smithells (Ed.), *Metals Reference Book*, 2nd Edition (London, 1955) p. 613.

²⁵) S. Dushman, *Scientific Foundation of Vacuum Technique* (London, 1949).

the then available evidence on the evaporation rates of alloys at high temperatures and demonstrates the utility of Raoult's Law for predicting the approximate behaviour of alloy systems as yet uninvestigated.

The materials of choice are stainless steel, high nickel alloys, oxygen free high conductivity (O.F.H.C.) copper²⁶, high alumina ceramics and borosilicate glasses. Steel will in general be undesirable for magnetic reasons but if admissible it should only be used with caution; see Norton²⁷.

Norton also gives curves for the helium permeability of fused silica and a variety of commercially available glasses. Todd²⁸ shows that water vapour is the primary constituent of the gas evolved from glass and that the quantity per unit area evolved when baking about 300° C varies linearly with respect to the square root of the time of bake-out. Waldschmidt²⁹ and Dushman (op cit.) review the question of constructional materials with reference to adsorption, outgassing and permeability of gases. Much of this knowledge is far from precise or complete but it serves as a rough guide to system design; see figs. 1 and 2.

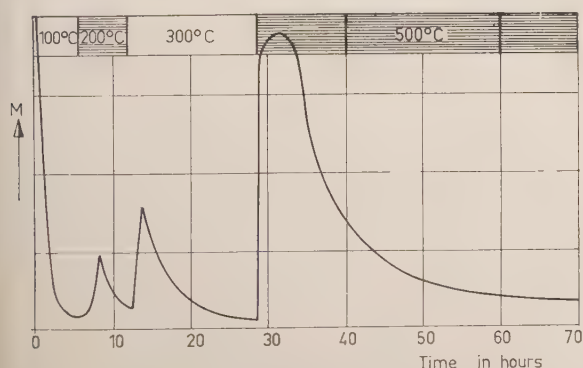


Fig. 1. Evolution of gas M (relative) of an iron rectifier vessel heated to various temperatures vs. time. (D. Deck: Mittlg. Brown Boveri 30 (1943) 46; see ²⁹)).

A typical example of the outgassing rate of a system fabricated largely from stainless steel is 10^{-15} torr l/sec·cm² ^{8,15} after about 10 hours

baking at 450° C (c.f. unbaked, stainless steel after 24 hours pumping; about 10^{-10} torr l/sec·cm²). Grove³⁰ reports lower rates (5×10^{-16} torr l/sec·cm²) after prolonged outgassing. It is unfortunate that faults in the material may only appear during the baking process, any leaks that arise in this way may be very difficult or impossible to repair in which case Grove is of the opinion that it is better and less costly to start with a new piece. However, see Ullman's³¹ remarks on the use of varnish as a very short-term remedy.

4. Fabrication including Joints and Windows

Good examples of successful fabrication procedures are given by Smith³², Marker³³, and Grove¹⁵). Inert gas arc welding and high temperature inert gas brazing are the best methods for the construction of the major items of equipment.

The requirement of a demountable joint which is not too costly to machine nor easily spoilt subsequently and which can be baked a large number of times (say 100) at temperatures of 450° C has led to a number of propositions. The once popular knife-edge joint and its various modifications, for example³⁴) and³⁵) has yielded place to the step-type flange³⁶) fig. 3 and the 24 carat gold ring seals³⁷) and³⁸); see fig. 4. The knife-edge joint gasket involves more stringent machining and is much more easily damaged than either of the others. The choice between the step-type and gold ring seals is at the moment largely one of personal preference³⁹),

³⁰) D. Grove, Private communication.

³¹) R. Ullmann, Vacuum Symposium Transactions, (Pergamon Press, London, 1957) p. 95.

³²) H. R. Smith, Vacuum Symposium Transactions (Pergamon Press, London, 1955) p. 27 *et seq.*

³³) R. C. Marker, Vacuum Techniques and Components used for Continuously Pumped Linear Accelerators. Namur, 1958.

³⁴) G. Hees, W. Eaton and J. Leck, Vacuum Symposium Transactions (Pergamon Press, London, 1955) p. 75; Vacuum 4 (1954) 438, published 1957.

³⁵) N. W. Robinson, J. Sci. Instrum. 34 (1957) 121.

³⁶) W. J. Lange and D. Alpert, Rev. Sci. Instr. 28 (1957) 726.

³⁷) W. N. Hickam, Rev. Sci. Instr. 20 (1949) 472.

³⁸) R. L. Sproull, Rev. Sci. Instr. 22 (1951) 410.

³⁹) J. Carmichael and D. Grove, Private communications.

²⁶) W. Espe, Nachrichtentechnik 6 (1956) 355 and 401.

²⁷) F. J. Norton, J. Appl. Phys. 28 (1957) 34.

²⁸) B. Todd, J. Appl. Phys. 26 (1955) 1238; 27 (1956) 1209.

²⁹) E. Waldschmidt, Metall 8 (1954) 749.

nevertheless it is probable that the gold seal is more useful and has some advantage for non-circular cross sections. The copper gaskets

seal. The Westinghouse research reports^{16,17,18}) make comments on all three types of joints mentioned here. Milleron⁴²) gives some prelim-

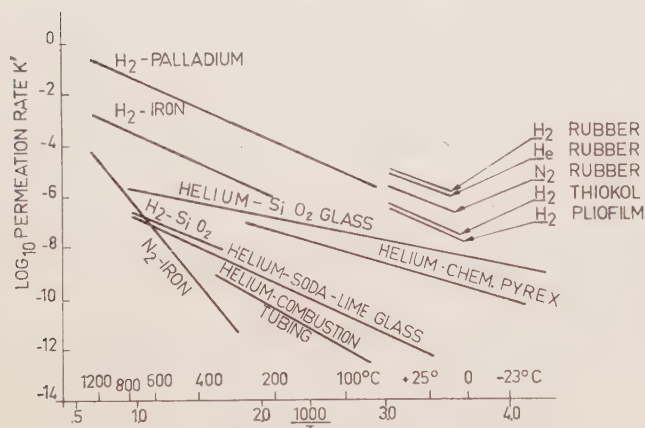


Fig. 2. Permeability of gases: Norton²⁷).

Rate of permeation of gas through solids. K' in cm^3 gas (NTP) per sec per cm^2 per mm thick per 1 atm. pressure difference.

(O.F.H.C.) must be annealed at 850–900°C which is a disadvantage. Grove¹⁵) reports successful operation (40 to 50 bake-outs at 450°C without failure) of the gold seals with

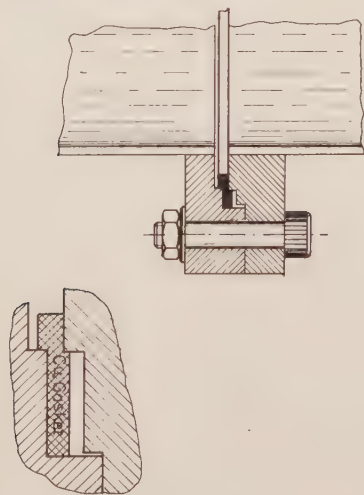


Fig. 3. Step type flange: Lange and Alpert³⁶).

dimensions up to 200 mm diameter with loadings of 50–100 kg/cm of gasket. Blears⁴⁰) has mentioned the successful use of this type of seal up to 500 mm diameter but the exact conditions of use are unknown. Patee⁴¹) claims some mechanical movement at 10^{-10} torr for his knife edge

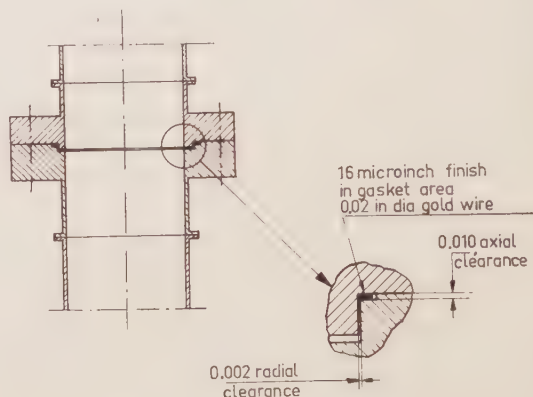


Fig. 4. Gold ring seal: Grove¹⁵).

inary and encouraging results on his copper "bead" seal.

The problems of observation windows and windows for the transmission of ultraviolet radiation are discussed by Grove¹⁵) and Lewin and Mark⁴³); see fig. 5.

Microwave window and ceramic-to-metal

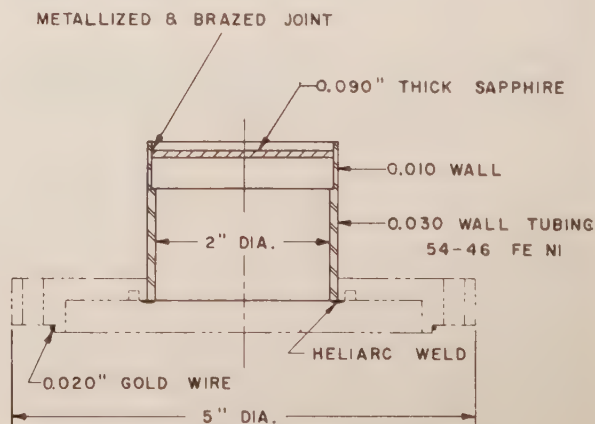


Fig. 5. Sapphire window assembly: Grove¹⁵).

seals for insulating one section of the chamber to another are also discussed by Grove¹⁵). One

⁴⁰) J. Blears, Private communication.

⁴¹) H. H. Patee, Rev. Sci. Instr. 25 (1954) 1132.

⁴²) N. Milleron, Report UCRL 5259 and San Francisco (1958).

⁴³) G. Lewin and R. Mark, San Francisco (1958). (This also appears under the title of Project Matterhorn, Scientific Paper No. 14 - NYO 8749.)

of the ceramic-to-metal joints found to be very successful was the RCA ram seal^{43,44}).

5. Valves

The development of suitable shut-off valves presents a serious problem. The valve should have the following rough specification: high

Mercury pumps have been little used outside purely laboratory installations e.g. Venema¹⁴) but oil pumps have been used extensively and successfully (e.g. refs.^{4,15-19}) in a variety of installations working into the 10^{-10} torr range. Low speed systems constructed with a glass diffusion pump and an Alpert type copper

TABLE 1

Open Conductance l/sec	Aperture mm	Closed Conductance l/sec	Remarks	Reference
0.3	—	10^{-11}	Closed conductance increases with use	Alpert ⁴⁵)
1	—	10^{-11}	Modified Alpert type	Bills and Allen ⁴⁶)
3-5	—	10^{-8} - 10^{-9}	M.I.T.	None
60	38	10^{-9}		Marker ³³)
80	50	5×10^{-9}	See fig. 6	15,16,17)

open conductance, low closed conductance, capability of withstanding repeated baking at 400 – 500°C and freedom from all organic materials. For some purposes it may be necessary for the valve to withstand the bake-out temperature when tightly shut.

Some valves described in the literature are given in table 1. The ones of small open conductance are useful for gas handling equipment.

Larger designs of all metal valves are under development (see ref.⁴²).

Small glass valves of different construction have been described⁴⁷) including those suitable for electromagnetic operation⁴⁸). These valves are in general only suitable for laboratory experiments.

6. Pumps and Traps

The methods of pumping at present available are: oil or mercury diffusion pumps, molecular, getter-ionisation and cryogenic.

For conventional methods of pumping (oil or mercury diffusion) problems of pumping and trap design must be considered together.

⁴⁴) I. E. Martin and A. C. Tunis, R.C.A. Engineer 3 (1957) 9.

⁴⁵) D. Alpert, Rev. Sci. Instr. 22 (1951) 536.

⁴⁶) D. G. Bills and F. G. Allen, Rev. Sci. Instr. 26 (1955) 654.

trap⁴⁹) have successfully maintained pressures below 10^{-9} torr for periods of up to several weeks

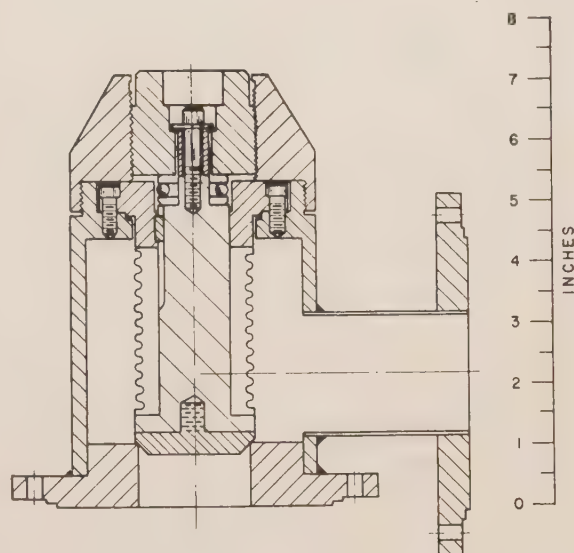


Fig. 6. 80 l/sec U.H.V. valve: Westinghouse¹⁷).

after bake-out after which the pressure rises to 10^{-8} torr or more, see fig. 7.

It has been found that it is unimportant whether a two or three-stage pump is used. The

⁴⁷) R. W. Decker, J. Appl. Phys. 25 (1954) 1441.

⁴⁸) T. P. Vogl and H. D. Evans, Rev. Sci. Instr. 27 (1956) 657.

⁴⁹) D. Alpert, Rev. Sci. Instr. 24 (1953) 1004.

trap which is not refrigerated is made from corrugated O.F.H.C. copper insert (other metals have been used with success) and gives in effect a large number of small tubes in parallel. The mechanism of the trap is not well understood but the "stay-down" time at pressures below 10^{-9} torr has been shown to be affected by the

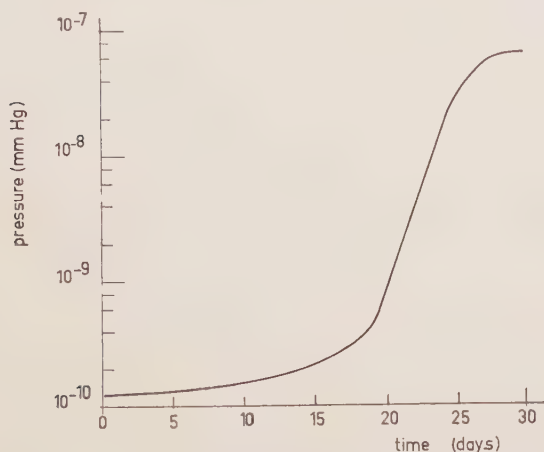


Fig. 7. A typical pressure versus time curve in a small ultra high vacuum system using a copper trap to isolate the system from the pumps: Westinghouse¹⁶).

surface area and the input conductance (Carmichael⁵⁰). The scale up of these small traps (conductance $\approx \frac{1}{4}$ l/sec) so that they may be used with larger pumps has not proved easy. When larger conductances were used with metal oil pumps the stay-down time was very short (as little as 1 hour in some cases); this was thought to be due to oil cracking in the metal pump but this explanation now seems untenable¹⁸). Grove⁵¹) reports some experiments with a 7 l/sec trap in conjunction with a two-stage glass oil pump and explains his observations in terms of the ion induced reemission theory of Varnerin and Carmichael⁵²). The copper trap has been shown to be ineffective for mercury vapour⁵³).

The use of liquid nitrogen traps with oil diffusion pumps has been reported by Ullmann³¹) and Grove¹⁵). Both use traps so designed that any oil migrating over the surface must pass through a region at liquid nitrogen temperatures. Ullmann uses a single trap of the "Post" type (after Dr. R. F. Post, U.C.R.L.) in conjunction with an oil pump of nominal speed 650 l/sec.

After bake-out of system and trap (total volume ≈ 0.5 m³) pressures in the region of 5×10^{-9} – 8×10^{-10} were obtained. It should be noted that this experiment was concluded only by varnishing the welds to cure some leaks. Grove's system used two traps in series with a 300 l/sec pump to get an overall speed at the exit of the second trap of 15 l/sec. After bake-out of the system including the two traps he claims that pressures in the range 3 – 8×10^{-10} torr may be maintained indefinitely; see fig. 8. Milleron⁴²) has produced a trap design which he claims may be scaled up for large oil pumps. In addition to two or more traps in series it may be an advantage to use two diffusion pumps in series.

For the automatic distribution of liquid nitrogen for systems involving many pumps see Barrett⁵⁴) and Wells⁵⁵).

Pumps working either as getter or ionisation pumps or in combination are being investigated for 10^{-9} torr region as they have the possibility of producing "clean" vacua. Clean should be understood to mean freedom from organic pump oils and the presence only of inorganic materials with a very low vapour pressure at room temperature. Thomas⁵⁶) and more recently Holland⁵⁷) have reviewed the work on this topic but without particular reference to U.H.V. It is not intended here to comment on Herb's Evapor Ion pump about which there is no evidence that it will work below 10^{-9} torr, although this should be possible if the pump itself and the titanium could be thoroughly outgassed.

⁵⁰) J. Carmichael, San Francisco (1958), To be published J. Appl. Phys.

⁵¹) D. Grove, Some Observations on the Behaviour of Copper Traps and UHV Systems of Moderate Size, Technical Memo No. 40 (Project Matterhorn).

⁵²) L. Varnerin and J. Carmichael, Westinghouse Report No. 71 F 191 – R6. On the same topic see also: L. Varnerin and J. Carmichael, J. Appl. Phys. **28** (1957) 913; J. H. Carmichael and E. A. Trendlenburg, Westinghouse Scientific Paper No. 403 FF 220 – P2.

⁵³) R. F. McFarland and D. G. McDonald, Rev. Sci. Instr. **29** (1958) 530.

⁵⁴) A. S. D. Barrett, San Francisco, 1958.

⁵⁵) G. F. Wells, Rev. Sci. Instr. **29** (1958) 893.

⁵⁶) E. Thomas, Vide **11** (1956) 88.

⁵⁷) L. Holland, J. Sci. Instrum. **36** (1959) 105.

A number of experiments of an academic nature have been reported on the use of small ionisation pumps of low or very low speed; (see for example⁵⁸⁻⁶¹). At present efforts are being made to provide low ultimate pressures and large gas handling capacities using gettering frequently in conjunction with ionisation^{62,36,19}). The latter is an interesting experiment of pulsed gettering by molybdenum in conjunction with a diffusion pump.

Hall⁶⁴) has made a pump, which is now commercially available (Vac Ion); it has an

of speed ranging between 100 and 10 000 l/sec are under development⁶⁵) but it is uncertain as yet whether these speeds are maintained in the U.H.V. region. These pumps are normally used without traps but Prugne and Gavin⁶⁶) have studied the possibilities for a getter pump trapped with liquid hydrogen or liquid helium for attaining low pressures in relatively large volumes.

There are some serious disadvantages associated with ionic pumping. The speed for the noble gases is very small compared with that

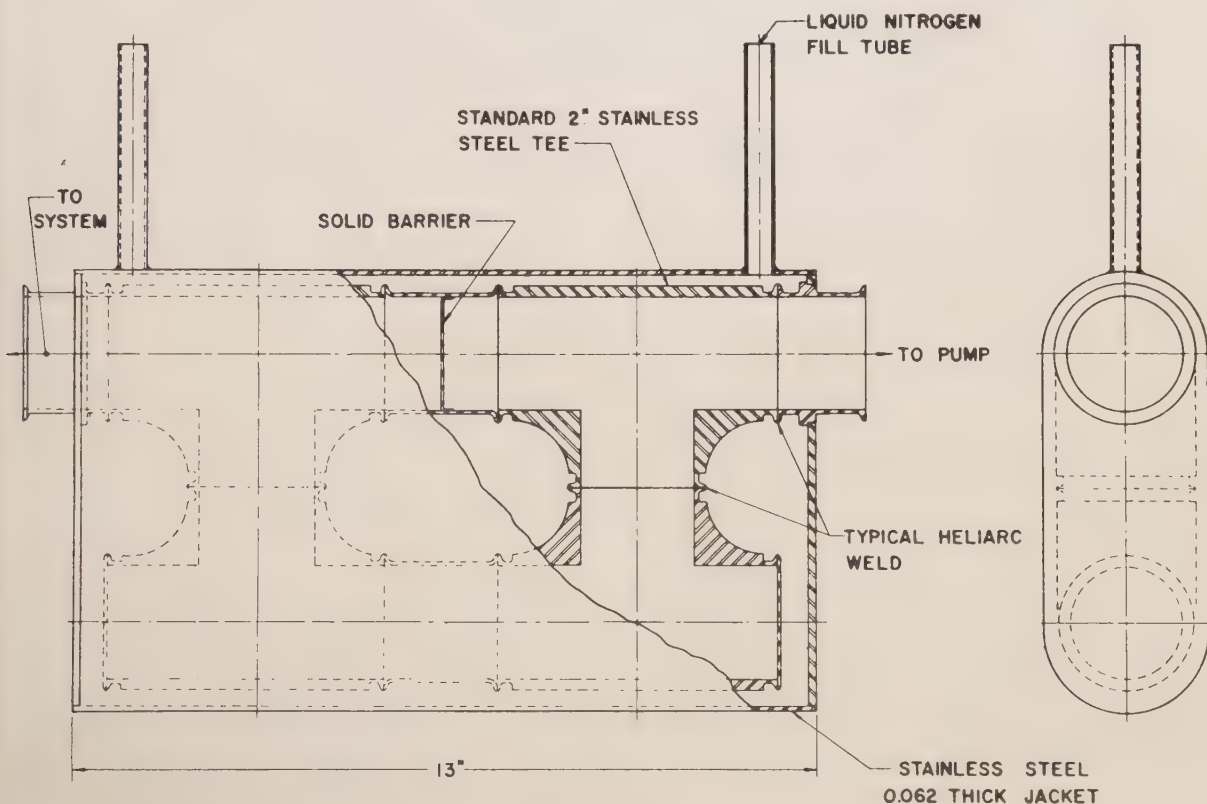


Fig. 8. Liquid nitrogen trap: Grove¹⁵). (Designed by D. Scag, Allis Chalmers Mfg. Co., U.S.A.)

ultimate pressure in the region of 2×10^{-10} torr and a speed of about 5 l/sec (air) at about 10^{-7} torr. The pump is like a large Penning type gauge with cathodes of titanium. Larger pumps

for the other permanent gases. Saturation effects have been observed, namely, a decrease in pumping speed as a function of the number of atoms pumped. Some aspects of this have been

⁵⁸) L. Varnerin and J. Carmichael, *J. Appl. Phys.* 26 (1955) 782.

⁵⁹) R. Bloomer and M. Haine, *Vacuum* 3 (1953) 128.

⁶⁰) C. Comsa and G. Musa, *J. Sci. Instrum.* 34 (1957) 291.

⁶¹) G. Reich and H. G. Nöller, *Vacuum Symposium Transactions* (Pergamon Press, London, 1958) p. 97.

⁶²) A. Schram, Namur, 1958.

⁶³) G. Reich, Namur, 1958.

⁶⁴) L. D. Hall, *Rev. Sci. Instr.* 29 (1958) 367.

⁶⁵) L. D. Hall, San Francisco, 1958, see also *Science* 128 (1958) 279; and R. L. Jepsen (Private communication).

⁶⁶) P. Prugne and P. Garin, Namur, 1958.

investigated by Carmichael and Trendlenburg⁶⁷); see also ⁵²).

Mercier and Benoist⁶⁸) have made calculations on molecular pumps which take into account the outgassing of the pump while in operation. Becker⁶⁹) has designed and made a molecular pump for which pressures of below 10^{-9} torr and a speed for air of about 140 l/sec are claimed. The ultimate pressure is limited by the outgassing of hydrogen from the pump which cannot be baked out. It should be noted that this type of pump works more effectively the greater the molecular weight of the gas being pumped. Thus for pumping hydrogen to a low base pressure it would be necessary to use a diffusion pump for backing.

The possibilities of cryogenic pumping are now being examined seriously as one of the methods of obtaining low ultimate pressures and exceptionally high speeds. The studies of Prugne and Gavin⁶⁶) have already been mentioned; a Ukrainian paper⁷⁰) refers to liquid hydrogen condensation pumps of two sizes for 13 000 and 4000 l/sec for the production of pressures of 10^{-8} – 10^{-9} torr. Ames *et al.*⁷¹) use liquid helium for the evacuation from atmospheric pressure to about 10^{-3} torr of a volume of 0.5 litre and then continue with the Vac Ion pump to get pressures below 10^{-9} torr. This they claim produces very "clean" vacua without the usual hydrocarbon contamination that is found even in ultra high vacuum systems. In this context see Reynolds⁷²) and Blears⁷³). Ullmann⁷⁴) has used a bakeable liquid helium cooled trap with activated charcoal for producing pressures of 10^{-10} torr or less for long periods. Serious consideration is

now being given to produce very high speed ($\approx 10^5$ l/sec) pumps using liquid helium.

7. Leak Detection

Alpert⁴) points out that for U.H.V. systems the detection of leaks by probing with acetone or trichlorethylene and using the Bayard-Alpert gauge is effective when the commercial helium leak detector is useless. This method he claims to be about a thousand times more sensitive ($\approx 10^{-13}$ torr l/sec) than the best commercial helium detectors when used in the conventional way (dynamically). Efforts are being made (e.g. Nerken⁷⁵) and Warnecke⁷⁶) to improve the leak detectors so that very small leaks may be found. Warnecke suggests about 10^{-15} torr l/sec may be found by accumulation in a reasonable time, in which case the detector may be effectively employed with large ultra high vacuum systems. If the sensitivity can be improved and this is technically possible (see for example in a slightly different context Reynolds⁷⁷) the major disadvantage of Alpert's method, that of temporarily clogging up the leak, will be overcome. The conventional helium leak detector is nevertheless a useful tool for checking all system components. If they pass this test there is a very high probability that they are good.

8. Residual Gases

To study the residual gases in an ultra high vacuum system various measurements have been carried out by the use of the omegatron and the more conventional mass spectrometer, (see for example refs.^{16,17,18,63}). These studies have been limited by the sensitivity of the instrument to a partial pressure determination of the particular component which is rather close to that of the total pressure (i.e. 10^{-9} torr; see for example Warnecke⁷⁶) and Wagener and Marth⁷⁷). The work of Reynolds⁷²) with a very sensitive mass spectrometer (detectable partial pressures $\approx 10^{-12}$ torr) showed how the system

⁶⁷) J. H. Carmichael and E. Trendlenburg, J. Appl. Phys. 29 (1958) 1570.

⁶⁸) C. Mercier and P. Benoist, J. Phys. Radium 17 (1956) 1A; 17 (1956) 182A.

⁶⁹) W. Becker, Namur, 1958; and Vakuum Technik 7 (1958) 149.

⁷⁰) B. Lazaryev, Ye. S. Bororyk and N. Tsyn, Ukrayin fiz. Zh. 2 (1957) 175.

⁷¹) I. Ames, R. Christensen and J. Teale, Rev. Sci. Instr. 27 (1958) 736.

⁷²) J. H. Reynolds, Rev. Sci. Instr. 27 (1956) 928.

⁷³) J. Blears, J. Sci. Instrum. Suppl. 1 (1951) 36.

⁷⁴) J. R. Ullmann, San Francisco, 1958.

⁷⁵) A. Nerken, Private communication.

⁷⁶) R. J. Warnecke Jr., Namur, 1958; and Vide 73 (1958) 25.

⁷⁷) G. S. Wagener and P. I. Marth, J. Appl. Phys. 28 (1957) 1027.

even after bake-out and pumping was sensitive to the previous history and treatment with other gases. With the further development of this work the choice of materials and their treatment may be better defined.

9. Conclusions

For a technique that is still in its infancy the following may be said. Pressures of below 10^{-9} torr may be produced, maintained and measured in fairly large all-metal systems ($\approx 1 \text{ m}^3$). It seems reasonable to believe that the same techniques may be applied to considerably larger systems without waiting for the developments that will be forthcoming in the next year or so.

The essential requirement of bake-out makes the choice of constructional materials considerably more limited than in the conventional vacuum system. The oil diffusion pump used in conjunction with liquid nitrogen traps is, at

present, the easiest and least uncertain approach to the question of what pumping unit to use. Demountable joints may be successfully made either with the gold ring seal or the step type flange although there appears to be an increasing bias to concentrate on the former. Fairly satisfactory moderately high conductance shut-off valves are likely to become available in the near future.

The economics of the U.H.V. system are not yet easy to assess but it is probable that the complete U.H.V. metal system is likely to cost about a factor of five times more than a conventional vacuum system of the same size and complexity.

Acknowledgement

The writer is particularly indebted to Drs. Carmichael, Grove and Trendlenburg for information and valuable discussions.

14. LES SOURCES D'IONS ET DE PLASMA

T. CONSOLI, P. HUBERT, R. LE QUINIO et D. VÉRON

Commissariat à l'Énergie Atomique, France

A short survey is given of existing ion and plasma sources with special reference to their application in thermonuclear work.

A. SOURCES D'IONS

1. Introduction

Le développement des machines à injection dans le domaine de la fusion contrôlée va donner une impulsion nouvelle à la recherche de sources d'ions à grande intensité. Cependant les publications de la Conférence sont généralement muettes sur le sujet et ce n'est que dans le rapport de J. S. Luce¹⁾ que l'on trouve mention de quelques nouvelles performances.

La production de faisceaux d'ions pour différents besoins est un problème ancien. Les développements ont porté surtout vers deux utilisations: injection dans les diverses sortes d'accélérateurs et séparation des isotopes. C'est pour la première qu'ont été construites les sources de plus grande intensité actuellement connue (ordre de grandeur: 1 A de protons).

2. Principes Généraux

Trois fonctions sont généralement distinguées dans une source:

1. La première est la fabrication des ions ou, plus généralement la fabrication des plasmas d'où seront tirés les ions—C'est celle qui donne le nom à la source suivant le mode de production: sources haute fréquence, à couplage par capacité ou self, sources à arc en champ magnétique, simple ou avec cathode réflectrice (P.I.G.)—Les sources les plus intenses semblent être toutes des sources à arc basse tension, à cathode chaude et fort champ magnétique longitudinal (5 à 7000 G). Il a été rapporté cependant que Harwell aurait construit une source de 1 A par haute fréquence.

2. La seconde fonction est l'extraction qui se fait toujours par un champ appliqué à l'extérieur. Deux facteurs la limitent. D'abord les propriétés du plasma d'où ne peuvent sortir en tout cas qu'un nombre d'ions tel que:

$$J_M = n_i e \left(\frac{k T_e}{M} \right)^{\frac{1}{2}}$$

où

J_M est la densité de courant maximum

n_i la densité d'ions à l'intérieur du plasma

M leur masse

k la constante de Boltzmann

e charge élémentaire

T_e la température électronique.

L'on voit la nécessité de fortes températures à l'intérieur du plasma, $T_e = 300\,000^\circ\text{C}$ dans le duoplasmatron de Von Ardenne²⁾, ce qui explique les champs magnétiques puissants utilisés pour le confinement.

La variation avec la masse des ions semble défavorable aux ions moléculaires, plus lourds. En fait, Lamb et Lofgren³⁾ n'ont trouvé qu'une différence de 10% entre deutérons et protons.

Le second facteur de limitation est la charge d'espace qui crée un champ opposé au champ appliqué. L'extraction maximum est alors de:

$$J_{Mco} = \frac{1}{9\pi} \sqrt{\frac{2e}{M}} \frac{V^{\frac{3}{2}}}{d^2}$$

¹⁾ J. S. Luce, Studies of intense gaz discharges A/Conf. 15 P/1790.

²⁾ Von Ardenne, Tabellen der Elektronenphysik Ionenphysik und Übermikroskopie p. 543 à 555.

³⁾ Lamb and Lofgren, Rev. Scient. Instr. 27 (1956) 907.

J_{Mec} = densité de courant maximum
 V = tension extérieure appliquée
 d = distance entre l'électrode d'extraction et la limite du plasma.

La plupart des auteurs ont vérifié cette formule, en particulier la loi en :

$$V^{\frac{3}{2}}$$

mais, là encore, Lamb et Lofgren³) n'ont trouvé qu'une loi en V .

De plus, on ne peut augmenter indéfiniment V et réduire d , à cause des claquages—Comme ceux-ci dépendent généralement du champ en :

$$\frac{V}{d}$$

on voit qu'il serait préférable d'avoir d le plus petit possible, même avec des tensions faibles.

Mais des considérations annexes, technologie ou bombardement des électrodes par les particules, limitent cette possibilité. Tenir, comme Von Ardenne, 60 kV sur 5 mm, semble tout à fait remarquable. Quant à la variation avec M , elle est encore défavorable aux ions lourds.

3. La troisième fonction est la focalisation qui a reçu des solutions variées : lentilles simples ou doubles, généralement de nature électrostatique, sauf dans le cas des sources à grande intensité où les lentilles magnétiques semblent préférées. Cette focalisation peut être très importante dans les machines à injection : il y a intérêt à avoir un faisceau le plus fin possible pour éviter l'injecteur, dans OGRA, par exemple.

Toutes ces fonctions ne sont d'ailleurs pas strictement indépendantes, la focalisation résultant beaucoup de la forme et de l'énergie du courant extrait, lequel est fonction de la nature du plasma. Par exemple, des oscillations de l'ordre du Mégahertz (le "Hash") sont souvent observées dans les décharges en champ magnétique. On les atténue beaucoup en faisant varier empiriquement ce dernier, mais elles influent toujours sur le faisceau sorti.

3. Réalisations

En dehors des Russes sur lesquels il y a peu de renseignements, trois centres semblent avoir développé les sources à grande intensité :

1. A. Livermore, Lamb et Lofgren³), dans le cadre du projet M.T.A. ont construit une source de protons d'environ 1 A—Mais la mise en veilleuse du projet lui-même a freiné le développement et il semble que des sources d'ions semblables soient utilisées maintenant dans des recherches de physique nucléaire, plus modestement à 100 mA, 100 kV.

2. A. Oak-Ridge, après l'élan initial dû aux calutrons (100 mA)⁴), le développement de sources pour cyclotron (200 mA)⁵), une source de peut être plusieurs ampères a été construite¹), sur le modèle des précédentes.

3. Mais, c'est surtout à Dresde, que Von Ardenne²) a le plus développé la physique des sources et qu'il est parvenu avec le duoplasmatron à des résultats particulièrement intéressants.

4. Ces sources sont toutes trois des arcs à basse tension (100 V environ) à cathode chaude et fort champ magnétique longitudinal. Mais l'extraction se fait perpendiculairement à celui-ci dans le cas d'Oak-Ridge et parallèlement dans les deux autres cas. De plus, une géométrie particulière de l'orifice d'extraction permet à Livermore et, de façon moins marquée, à Dresde, une diffusion préalable du plasma, facilitant la sortie des ions. Enfin, seulement à Dresde, une électrode "intermédiaire" permet une concentration du plasma devant l'extraction. Celle-ci et l'étude soignée des autres électrodes permet d'obtenir 500 mA de protons d'un trou de 1 mm².

En fait les sources américaines ont toutes deux des "grilles" à l'extraction et sont plus ainsi une juxtaposition de petites sources venues d'un plasma commun et disposées soit en cercle²) soit en colonne¹).

C'est ce qui explique leur faible densité de courant par rapport à celle du duoplasmatron et c'est sans doute pourquoi les américains eux-mêmes vont peut-être s'orienter vers cette nouvelle voie⁶).

⁴) Oak Ridge Tid 5218 et 5219.

⁵) Livingston and Jones, Rev. Scient. Instr. 25 (1954) 552.

⁶) Communication orale d'un membre d'Oak Ridge.

5. La source utilisée dans Félix⁷⁾ est une source pulsée à gaz occlus dans du titane, dérivée directement de l'un des canons à plasma de Bostick. On annonce une intensité de 400 mA. Ces performances doivent pouvoir être encore améliorées.

6. Enfin, dans OGRA, les Russes semblent employer une source peu différente de celle des calutrons d'Oak-Ridge⁸⁾.

7. Les dispositifs utilisés dans les magnétrons à ions constituent des sources d'ions particulièrement simples dans lesquelles la grande surface d'extraction permet d'obtenir aisément des courants de l'ordre de 1 ampère.

Il est à craindre que ces sources pêchent par un transfert de gaz excessif dans l'espace d'utilisation des ions.

8. Toutes les performances décrites dans la littérature sont généralement avantageuses. En fait, elles sont souvent difficilement reproductibles par suite de la modification et de l'usure rapide des matériaux. La vie d'une source de grande intensité en régime continu est généralement courte. Il est souvent suffisant de travailler en pulsé ce qui limite l'échauffement mais il est alors difficile de comparer les intensités obtenues avec des durées d'impulsion et des taux de répétition différents.

4. Problèmes Particuliers

La composition d'un faisceau issu d'un gaz unique est toujours complexe (H^+ , H_2^+ , H_3^+ , par exemple) et les expérimentateurs se sont toujours efforcés d'obtenir le rendement maximum pour une espèce de particules déterminée. En particulier, les sources précédentes donnaient de 75 à 95% de protons.

Cependant J. S. Luce¹⁾ affirme nettement qu'il est facile de modifier le rapport du nombre d'une espèce d'ions à une autre en changeant simplement la pression du gaz neutre dans la source. En fait, le problème est plus complexe puisque J. S. Luce lui-même donne des chiffres fonction de l'intensité de la décharge (source à arc) et cite l'influence de la température des parois.

En tout cas, ces conditions pour l'existence d'ions moléculaires (abaissements nécessaires de la pression et de l'intensité d'arc par exemple), apportent une limite supplémentaire à la production de faisceaux intenses, limite probablement plus importante que celles déjà considérées.

De plus, ceci n'apparaît vrai que dans une géométrie donnée puisque pour une source de faible intensité, Lorrain⁹⁾ a trouvé des résultats différents.

Dans l'emploi de machines à injection, d'autres difficultés apparaissent encore: l'énergie à communiquer aux particules (600 à 900 keV pour réduire l'échange des charges). Le problème est alors de disposer d'alimentation d'une telle puissance, avec les isollements et les stabilités de tension nécessaires; il est aussi, de focaliser les faisceaux intenses et de grande énergie obtenus.

C'est sans doute pourquoi, dans le DCX, les expériences décrites¹⁰⁾ font l'état d'un courant injecté de 200 μA et la prochaine étape semble n'être que de quelques milliampères.

5. Conclusion

En conclusion, on peut envisager de travailler avec les sources d'ions actuellement connues à condition de confirmer leurs possibilités en ions moléculaires. Leur développement ultérieur sera subordonné, en plus du progrès technologique (refroidissement des parois, matériaux nouveaux etc.) à une compréhension approfondie des phénomènes internes au plasma et au faisceau — sections efficaces des phénomènes élémentaires, compensation de charge d'espace etc...

Les sources pulsées pourront probablement avoir des performances supérieures si leur usage paraît suffisamment intéressant pour justifier un effort dans ce sens.

⁷⁾ Eklers, Gow, Ruby and Wilcox, *Rev. Scient. Instr.* **29** (1958) 614.

⁸⁾ Kourchatov, *Atomnaya Energia* **5** (1958) 105.

⁹⁾ P. Lorrain, *Canadian journal of research* **25** (1947) sec A, 6.

¹⁰⁾ Barnett, Bell, Luce, Shipley and Simon, *A/Conf.* **15/P/344**.

B. LES SOURCES DE PLASMA UTILISEES DANS LES RECHERCHES SUR LES PLASMAS THERMONUCLEAIRES

1. Introduction

Dans les expériences du domaine de la fusion contrôlée, le plasma que l'on essaie de chauffer peut être obtenu soit par accumulation d'ions et d'électrons injectés séparément, soit par une décharge ionisant le gaz dans l'enceinte principale, soit par injection de plasma produit dans une décharge extérieure.

La première méthode n'est pas envisagée ici. La seconde peut conduire à des développements très vastes puisque toute décharge dans un gaz constitue une source de plasma. Toutefois, son intérêt semble limité à des cas où son fonctionnement est intimement lié au principe de la machine utilisatrice, de sorte que sa discussion ne sera pas entreprise non plus. Enfin, la troisième méthode utilise des sources de plasma pulsées, d'invention récente, constituant une application des principes magnétohydrodynamiques et dont nous allons résumer le principe et les propriétés.

Ces sources comportent un mécanisme d'accélération communiquant au plasma une énergie cinétique pouvant être importante, de sorte qu'on les désigne souvent sous le nom de canons à plasma.

2. Canons à Plasma avec Electrodes

Les principaux travaux sur ce sujet sont dus à Bostick¹⁾, Artsimovitch²⁾, Morozov³⁾ et Finkelstein⁴⁾. Ils sont conduits aux types de sources suivantes:

SOURCE BOUTON

Elle consiste en deux fils de titane de 1 mm de diamètre, chargés d'hydrogène, placés à quelques millimètres l'un de l'autre dans un isolant. La décharge produite entre les bouts libres est soufflée par son propre champ magnétique.

SOURCE RAIL

Elle comporte deux électrodes parallèles, plongées dans un champ magnétique perpen-

diculaire à leur plan. La décharge est initiée à une extrémité par un arc jaillissant entre deux électrodes auxiliaires dont la nature autorise diverses variantes. L'accélération résulte de l'action mutuelle du courant dans l'arc et du champ magnétique total. La forme allongée des rails permet à la force de s'exercer plus longtemps que dans la source bouton. Bostick conseille de réaliser les rails avec les deux moitiés d'un tube cylindrique fendu suivant un plan méridien.

SOURCE COAXIALE

Une des électrodes est l'extrémité d'une tige placée suivant l'axe d'un cylindre; l'autre est, soit un cylindre, soit un empilement alterné de rondelles de titane chargées en deutérium et de rondelles isolantes. Cette source de plasma est l'élément principal de la source d'ions décrite dans la référence ⁵⁾.

SOURCE ROTATIVE

C'est une source coaxiale plongée dans un champ magnétique axial qui imprime au plasma un mouvement de rotation.

3. Canons à Plasma sans Electrodes

Un seul appareil de ce type a été décrit jusqu'à présent. Il s'agit du canon de J. Marshall⁶⁾ qui utilise un tube cylindrique allongé, le long duquel se succèdent les éléments suivants:

(a) une vanne mécanique ultra rapide qui introduit dans l'enceinte vide environ 1 cm³ NTP de gaz en un temps de l'ordre de 100 μ s.

(b) section d'ionisation et de préaccélération

¹⁾ W. H. Bostick, Phys. Rev. **104** (1956) 292; Phys. Rev. **105** (1957) 46; Phys. Rev. **106** (1957) 104; Conf. Genève (1958) P/367; Conf. on extremely high temperature, Air Force Cambridge Research Center (1958) p. 169.

²⁾ L. Artsimovitch *et al.*, Soviet JETP **33** (1957) 3.

³⁾ A. I. Morozov, Soviet JETP **5** (1957) 215.

⁴⁾ D. Finkelstein *et al.*, The Physics of Fluids (May-June 1958) p. 188.

⁵⁾ Eklers, Gow and Ruby, Rev. Scient. Instr. **29** (1958) 614.

⁶⁾ J. Marshall, Conf. Genève (1958) P/355.

utilisant la décharge très rapide d'un condensateur dans une bobine à spire unique;

(c) une section d'accélération où le plasma est poussé par un "piston magnétique", c'est-à-dire par une "bosse" de champ magnétique axial se propageant avec une vitesse convenable.

On peut imaginer qu'une version améliorée d'un tel canon pourrait comporter, après la valve, un bobinage analogue à celui préconisé par V. Josephson pour créer des ondes de choc (réf. 7)).

PERFORMANCES ET UTILISATION

Les sources type bouton sont celles dont les performances nous sont les mieux connues. C'est avec elles, en effet, que Bostick a démontré les curieuses propriétés des plasmoides. Ces propriétés consistent essentiellement en ce que:

(a) la bouffée de plasma projetée par le canon conserve une certaine individualité au lieu de se disperser. Cette cohésion s'explique par les effets magnétohydrodynamiques dûs aux courants circulant à l'intérieur du plasmode;

(b) un plasmode est capable de voyager perpendiculairement aux lignes de force d'un champ magnétique. Cette propriété s'explique par une polarisation, créant un champ électrique dirigé de telle façon que la vitesse de déplacement correspond à une dérive dans un système de champs électrique et magnétique croisés;

(c) les plasmoides interagissent fortement entre eux lorsqu'ils se rencontrent. Cet effet s'explique par l'interaction des champs propres à chaque système.

Les plasmoides contiennent de 10^{15} à 10^{17} particules. On annonce des vitesses dirigées atteignant 2×10^7 cm/sec†, ce qui correspond pour des deutons, à une énergie de 400 eV alors que l'énergie d'agitation thermique reste de l'ordre de 10 eV.

Il ne semble pas que les sources bouton aient été utilisées pour l'injection dans des machines particulières. Par contre, les sources coaxiales ont reçu de multiples applications dans les expériences de Post à Livermore (réf. 8)). En

dehors de la référence 5), nous n'avons pas trouvé de document exposant les performances de ces sources. Par contre, divers renseignements se trouvent dans les publications relatives aux Pyrotrons.

La machine "Cucumber" par exemple, utilise une source injectant les particules dans l'axe avec une densité suffisante pour que les collisions assurent la capture.

La machine "3 stage Toy-top" utilise une injection transversale avec trois sources convergeant au centre. La capture est alors assurée par l'interaction des jets de plasma.

La machine "Saturne" utilise le même procédé, avec une centaine de sources.

On constate donc que le plasma fourni par les sources coaxiales est assez dense (10^{16} à 10^{18} particules par impulsion), qu'il est capable de traverser un champ magnétique et d'interagir avec un jet venant en sens inverse. Par contre, on ignore sa vitesse qui est probablement assez faible puisque Post estime, qu'avant compression, le plasma obtenu a une température de 5 eV seulement††.

Jusqu'à présent, les canons à rails semblent avoir été étudiés uniquement pour eux-mêmes. Les résultats annoncés sont pratiquement identiques à ceux des sources boutons. Il s'agit probablement d'une étape provisoire puisque leur principe devrait assurer une plus grande efficacité.

Tous les canons à électrodes présentent le défaut de fournir un plasma fortement pollué. En particulier, les canons à titane donnent une forte proportion d'ions titane. Cet inconvénient n'est pas forcément rédhibitoire puisque des mécanismes ultérieurs de purification peuvent jouer. On peut craindre que les performances des canons à rails soient limitées par l'évaporation des électrodes.

C'est la raison pour laquelle les canons sans

† Dans les sources à rails, il serait même possible, d'après Bostick, d'atteindre une vitesse de 10^8 cm/sec (Réf. 4)).

†† Dans la source coaxiale exposée à Genève lors de la 2ème Conférence Atomique Internationale, cette température, d'après une information verbale due à Post, dépasserait 50 eV.

7) V. Josephson, J. Appl. Phys. T 29 (1958) 30.

8) R. F. Post, Conf. Genève (1958) P/377.

électrodes sont assez séduisants. Toutefois, les performances du canon de Marshall sont encore assez modestes puisqu'il annonce une vitesse de 5×10^6 cm/s avec environ 10^{19} particules accélérées. On doit alors souligner qu'il s'agit d'un état intermédiaire de la machine puisque l'auteur admet qu'elle pourrait donner les mêmes résultats sans piston magnétique.

Le seul usage annoncé du canon de Marshall est l'injection dans IXION, fonction pour laquelle la vitesse atteinte avait certainement peu d'importance.

4. Conclusions

Les propriétés particulières des jets de plasma sont extrêmement séduisantes pour permettre une injection dans une configuration magnétique.

Dés résultats supérieurs à ceux qui ont été obtenus jusqu'à présent pourraient être espérés si l'on savait obtenir, dès l'injection, un plasma

propre ayant une température ionique de quelques keV et une température électronique quelque peu inférieure.

Il n'est pas déraisonnable d'envisager que cet objectif pourrait être atteint par la collision de jets accélérés jusqu'à 10^8 cm/s. Ce n'est toutefois pas une certitude car les mécanismes de collision ne sont pas suffisamment connus pour hasarder une prédiction sur le sort de l'énergie cinétique dirigée au cours d'une telle collision.

Le canon le plus prometteur à ce point de vue est probablement le canon de Marshall, bien qu'il soit à craindre que son principe rende difficile et peut-être impossible l'injection dans un champ transversal.

L'intérêt du canon à plasma accéléré par rapport à un accélérateur de particules est évident lorsque l'on considère que l'injection de 10^{19} particules équivaut à un courant de 1 ampère pendant plus d'une seconde.

15. SOME REMARKS ON ENERGY STORAGE

D. TH. J. TER HORST

Short Circuit Laboratory KEMA, Arnhem, Netherlands

A short survey is given on storage of energy in capacitor banks, magnetic coils, rotating machinery and batteries.

The costs of storage of energy and the time necessary to release the energy to a gas discharge are compared.

1. Introduction

In order to store energy in such a way that the energy can be converted instantaneously into electrical energy to supply a gas discharge various methods are possible including storage in capacitor banks, storage in magnetic coils, storage as kinetic energy in rotating machinery and storage in batteries.

Each of the ways mentioned may have its features, each of them are well-known for a long time already.

In this note some attention will be paid to the cost of energy storage. An effort will be made to compare the cost of energy storage per joule if either a bank of capacitors, a coil, rotating machinery or a battery is used.

2. Energy Storage as Electrostatic Energy in Capacitor Banks

The energy content W of a capacitor is given in first approximation by the expression

$$W = 1.11 \frac{10^{-12}}{8\pi} E^2 \epsilon O d. \quad (1)$$

E is the voltage gradient in the dielectric material, expressed in V/cm; ϵ the dielectric constant, O the total area of metal sheet and d the thickness of one dielectric layer; O is expressed in cm², d in cm, W in joule.

The value of the voltage gradient E in the dielectric layer of a capacitor designed for energy storage can be seen as one of the yardsticks for the quality. A low value for E represents a high quality capacitor. The voltage gradient is usually of the order of 70 V/micron.

However, the deviation in this value for capacitors of different quality can be of the order of a factor 2.

The value of ϵ depends on the material of the dielectric layer and on the impregnation, but in capacitors of the size and voltage involved in these remarks a value $\epsilon \approx 3$ may represent some average value.

With $E = 700$ kV/cm and $\epsilon \approx 3$, W can be found from (1) as:

$$W = 6.47 \times 10^{-2} O d \text{ joule}. \quad (1')$$

The cost of a capacitor is mainly determined by its weight. The weight G of the active material is

$$G = O d s$$

s being the average spec. weight of the material.

Taking into account a surplus of about 30% for tank, impregnation, bushings a.s.o. one finds

$$G = 1.3 O d s \text{ gram}. \quad (2)$$

The cost per joule are therefore with (1') and (2)

$$\frac{G_p}{W} = 20.1 sp,$$

where p is the price per gram active material.

An average specific weight of 2 and a price per kg of Dutch fl. 15.—† for the bare capacitor unit might be reasonable and one finds

$$G/W = 20.1 \times 2 \times 1.5 \times 10^{-2} = \text{Dutch fl. } 0.60/\text{joule}.$$

This price per joule can vary considerably when the voltage stress in the dielectric layer is altered. Moreover this price covers the cost

† 1 Dutch fl. $\approx$ $\frac{1}{4}$ U.S. \$.

of the capacitor units only. For a stored energy of the order of 10^5 joule the cost per joule of the equipment necessary for the erection of the complete bank including the switching might be of the same order.

A high value for the voltage stress gives rise to ionization phenomena in the dielectric layer particularly at higher frequencies and small damping. If therefore a high value for the ringing frequency and a high number of subsequent discharges of the bank is required the voltage gradient should be kept to a reasonable value.

A curve showing the price of a capacitor as a function of the voltage stress in the dielectric

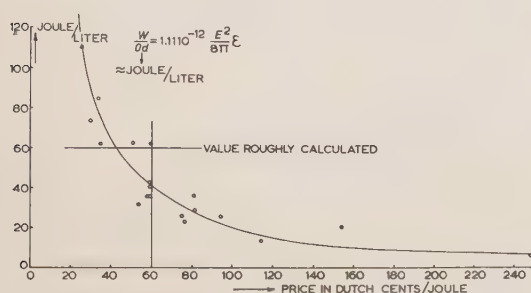


Fig. 1

layer is shown in fig. 1. In this figure the voltage stress is expressed as the stored energy per unit volume. This quantity is proportional to the square of the voltage stress.

The feature of energy storage in capacitor banks has the advantage that a large percentage of the energy can be delivered to a gas discharge within a very short time. The inductance of capacitors can be made very small, even so small that the inductance of the external circuit connecting the capacitor or the capacitor bank with the discharge should be designed carefully in order to minimize its influence on the value and frequency of the discharge current. A ringing frequency of the bare capacitor unit of the size of say 1500 joule/unit can be made as high as 400 kc/sec. It should be possible, at least in principle, to increase this value considerably. This could be of some advantage for research in plasma physics.

3. Energy Storage as Magnetic Energy in Coils

Energy storage as magnetic energy in coils is considered by Walker and Early¹⁾. These investigators studied the storage of energy in the magnetic field of a large air-cored very low impedance transformer.

It is stated that the energy can be delivered in less than 5 msec to a non-inductive load.

The time involved in this way of energy supply to a gas discharge is considerably greater than can be obtained with energy supply stored in capacitor banks.

As far as costs are concerned this method of energy storage might be of importance in case large quantities of energy are required.

Taking into account the cost of the charging equipment Walker and Early¹⁾ showed that in case of using a capacitor bank the price per joule is a constant as a function of the quantity of energy required. However, the price per joule is decreasing as a function of total energy required in case of energy storage as magnetic energy in coils.

It is rather a method of speculation to predict from the formulae given by Walker and Early, at which quantity of energy the price per joule produced by means of coils is smaller than the price per joule in case the energy is produced by means of capacitors banks.

A rough calculation based on the present cost of material and equipment showed this could be the case at a quantity of energy beyond a megajoule. However, this rough estimation might vary strongly with the cost of material and labour in each country.

The switching of the energy in case of storage as magnetic energy in coils might be quite more difficult than in the case of storage in capacitor banks¹⁾.

4. Storage as Kinetic Energy of Rotating Machinery

Regarding the energy storage as kinetic

¹⁾ R. C. Walker and H. C. Early, Rev. Scient. Instr. **29** (1958) 1020; Commun. Electronics **31** (1957) 320.

energy in rotating machinery some experience is gained in laboratories engaged in the testing of the rupturing capacity of electrical switch-gear under short circuit conditions. There are several of these laboratories particularly in the United Kingdom, France and the German Federal Republic.

The majority of the machines used for this purpose are turbo-alternators of a frame size for a normal continuous rating of 50–100 MW. Most of them are high speed, two pole machines with 3000 RPM, delivering a current with a frequency of 50 c/s.

The moment of inertia of such a high speed machine of a frame size for a rated power of 100 MW is of the order of 400 kgmsec^{-2} and the kinetic energy stored in the rotating part is about 2×10^8 joule.

In short circuit laboratories it is normal practice to allow for these machines under short circuit conditions a reduction in speed of the order of 10% during 0.1 second. Thus during one loop of 50 c/s 2% of the kinetic energy is converted into electrical energy, being 2×10^6 joule. With two machines running in parallel an amount of energy of the order of 4×10^6 joule would be available with a rise time of 5–10 milliseconds. From an engineering point of view one could allow for a considerable greater reduction in speed. Assuming the internal machine selfinduction being equal to the external selfinductance and the internal machine resistance equal to 10% of the internal selfinductance an amount of energy of $\approx 10^7$ joule/machine could be released with a rise time of about 10 msec.

The costs of such a machine, including driving motor, starter equipment, lubrication, air cooling equipment, an exciter and bed-plate but exclusive building and foundation, may amount to Dutch fl. 1.5×10^6 and the cost of energy would be of the order of Dutch fl. 0.75/joule. It should be noted that the energy obtained in this way is only available with a frequency for the current of the order of 50 c/s. The order of magnitude of current could amount to 200 kA peak.

5. Storage of Energy as Chemical Energy in Batteries

Batteries of the size as used in submarines can deliver large currents during short circuit. The short circuit laboratory of KEMA at Arnhem had an opportunity to carry out some short circuit tests on an 8 cell section of such a submarine battery of 2 V/cell. All 8 cells were connected in series. The total energy content was 4.27×10^8 joules.

During short circuit a steady current of 45 kA was obtained. The time constant of rate of rise of current was approximately 6 milliseconds. This value might be mainly due to the external copper connections and the circuit breakers one of which had to make the short circuit and the other to clear the circuit afterwards.

The external voltage drop during short circuit was of the order of 15%. The same test could be repeated several times. The duration of each short circuit was about 0.5 second. The reduction in current in subsequent tests was negligible. The repetition time, however, was rather long, of the order of some 20 minutes. However, short circuiting the cells permanently the decrement of the current was rather slow.

During 10 minutes the current decreased to about 20 kiloampères. It should be possible to shorten the repetition time considerably, without a marked reduction of current in subsequent tests.

This way of energy storage results in a limited value for the rate of rise of current, similar as mentioned in section 4.

As the raw material for making such batteries is relatively cheap, the price per joule stored energy might be as low as Dutch fl. 0.05.

6. Conclusions

Storage of energy in a capacitor bank as magnetic energy in coils and as kinetic energy in rotating machinery seems to be of the same order of cost. Storage in batteries is much cheaper.

As far as the energy stored in capacitor banks

is concerned, this method allows for fast rising discharge current. The rate of rise of discharge current produced by magnetic energy stored coils is lower. The rate of rise of discharge current produced by rotating machinery is usually of the order of some milliseconds,

roughly equal to the rate of rise of current produced by batteries.

It should be remarked that batteries need a careful designed protection equipment in order to safeguard not only the batteries themselves, but also the personnel.

16. HIGH MAGNETIC FIELDS

SETH BERGLUND

Institute of Physics, Uppsala, Sweden

A brief introduction is given to the literature concerning design and construction of coils producing high magnetic fields (i.e. up to 100 000 gauss and more)

1. Introduction

The high magnetic fields here considered for plasma discharge devices are those produced by coils outside the plasma itself, plasma current fields are not discussed. No division of fields in high and weak ones is tried, but as the future need for really high fields—100 000 gauss and more—seems fairly obvious and the question of mechanical strength in coil constructions then becomes of utmost importance, a brief introduction to the literature in this matter is given. Furthermore there is a short survey of devices and constructions of interest in connection with the above, presented in the fusion papers of the Geneva conference. At last design details of coils for a toroidal set-up at the Physical Institute of Uppsala, Sweden, are given.

2. DC Magnetic Fields

The magnetic fields required for high temperature plasma work are not always of really high field strength but instead of fairly large volume. The production of such fields seems in the first hand to be a matter of electrical power requirements and cooling, for which reason the coils should give as much useful magnetic field as possible for a given power dissipation.

J. D. Cockroft¹⁾ has given a useful relationship between field strength and d.c. power dissipation in a solenoid

$$B = \mu_0 \left(\frac{W\lambda}{a\rho} \right)^{\frac{1}{2}} G,$$

† Some data of used magnetic fields are given in the following 1958 Geneva papers: 345 (Kolb), 378 (Mirror machines), 356 (Scylla), 344 (DCX), 2298 (OGRA), 360, 363, and others (Stellarator).

where W is the power dissipation, λ the fraction of conductor with resistivity ρ in a cross section of a coil, A the inside radius, and G an efficiency factor determined by the geometry of the coil. Numerical values of G are given by Cockroft and also by Bitter²⁾. The latter has also given the problem of water cooling some consideration.

Typical examples of coils of the above mentioned sort are those for producing stabilising B_z -fields, where the field strengths normally do not far exceed 1000 gauss.

3. Mechanical Stresses

In certain cases of reported fusion work, however, fields† as high as 100 000 gauss or more have come in use, giving the question of mechanical strength of the coil material the utmost importance. Since most devices, requiring such high fields are of the pulsed type, the availability of a large energy source is important, whereas the question of cooling is of a more secondary nature.

The normal cross section of coil windings is rectangular and one can distinguish between



Fig. 1. Different coil windings. From left to right the bobbin, the spiral, the helix and the single turn.

four different simple arrangements of windings, fig. 1.

Under the influence of their own magnetic

¹⁾ J. D. Cockroft, Trans. Roy. Soc. (London) 227 (1928) 317.

²⁾ F. Bitter, Rev. Sci. Instr. 7 (1936) 482.

field such coils tend to expand radially and to be compressed in the axial direction.

The radial explosive force is³⁾

$$F_r = \frac{1}{2} I^2 \frac{\partial L}{\partial r},$$

and the axial contractive is

$$F_z = \frac{1}{2} I^2 \frac{\partial L}{\partial z}.$$

Furthermore

$$\frac{1}{2} I^2 L = \frac{1}{2\mu_0} \int B^2 dv,$$

from which it follows that the coil pressure increases as the square of the field strength and is independent of the size of the coil.

Associated with high currents in coils there is thus a high energy density, accompanied by intense pressures, and it should especially be noted that the outward pressure from the axial magnetic field of a coil is mathematically identical with the magnetic energy density $P_m = B^2/2\mu_0$.

Fig. 2 gives a plot of this pressure, expressed in kp/cm^2 versus field strength, given in gauss.

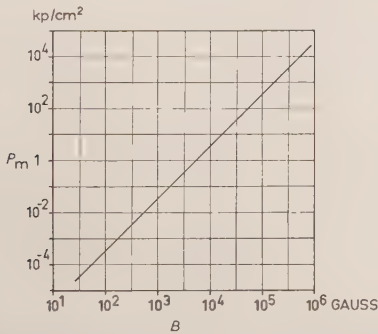


Fig. 2. The outward pressure from the axial magnetic field of a coil versus field strength.

Calculations of pressure distribution in finite coils of the bobbin type, the type of most interest in this connection, have been given by J. D. Cockroft⁴⁾. The radial outward pressures are cumulative, being zero on the inside surface of the coil and generally reaching a maximum in the interior. The basic design problem should be to concentrate the magnetic energy in the experimental volume and try to make the magnetic forces relatively isotropic. In a

design of a wire-wound coil reinforced peripherally by a steel band, Kapitza⁴⁾ has given the winding an hour-glass inside contour, so as to conform with the surfaces of constant pressure. Designs by Bitter⁵⁾ and by Foner and Kolm⁶⁾ also follow these lines. The maximal pressure anywhere inside a multilayer coil is somewhat smaller than the maximum P_m , but a lot of magnetic energy is still wasted inside the windings. The stored energy of the magnetic field is of course an irreducible minimum loss of electrical energy per experiment, though the total energy requirement must in most cases be expected to be much larger, owing to the generation of magnetic energy in the source and Ohmic heat in leads and in the coil.

As the maximal tensile stress on the coil material approaches $B^2/2\mu_0$, it is interesting to equate the strength of metals that might be used with the above expression. Furth, Levine and Waniek⁷⁾ find that the strength of hard copper corresponds to a field of up to 300 000 gauss.

At field strengths so great that the tensile strength of the metal is surpassed, the metal behaviour becomes essentially fluid, giving complications such as the growth of ridges or folds on the coil surfaces on account of instabilities in field-fluid interfaces.

The fact that the metal becomes fluid indicates that the magnetic energy density is comparable with the binding energy density of the lattice; in such cases it cannot either be much below the energy density required to melt the metal. The above mentioned authors compare the melting points of metals with field strengths and so find for hard copper, that approximately 500 000 gauss gives an energy density large enough for melting.

It is found that the Joule heat density diffusing

³⁾ H. P. Furth and R. W. Waniek, *Rev. Sci. Instr.* **27** (1956) 195.

⁴⁾ P. Kapitza, *Proc. Roy. Soc. (London)* **A 115** (1927) 658.

⁵⁾ F. Bitter, *Rev. Sci. Instr.* **8** (1937) 318.

⁶⁾ S. Foner and H. H. Kolm, *Rev. Sci. Instr.* **28** (1957) 799.

⁷⁾ H. P. Furth, M. A. Levine and R. W. Waniek, *Rev. Sci. Instr.* **28** (1957) 949.

into a coil must be roughly equivalent to the magnetic energy density, so there is an upper limit for magnetic density in coil materials set by their melting points. These limitations are, however, not so severe for multilayer coils as for single turn coils, especially as the Joule heating associated with the magnetic field can be distributed in multilayer coils so as to avoid high local temperatures.

4. Force-free Coils

From the above it is clear that heating gives a higher limit of field strength than mechanical forces for the kinds of coils here discussed. Lüst and Schlüter have suggested the possibility on an astronomical scale that magnetic fields might maintain their form through a balance of purely magnetic forces, and so are "force-free" in the sense that they do not tend to alter their shape. Very interesting results have also been attained in the search for force-free configuration of coil windings^{7,8}).

To give a force-free configuration the current density vector $\mathbf{i}$ and the field vector $\mathbf{B}$ must evidently agree in every point. This is not possible in practice and even when a mathematically force-free solution has been found, its physical approximation is limited because of the finite size of conductors.

A simple cylindrical force-free winding might be made up of several layers, with the pitch of the windings changing gradually from a direction nearly along the axis to one which spirals almost perpendicularly to the axis. The transition from the straight internal field to the circular external field is done in small steps, with the result that the pressure on each layer of the winding is comparatively low. The pressures diminish linearly with the ratio of individual conductor to coil dimension, and thus high voltages rather than high currents are required for good "force-free" coils.

A force-free configuration requires a lot of energy for its realization as the coil region is filled with magnetic field, so that there is a smooth continuum of magnetic pressure instead of material pressure, a fact that must of

course be taken into consideration when choosing between different coil designs.

At last it must be emphasized that it does not seem possible in practice to realize totally force-free coils. Parker⁹) has shown that the net inward force, which must be exerted on a coil system always is of the order of

$$F = L^2 \frac{\langle B^2 \rangle}{2\mu_0}$$

independent of the conductor configuration, L being the scale of the current-carrying elements. An evident gain is, however, that the force can be spread from a small surface over to a large surface.

5. The 1958 Geneva Papers

The purely technical information on the various devices presented in the Geneva papers is in most cases very scarce. The following survey will thus contain what has been constructed in strong magnetic fields rather than how these have been constructed. It is of course by no means to be considered as complete.

As far as the pinch type devices are concerned the magnetic fields of interest here are the stabilizing ones. These fields are all well below 10^4 gauss and thus the mechanical forces associated with them are well within the range of conventional techniques (cf. fig. 2). The problems are here mainly those of power supplies, cooling and—in pulsed devices—switching, which are not to be dealt with in this text.

In devices of the Stellarator type, coils have been used, producing fields in the range 30 000–40 000 gauss in fairly large volumes working on a pulsed basis with periods up to half a second. The principle of construction is to imbed the current-carrying, watercooled copper turns in their insulation of moulded polyethylene, teflon or equivalent material, while the mechanical forces are taken up by a close-fitting steel frame-work.

Similar coils are probably used in the continuously operated DCX device with its field

⁸) Furth, Levine and Waniek, *Scientific American* 198 (February 1958) 28.

⁹) E. N. Parker, *Phys. Rev.* 109 (1958) 1440.

strengths of up to 20 000 gauss. The 2.7 MW power consumption of these coils gives an idea of the power supply and cooling necessary when maintaining such field strengths even in moderate volumes. Analogous problems are no doubt present at the big Russian OGRA device with a field of 5000 gauss in a tube of 1.4 m diameter and 12 m length.

The most powerful magnetic fields have been used in connection with the mirror machines and the associated shock and compression experiments. In the Livermore papers coils giving fields up to 2×10^5 gauss are reported. Kolb at Naval Research Lab., Washington, reports successful use of coils with maximum fields of 5×10^5 gauss, in 20 cm long tubes with 3.6 cm external diameter, designed to withstand magnetic pressures corresponding to 7×10^5 gauss (i.e. 20 000 kg/cm²). The principles of construction of these coils are similar to those described in the paper by Furth, Levine and Waniek⁷⁾ i.e. a single turn or several single turns connected in parallel.

Finally it can be mentioned that Little and Thomson¹⁰⁾ at Los Alamos have described work with a two turn coil, 12 cm in giving diameter up to 1.2×10^5 gauss which at last broke down due to mechanical exhaustion.

6. Coil Design Uppsala

A fairly large toroidal installation now in use at the Institute of Physics, Uppsala, Sweden, with a capacitor bank storing up to 10^6 joule,

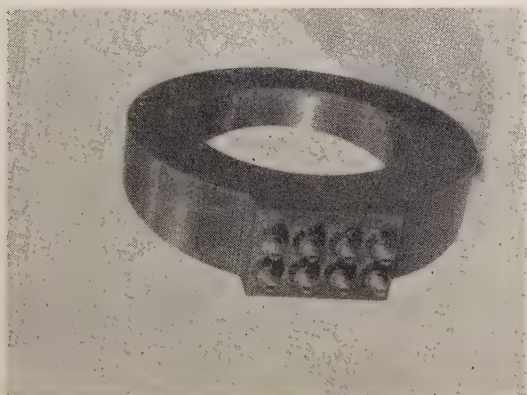


Fig. 3. Coil for producing B_z -field. Inner diameter 28 cm, outer diameter 43 cm; see text for further details.

is equipped with magnetic coils that might be of interest in this connection. As the apparatus is of the pulsed type with a slow repetition rate, no cooling except for conduction through the apparatus body is needed, but the mechanical

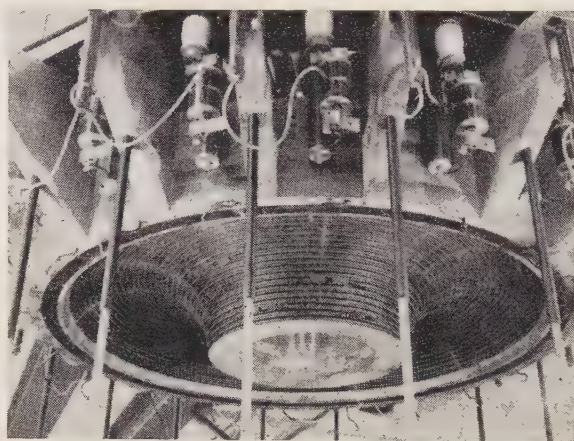


Fig. 4. The upper part of the two wooden blocks, which hold the B_θ -field coils in place and give mechanical strength to the assembly. The diameter of the conductors varies from 59 cm at the center to 150 cm at the outside. The two blocks are identical in shape.

forces are large and have constituted the main design problems.

There are really two different kinds of coils giving magnetic fields perpendicular to each other. First there are 40 removable coils for a magnetic field along the axis of the torus body, a B_z -field (see fig. 3). The coil bodies contain 4 separate conductor windings of the spiral type with 6 turns in each. The windings are made from flat strips of hard copper and are assembled by casting in epoxy resin reinforced with quartz powder. As can be seen from the picture the coils are given a wedgelike shape so as to fit together around the torus. They have an outer diameter of 43 cm.

The coils are designed for 15 kV and 30 kA. Tests on loose coils with the separate windings all in parallel have shown that the structure can stand several periods of 50 Hz current up to more than 100 kA, in which case the mechanical pressure is much greater than the maximum attained in the apparatus.

¹⁰⁾ E. M. Little and D. B. Thomson, Magnetic compression of shock preheated plasma, LA-2202.

The second set of coils gives a field, here called a B_θ -field, which is perpendicular to the first one. These coils consist of 80 single turns of thick insulated copper cable, which are pressed down into turned grooves in the two wooden blocks designed to take up the mechanical forces. Construction details are seen from the picture, fig. 4, showing the upper wooden block from underneath.

Obviously there are many possibilities of coupling the coils together, but normally one coil of the first type is connected with two coils

of the second type, both fields being pulsed simultaneously.

Interesting to note is that the coil arrangement resembles a "force-free" configuration, as the inner coils tend to expand and the outer ones to contract. This effect is partly relied on for mechanical strength of the assembly, though the entire system is tightly pressed between the close-fitting wooden blocks, which are held together by metal bolts.

The ASEA company has designed and constructed the entire coil assembly.

AUTHOR INDEX

- AMIOU, PIERRE, The selection of stainless steels for use in liquid hydrogen bubble chambers (Letter to the Editor) 118
- BÄCKSTRÖM, GUNNAR, Experimental methods for the study of neutron capture gamma rays (survey article) 5
- BALDINGER, E., P. SANTSCHI and P. WEHRLI, High speed transistorized scale-of-two (Letter to the Editor) 117
- BERGLUND, SETH, High magnetic fields 386
- BERLOWITSCH, E. J., *see* BONITZ
- BICKERTON, R. J., Brief review of the toroidal stabilised pinch 273
- BIELEIN, H., K. GÜTHNER, H. VON ISSENDORFF und H. WEGENER, Eine Apparatur zur Messung der "longitudinalen Elektronenpolarisation durch Mott-Streuung (see also Erratum p. 244) 79
- BIERMANN, L., Remarks on the stellarator-scheme 279
- BINEAU, M., T. CONSOLI, P. HUBERT, F. PRÉVOT, P. RICATEAU et A. SAMAIN, Les machines à miroirs magnétiques. Résumé de l'état de la question et esquisse d'un programme de recherches . . . 282
- BINEAU, M., T. CONSOLI, CH. MAISONNIER, F. PRÉVOT et P. RICATEAU, Les machines à miroirs magnétiques 290
- BONITZ, M. und E. J. BERLOWITSCH, Bemerkungen zu einem schnellen 20-Kanal-Impulshöhenanalysator 133
- BRAAMS, C. M., W. J. SCHRADER and C. J. TERLOUW, Confinement of plasma by radio-frequency electromagnetic fields 327
- BRAAMS, C. M., *see* RIETJENS
- BROOKS, F. D., A scintillation counter with neutron and gamma-ray discriminators 151
- COLLEY, D. C., J. B. KINSON and L. RIDDIFORD, 9-inch liquid hydrogen bubble chamber in a pulsed magnetic field 26
- COLLINS, L. E. and A. C. RIVIERE, A negative hydrogen ion injector for a tandem electrostatic accelerator 121
- CONSOLI, T., P. HUBERT, R. LE QUINIO et D. VÉRON, Les sources d'ions et de plasma 376
- CONSOLI, T., *see* BINEAU
- DUBBELDAM, P. S., C. C. JONKER and F. J. HEEMSKERK, Measurement of the polarization of D-D neutrons by a solenoid 234
- EKSPONG, A. G. and B. E. RONNE, A microscope for the scanning of nuclear research emulsions . . . 129
- FACCHINI, U., E. GATTI and F. PELLEGRINI, A new method for track angle measurement in a gridded ionization chamber 221
- FOLKIERSKI, A., Report on high-speed photography 346
- GATTI, E. and V. SVELTO, Theory of time resolution in scintillation counters 189
- GATTI, E., *see* FACCHINI
- GERHOLM, T. R. and B.-G. PETTERSSON, Instrument for electron-gamma directional correlation experiments with magnetic momentum selection . . 107
- GRUHLE, WOLFGANG, Impuls-Zeitformer für schnelle Koinzidenzstufen 112
- GÜTHNER, K., *see* BIELEIN
- HEEMSKERK, F. J., *see* DUBBELDAM
- HELSELBERG JENSEN, T., *see* WANDEL
- HUBERT, P., *see* BINEAU
- HUBERT, P., *see* CONSOLI
- HUO, M. and G. W. HUTCHINSON, An energy sensitive Čerenkov detector with improved optical focusing 30
- HUTCHINSON, G. W., W. H. WELLS and D. H. WHITE, Fast scaling circuits 228
- HUTCHINSON, G. W., *see* HUO
- JONKER, C. C., *see* DUBBELDAM
- JORDAN, H. L., H. KEVER and K. SCHINDLER, Notes on fast magnetic compression of plasmas 322
- KASHY, E., R. R. PERRY and J. R. RISSER, A method for the preparation of very thin foils 167
- KELLER, R., Le mouvement anharmonique des ions dans un cyclotron et le diagramme de phase . . 181
- KEVER, H., *see* JORDAN
- KINSON, J. B., *see* COLLEY
- KLEIN, A. G., *see* PRYOR
- KOCKUM, J. and N. STARFELT, The response to high-energy gamma rays of a NaI(Tl) scintillation spectrometer 171
- KOFOED-HANSEN, O., *see* WANDEL
- KOSTIN, MORTON D., Gas scattering in a strong focusing synchrotron 99
- LATHAM, R. and J. A. NATION, Report on linear pinch devices 261
- LE QUINIO, R., *see* CONSOLI
- LINHART, J. G. and A. SCHOCH, Relativistic electron beam devices for fusion 332
- LÜST, R., Notes on some theoretical investigations of the physics of high temperature plasmas 247
- MAISONNIER, CH., *see* BINEAU
- MALAMUD, E. and A. SILVERMAN, Magnetic measurements on the Cornell alternate gradient synchrotron 67
- MARSEGUERRA, M. and G. PAULI, Neutron transmission probability through a curved revolving slit . . 140

- MORTON, A. H. and W. I. B. SMITH, Improved cyclotron performance from control of initial ion motion 36
- MULLER, F. A., A fast scaling stage 115
- MÜLLER, W. und J. REMBSE, Überlegungen zum Bau eines Wendellinearbeschleunigers für Protonen 202
- MÜLLER-WARMUTH, W. und P. SERVOZ-GAVIN, Hochstabilisierende Protonenresonanz-Magnetfeldregelung mit elektronischem Integrator 90
- MUNDAY, G. L., Ultra high vacuum technology . . . 367
- NATION, J. A., *see* LATHAM
- NYSÄTER, HANS M., Accurate magnetic field and field gradient measuring instrument for dynamic low fields in a synchrotron magnet 44
- OGILVIE, K. W. and D. PAIX, A pulse-height storage circuit of high stability 164
- O'NEILL, W. J., E. SUNDAHL and H. OSTRANDER, An absorber system for scattering experiments. . . 50
- OSTRANDER, H., *see* O'NEILL
- PAIX, D., *see* OGILVIE
- PAULI, G., *see* MARSEGUERRA
- PELLEGRINI, F., *see* FACCHINI
- PERRY, R. R., *see* KASHY
- PETTERSSON, B.-G., *see* GERHOLM
- PORAT, D. I., *see* RAMAVATARAM
- PRÉVOT, F., *see* BINEAU
- PRYOR, A. W. and A. G. KLEIN, Statistical design basis for fast scaling systems 1
- RAMAVATARAM, K. and D. I. PORAT, Measurement of surface density of thin foils 239
- REMBSE, J., *see* MÜLLER
- RICATEAU, P., *see* BINEAU
- RIDDIFORD, L., *see* COLLEY
- RIETJENS, L. H. TH. and C. M. Braams, Detection of heavy particles in thermonuclear reaction experiments 363
- RISSE, J. R., *see* KASHY
- RIVIERE, A. C., *see* COLLINS
- RONNE, B. E., *see* EKSPOG
- SAMAIN, A., *see* BINEAU
- SANDSTAD, J., Regularizing action of scalers (Letter to the Editor) 243
- SANTSCHI, P., *see* BALDINGER
- SCHINDLER, K., *see* JORDAN
- SCHOCH, A., *see* LINHART
- SCHRADER, W. J., *see* BRAAMS
- SERVOZ-GAVIN, P., *see* MÜLLER-WARMUTH
- SHARP COOK, C., Calculation of the penetration of gamma rays through the edges of a $\frac{1}{2}$ -inch diameter lead collimator 103
- SILVERMAN, A., *see* MALAMUD
- SILVERSTEIN, EDWARD A., Calculation of the *G* factor for gas scattering experiments 53
- SMITH, W. I. B., *see* MORTON
- STARFELT, N., *see* KOCKUM
- SUNDAHL, E., *see* O'NEILL
- SUNDSTRÖM, TAGE and SVEN SVENNERSTEDT, An experimental study of the linear pinch stability . . . 213
- SVELTO, V., *see* GATTI
- SVENNERSTEDT, SVEN, *see* SUNDSTRÖM
- TER HORST, D. TH. J., Some remarks on energy storage 382
- TERLOUW, C. J., *see* BRAAMS
- VÉRON, D., *see* CONSOLI
- VON ISSENDORFF, H., *see* BIENLEIN
- WANDEL, C. F., T. HESSELBERG JENSEN and O. KOFOED-HANSEN, A compilation of some rates and cross sections of interest in controlled thermonuclear research 249
- WEGENER, H., *see* BIENLEIN
- WEHRLI, P., *see* BALDINGER
- WELLS, W. H., *see* HUTCHINSON
- WHITE, D. H., *see* HUTCHINSON
- WULF, H., Plasma diagnostics by spectroscopical means 352

BOOK REVIEWS

- KISTEMAKER, J., J. BIGELEISEN and A. O. C. NIER (Editors), Proceedings of the International Symposium on Isotope Separation (G. ANDERSSON) 52
- DIELS, K. and R. JAECKEL, Leybold Vakuum-Taschenbuch für Laboratorium und Betrieb (SETH BERGLUND) 120
- KOCH, J. (Editor), R. H. V. M. DAWTON, M. L. SMITH and W. WALCHER, Electromagnetic Isotope Separators and Applications of Electromagnetically Enriched Isotopes (G. ANDERSSON) 120
- FLÜGGE, S., (Editor), Encyclopedia of Physics, Vol. XL, Nuclear Reactions I (T. R. GERHOLM) 244

SUBJECT INDEX

Accelerators

Improved cyclotron performance from control of initial ion motion, A. H. MORTON and W. I. B. SMITH	36
Accurate magnetic field and field gradient measuring instrument for dynamic low fields in a synchrotron magnet, HANS M. NYSÄTER.	44
Magnetic measurements on the Cornell alternate gradient synchrotron, E. MALAMUD and A. SILVERMAN	67
Gas scattering in a strong focusing synchrotron, MORTON D. KOSTIN	99
Le mouvement anharmonique des ions dans un cyclotron et le diagramme de phase, R. KELLER	181
Überlegungen zum Bau eines Wendellinearbeschleunigers für Protonen, W. MÜLLER und J. REMBSE.	202

Counters, Chambers and Detectors

9-inch liquid hydrogen bubble chamber in a pulsed magnetic field, D. C. COLLEY, J. B. KINSON and L. RIDDIFORD	26
An energy sensitive Čerenkov detector with improved optical focussing, M. HUQ and G. W. HUTCHINSON	30
An absorber system for scattering experiments, W. J. O'NEILL, E. SUNDAHL and H. OSTRANDER	50
The selection of stainless steels for use in liquid hydrogen bubble chambers (Letter to the Editor), PIERRE AMIOT	118
A scintillation counter with neutron and gamma-ray discriminators, F. D. BROOKS	151
A method for the preparation of very thin foils, E. KASHY, R. R. PERRY and J. R. RISSE.	167
The response to high-energy gamma rays of a NaI(Tl) scintillation spectrometer, J. KOCKUM and N. STARFELT	171
Theory of time resolution in scintillation counters, E. GATTI and V. SVELTO	189
A new method for track angle measurement in a gridded ionization chamber, U. FACCHINI, E. GATTI and F. PELLEGRINI	221

Electronic Equipment

Statistical design basis for fast scaling systems, A. W. PRYOR and A. G. KLEIN	1
Impuls-Zeitformer für schnelle Koinzidenzstufen, WOLFGANG GRUHLE	112
A fast scaling stage, F. A. MULLER.	115
High speed transistorized scale-of-two (Letter to the Editor), E. BALDINGER, P. SANTSCHI and P. WEHRLE.	117
A pulse-height storage circuit of high stability, K. W. OGLVIE and D. PAIX.	164
Fast scaling circuits, G. W. HUTCHINSON, W. H. WELLS and D. H. WHITE	228
Regularizing action of scalars (Letter to the Editor), J. SANDSTAD	243

Erratum

H. BIENLEIN <i>et al.</i> , Nuclear Instruments 4 (1959) 79	244
---	-----

Fusion

An experimental study of the linear pinch stability, TAGE SUNDSTRÖM and SVEN SVENNERSTEDT	213
Notes on some theoretical investigations of the physics of high temperature plasmas, R. LÜST	247
A compilation of some rates and cross sections of interest in controlled thermonuclear research, C. F. WANDEL, T. HESSELBERG JENSEN and O. KOFOED-HANSEN	249
Report on linear pinch devices, R. LATHAM and J. A. NATION	261
Brief review of the toroidal stabilised pinch, R. J. BICKERTON	273
Remarks on the stellerator-scheme, L. BIERMANN	279
Les machines à miroirs magnétiques. Résumé de l'état de la question et esquisse d'un programme de recherches, M. BINEAU, T. CONSOLI, P. HUBERT, F. PRÉVOT, P. RICA TEAU et A. SAMAIN	282
Les machines à miroirs magnétiques, M. BINEAU, T. CONSOLI, CH. MAISONNIER, F. PRÉVOT et P. RICA TEAU	290
Notes on fast magnetic compression of plasmas, H. L. JORDAN, H. KEVER and K. SCHINDLER	322
Confinement of plasma by radio-frequency electro-magnetic fields, C. M. BRAAMS, W. J. SCHRADER and C. J. TERLOUW	327
Relativistic electron beam devices for fusion, J. G. LINHART and A. SCHOCH.	332
Report on high-speed photography, A. FOLKIERSKI	346
Plasma diagnostics by spectroscopical means, H. WULFF	352

Fusion (*continued*)

- Detection of heavy particles in thermonuclear reaction experiments, L. H. TH. RIETJENS and C. M. BRAAMS . . . 363
 Ultra high vacuum technology, G. L. MUNDAY . . . 367
 Les sources d'ions et de plasma, T. CONSOLI, P. HUBERT, R. LE QUINIO et D. VÉRON . . . 376
 Some remarks on energy storage, D. TH. J. TER HORST . . . 382
 High magnetic fields, SETH BERGLUND . . . 386

Instruments for Nuclear Spectroscopy

- Eine Apparatur zur Messung der longitudinalen Elektronenpolarisation durch Mott-Streuung, (see also Erratum p. 244), H. BIENLEIN, K. GÜTHNER, H. VON ISSENDORFF und H. WEGENER . . . 79
 Hochstabilisierende Protonenresonanz-Magnetfeldregelung mit elektronischem Integrator, W. MÜLLER-WARMUTH WARMUTH und P. SERVOZ-GAVIN . . . 90
 Instrument for electron-gamma directional correlation experiments with magnetic momentum selection, T. R. GERHOLM and B.-G. PETTERSSON . . . 107
 Bemerkungen zu einem schnellen 20-Kanal-Impulshöhenanalysator, M. BONITZ und E. J. BERLOWITSCH . . . 133
 Measurement of the polarization of D-D neutrons by a solenoid, P. S. DUBBELDAM, C. C. JONKER and F. J. HEEMSKERK . . . 234

Instruments for the Study of Nuclear Research Emulsions

- A microscope for the scanning of nuclear research emulsions, A. G. EKSPONG and B. E. RONNE . . . 129

Ion Sources

- A negative hydrogen ion injector for a tandem electrostatic accelerator, L. E. COLLINS and A. C. RIVIERE . . . 121

News Items

- Symposium on "Current Developments in the Production of High Vacua" (London, 1959) . . . 51
 Conference on "Some Aspects of Magnetism" (Sheffield, 1959) . . . 52

Nuclear Spectroscopy Measurements

- Experimental methods for the study of neutron capture gamma rays (survey article), GUNNAR BÄCKSTRÖM . . . 5
 Measurement of surface density of thin foils, K. RAMAVATARAM and D. I. PORAT . . . 239

Nuclear Spectroscopy Methods

- Calculation of the G factor for gas scattering experiments, EDWARD A. SILVERSTEIN . . . 53
 Calculation of the penetration of gamma rays through the edges of a $\frac{1}{2}$ -inch diameter lead collimator, C. SHARP COOK . . . 103
 Neutron transmission probability through a curved revolving slit, M. MARSEGUERRA and G. PAULI . . . 140

